Macroeconomics of Exchange Rates

Lecture 2

Exchange Rates in General Equilibrium

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Roadmap

1. Real Exchange Rate and Expenditure Switching
   - Import demand and net exports
   - Market clearing and aggregate consumption
2. Real Exchange Rate and International Risk Sharing
   - Complete markets and financial autarky
   - General incomplete markets
3. Exchange Rates in General Equilibrium
   - Intertemporal budget constraint
   - Monetary policy

EXPENDITURE SWITCHING

Exports and Imports

- Net exports:

$$NX_t = \mathcal{E}_t P^*_{Ht} C^*_{Ht} - P_{Ft} C_{Ft}$$

  - C_{Ft} and C^*_{Ht} are aggregate import quantities at home/abroad
  - P_{Ft} and P^*_{Ht} are local-currency import price indexes

- Log-linearizing (around $\bar{NX} = 0$; denoting $nx_t \equiv NX_t/\bar{Y}$):

$$nx_t = \gamma[c^*_{Ht} - c_{Ft} - s_t]$$

  - $s_t \equiv p_{Ft} - p^*_{Ht} - e_t$ is ToT

- Import demand in log deviations:

$$c_{Ft} = -\theta(\underbrace{p_{Ft} - p_t}_{(1-\gamma)(p_{Ft}-p_{Ht})=(1-\gamma)(s_t+q_{Ht})}) + c_t + \epsilon_t$$

  - θ is the elasticity of the import demand schedule
  - aggregate consumption c_t is a shifter; ϵ_t is preference shock

Net Exports

- A rather general expression for net exports (holds independently of price setting and price stickiness):

$$nx_t = \gamma[\theta q_t + (\theta - 1)s_t - (c_t - c_t^*) + \tilde{\epsilon}_t]$$

  - both RER q_t and ToT s_t matter for NX
    Marshall-Lerner condition: $\theta + (\theta - 1) = 2\theta - 1 > 0$
  - consumption boom $\Rightarrow$ increase in imports, deterioration of NX

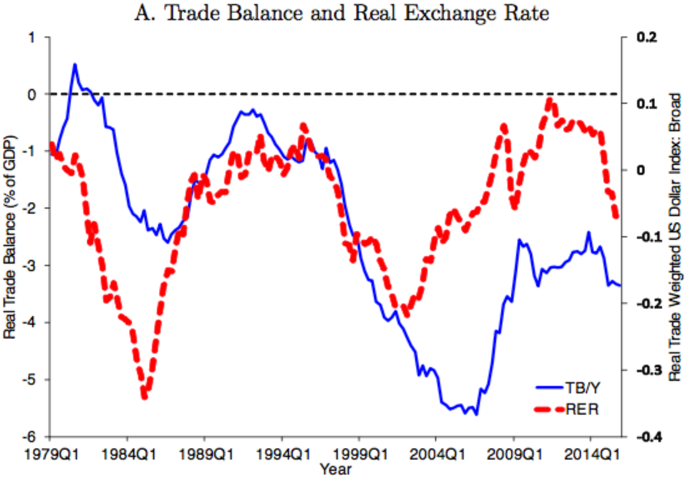
- Expenditure switching mechanism: ER $\Rightarrow$ NX

$$nx_t = \gamma\big[\vartheta q_t - (c_t - c_t^*)\big], \quad \vartheta \equiv 2\theta(1-\alpha)\frac{1-\gamma}{1-2\gamma} - \frac{1-2\alpha(1-\gamma)}{1-2\gamma}$$

  - expressing s_t via q_t using Prop. 2. Now ML cond: $\vartheta > 0$
  - in the data, nx_t weakly correlated with q_t and significantly less volatile; less so over LR (over 5 years); J-curve

Illustration

(Alessandria and Choi, 2019)



Market Clearing and Agg. Consumption

The expenditure switching mechanism

- Domestically-produced output allocation: $Y_t = C_{Ht} + C_{Ht}^*$
- Log-linearized: $y_t = (1-\gamma)c_{Ht} + \gamma c_{Ht}^*$
- Equilibrium relationship b/w relative consumption and RER:

$$\boxed{c_t - c_t^* = \frac{1}{1-2\gamma}\big[(y_t - y_t^*) - 2\gamma\varkappa q_t\big],} \qquad \varkappa \equiv \frac{2\theta(1-\alpha)(1-\gamma)}{1-2\gamma}$$

  - using demand schedules for c_{Ht}, c_{Ht}^* and Prop. 2; under sticky prices, this equation is the "error correction term"
  - y_t, y_t^* — exog. endowment or endog. agg. supply of goods (incl. due to reduction in markups from monetary expansions)
  - in autarky, $\gamma = 0$ and $c_t = y_t$; in general, y_t affects (c_t, c_t^*, q_t)
  - with y_t constant, q_t and c_t are linked by expenditure switching
  - important implications for Backus-Smith!

