Discussion of

Granular Identification

by Xavier Gabaix and Ralph Koijen

Oleg Itskhoki
Princeton University

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- Related ideas in the literature:

1. Levchenko, di Giovanni and Méjean:

$$u_{it} = \delta_{it} - \bar{\delta}_{Ut} \qquad \Rightarrow \qquad \text{Granular contribution} = \frac{\mathrm{var}(\delta_{it} - \bar{\delta}_{Ut})}{\mathrm{var}(\delta_{it})}$$

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     Given the same change in average $\bar{p}_t = \int_\Omega p_{\omega,t} \mathrm{d}\omega$, use the change in dispersion in $p_{\omega,t}$ as an instrument for demand

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  - AIK (2018): $\Delta p_{it} = \gamma \Delta \bar{p}_{-it} + \alpha \Delta mc_{it} + \varepsilon_{it}$

Identifying Assumptions I

- For concreteness consider a simple example:
  - Firm-level supply: $\delta_{it} = \eta_t + u_{it}$,
    η_t is aggregate cost shifter; u_{it} are idiosyncratic cost shocks
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- Regress p_t on $\bar{\delta}_{Wt}$ using $z_t = \delta_{\Gamma t}$ as an instrument results in a consistent estimator of α under the GIV validity condition:

$$\mathbb{E}\{\varepsilon_t z_t\} = 0 \quad \Leftrightarrow \quad \sum_{i=1}^N \mathbb{E}\{\varepsilon_t u_{it} S_{it}\} = 0,$$

  which is, in general, stronger than $\mathbb{E}\{\varepsilon_t u_{it}\} = 0$

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- When is $\mathbb{E}\{\varepsilon_t u_{it}\} = 0$ sufficient?
  - E.g., when $\{S_{it}\}$ can be taken as exogenous constants from the point of view of the model
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- When one cannot expect $\mathbb{E}\{\varepsilon_t u_{it}\} = 0$ to be sufficient?
  - When idiosyncratic shocks are correlated with firm size:

$$\sum_{i=1}^{N} \mathbb{E}\{\varepsilon_t u_{it} S_{it}\} = \sum_{i=1}^{N} \mathbb{E}\{\varepsilon_t u_{it} | S_{it}\} \mathbb{E} S_{it} \neq 0$$

if $\quad \mathbb{E}\{\varepsilon_t u_{it} | i \in \text{Small}\} \neq \mathbb{E}\{\varepsilon_t u_{it} | i \in \text{Large}\} \neq 0$,

e.g. in a recession small firms shrink more than large firms

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$$\hat{u}_{it} = \delta_{it} - (1 + \hat{\beta}_{it})\hat{\eta}_t \quad \longrightarrow \quad \hat{z}_t \equiv \hat{u}_{\Gamma t}$$

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   - "bad" granularity in $\beta_{it}\eta_t$ vs "good" granularity in u_{it}?

Specification tests

1. "Over-id" tests of misspecification:
   - $u_\Gamma = \bar{\delta}_W - \bar{\delta}_U$ is the efficient instrument, in the absence of misspecification and measurement error
   - however, there are many valid instruments: in particular, any $z_{ab} \equiv (a-b)'\boldsymbol{\delta}$, where $a \neq b$ are two vectors with $(a-b)'\boldsymbol{\iota}=0$
   - e.g., one can test if using only large firms in the GIV yields robust results (to avoid noise from small firms)
   - "placebo": use $\bar{\delta}_W$ ($\bar{\delta}_U$) as invalid instrument(s) instead of $z=\bar{\delta}_W-\bar{\delta}_U$. The concern is that $z \approx \bar{\delta}_W$ if $\bar{\delta}_U$ is mostly noise
2. Micro-identification and functional forms:
   - Any micro environment (not necessarily granular) provides data $x_{it} = f(\eta_t, u_{it})$ full of micro-level orthogonal "noise" u_{it}
   - One only needs to purge x_{it} of aggregate η_t and use $\{u_{it}\}$ to construct Micro-IV: $z_t = G\big(\{\hat{u}_{it}\}\big)$, where $\hat{u}_{it} = H\big(\{x_{it}\}\big)$
   - However, $f(\cdot)$ is rarely known. How good is $f_{it} \approx \eta_t + u_{it}$ in general? E.g., what if $x_{it} = \eta_t \cdot u_{it}$ or $x_{it} = \eta_t + g(\eta_t, S_{it}) \cdot u_{it}$?

Summary

- Beautiful idea
- Will be part of the identification toolkit in economics
- Main open question to me:
  - is it a niche identification technique or universal tool for macro
- Asymptotic inference theory is a necessary first step.
  But small sample properties are essential, in *realistic* granular environments