The Logic of Market Clearing

1. Supply-side expansion: increase in y_t
   - $c_t - c_t^*$ increases (if $\gamma < 1/2$) less than proportionally (if $\gamma > 0$)
   - q_t tends to increase (RER depreciation) reflecting relative abundance of home goods $\Rightarrow$ Backus-Smith puzzle

2. Expenditure switching: increase in q_t holding y_t constant
   - depreciation $\Rightarrow$ relative demands for home good $\uparrow$ everywhere
   - increase in $c_t - c_t^*$ cannot be consistent with market clearing

   $\Rightarrow$ must be associated with a decline in $c_t - c_t^*$ (weak if γ is small)

3. Keynes' Transfer problem: decline in c_t holding y_t constant
   - $c_t \downarrow$ due to wealth decline or "patience" preference shock
   - surplus y_t now needs to be allocated for consumption globally

   $\Rightarrow$ requires depreciation ($q_t \uparrow$), and a large one if γ small

INTERNATIONAL RISK SHARING

International Risk Sharing

- Fundamental asset pricing equation: $\mathbb{E}_t\{\mathcal{M}^h_{t+1}\mathcal{R}^j_{t+1}\} = 1$
  - $\mathcal{R}^j_{t+1} \equiv (\mathcal{P}^j_{t+1} + \mathcal{D}^j_{t+1})/\mathcal{P}^j_t$ is the rate of return on asset j
  - $\mathcal{M}^h_{t+1}$ is nominal SDF of agent h who can trade asset j
  - with CRRA utility, representative SDF: $\mathcal{M}_{t=1} = \beta\big(\frac{C_{t+1}}{C_t}\big)^{-\sigma}\frac{P_t}{P_{t+1}}$

- Denote with J_t and J^*_t the sets of assets j available at time t to home and foreign households, respectively
  - converting from foreign to home currency: $\mathcal{R}^j_{t+1} = \mathcal{R}^{j*}_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$

- For all assets $j \in J_t \cap J^*_t$ (i.e., traded by both home & foreign):

$$\mathbb{E}_t\left\{\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} - \left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\right]\cdot\frac{\mathcal{R}^{j*}_{t+1}}{P^*_{t+1}/P^*_t}\right\} = 0$$

  - traded assets bring home & foreign real SDFs closer together

- If $J_t \cap J_t^*$ spans every state of the world, we have:

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} = \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \quad \Rightarrow \quad \left(\frac{C_t}{C_t^*}\right)^{\sigma} = \chi_0 \cdot \mathcal{Q}_t$$

  - efficient risk-sharing logic: $\frac{u'(C_t)}{P_t} \propto \frac{u'(C_t^*)}{\mathcal{E}_t P_t^*}$ vs $C_t/C_t^* = const$
  - another implication: $\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \frac{\mathcal{M}_{t+1}}{\mathcal{M}_{t+1}^*}$ (marginal value of currency)

- Log-linearized Backus-Smith condition:

$$\sigma(c_t - c_t^*) = q_t$$

  - risk sharing (transfers) vs expenditure switching in goods mkt
  - only "utility shocks" can break this tight link (BS puzzle)

- Solution for RER using market clearing:

$$q_t = \xi \cdot (y_t - y_t^*), \qquad \xi \equiv \frac{\sigma}{(1-2\gamma) + 2\gamma\sigma\varkappa}$$

Financial Autarky

- Assume $J_t \cap J_t^* = \emptyset$, while goods trade is possible $\Rightarrow NX_t \equiv 0$

- Then $nx_t = 0$ implies:

$$c_t - c_t^* = \vartheta q_t, \qquad \vartheta = 1 + 2(\theta - 1)(1 - \alpha)\frac{1-\gamma}{1-2\gamma} > 0$$

- Cole-Obstfeld case: $\sigma\vartheta = 1$ (in particular, $\sigma = \theta = 1$)
    - in this case, $nx_t = 0$ is collinear with Backus-Smith
    - complete-market allocation achieved independently of $J_t \cap J_t^*$
    - perfect insurance guaranteed by ToT adjustment

- General solution, using market clearing:

$$q_t = \zeta \cdot (y_t - y_t^*), \qquad \zeta \equiv \frac{1}{(1-2\gamma)\vartheta + 2\gamma\varkappa}$$

    - qualitatively the same as complete markets (BS puzzle)
    - only "preference shocks" ϵ_t can break this tight link

General Incomplete Markets

- $J_t \cap J_t^*$ is non-empty, yet does not provide full spanning

- General log-linearized risk sharing condition:

$$\boxed{\mathbb{E}_t\{\sigma(\Delta c_{t+1} - \Delta c_{t+1}^*) - \Delta q_{t+1}\} = \hat{\psi}_t}$$

  - as if "single-bond economy"; relationship in expectations only
  - $\hat{\psi}_t$ is either risk-premium or risk-sharing friction ("UIP shock")
  - without $\hat{\psi}_t$ shocks, $x_t \equiv \sigma(c_t - c_t^*) - q_t$ is a martingale
  - BS $\mathrm{corr}(\Delta c_t - \Delta c_t^*, \Delta q_t) < 0$ is possible w or w/out $\hat{\psi}_t$

- Using market clearing:

$$\mathbb{E}_t \Delta q_{t+1} = \xi \cdot \mathbb{E}_t\{\Delta y_{t+1} - \Delta y_{t+1}^*\} - \frac{\hat{\psi}_t}{1 + 2\gamma\sigma\varkappa/(1-2\gamma)}$$

  - is $\hat{\psi}_t = 0$ and $y_t - y_t^*$ is RW, then q_t is also random walk
  - q_t and $\lim_j \mathbb{E}_t q_{t+j}$ are indeterminate from risk-sharing alone; without $\hat{\psi}_t$, q_t is generally non-stationary

EXCHANGE RATES IN GENERAL EQUILIBRIUM

Country Budget Constraint

- Country budget constraint can be written as:

$$\mathcal{B}_t = \mathcal{R}_t \mathcal{B}_{t-1} + NX_t$$

  - $\mathcal{B}_t$ is NFA and $\mathcal{R}_t$ is gross return on NFA, given $\{B_t^j, \mathcal{R}_{t+1}^j\}_{J_t}$
  - $\mathcal{B}_t = \sum_{j\in J_t} \mathcal{P}_t^j B_t^j$ and $\mathcal{R}_{t+1}\mathcal{B}_t = \sum_{j\in J_t} (\mathcal{P}_{t+1}^j + \mathcal{D}_{t+1}^j) B_t^j$

- Intertemporal budget constraint (IBC) is then:

$$\mathcal{R}_t \mathcal{B}_{t-1} + \sum_{k=0}^{\infty} \frac{NX_{t+k}}{\mathcal{R}_{t,t+k}} = 0, \qquad \mathcal{R}_{t,t+k} \equiv \prod_{\ell=1}^{k} \mathcal{R}_{t+\ell}$$

  - $NX_t < 0$ needs to be compensated either by $NX_{t+k} > 0$ or by favorable returns on NFA (valuation effects)
  - what is more important: risk sharing or budget constraint?

- Log-linearized flow budget constraint:

$$\boxed{\beta b_{t+1} - b_t = nx_t = \gamma[\vartheta q_t - (c_t - c_t^*)]}$$

  - $b_t \equiv \bar{\mathcal{R}} \mathcal{B}_{t-1}/\bar{Y}$ is NFA/GDP-ratio, $\beta = 1/\bar{\mathcal{R}} < 1$
  - together with risk sharing $\Rightarrow$ dynamic system in (q_t, b_{t+1}) with initial b_0 and terminal (no-bubble) $\lim_{j\to\infty} \beta^j b_{t+j} = 0$.

Real Exchange Rate

- Assume $\tilde{y}_t \equiv y_t - y_t^*$ and $\hat{\psi}_t$ follow AR(1)s with $\rho \le 1$:

$$q_t = -\frac{1}{\vartheta + \frac{2\gamma\varkappa}{1-2\gamma}} \frac{1-\beta}{\gamma} b_t + \frac{1}{1+\frac{2\gamma\sigma\varkappa}{1-2\gamma}} \frac{\beta}{1-\beta\rho} \hat{\psi}_t + \left[\frac{\beta(1-\rho)}{1-\beta\rho}\xi + \frac{1-\beta}{1-\beta\rho}\zeta \right] \tilde{y}_t$$

- Without "financial" shocks $\hat{\psi}_t$, we can further show that:

$$\Delta q_t = \zeta \Delta \tilde{y}_t + \frac{\beta(1-\rho)}{1-\beta\rho}(\xi - \zeta)\Big(\tilde{y}_t - \frac{1}{\beta}\tilde{y}_{t-1}\Big)$$

  - special cases: $\beta \to 1$ and $\rho \to 1$ and Cole-Obstfeld ($\xi = \zeta$)
  - low volatility and strong positive correlation with c_t and y_t

- With $\hat{\psi}_t$, $\Delta q_t \sim ARMA(1,1)$ with AR root ρ and MA root $\frac{1}{\beta}$
  - indistinguishable from RW when β and ρ are close to 1; also extra RER volatility and weak negative BS correlation
  - small persistent $\hat{\psi}_t$ $\Rightarrow$ persistent departures from $\mathbb{E}_t \Delta q_{t+j} = 0$ met with large surprise jumps in q_t to satisfy IBC
  - home bias and incomplete ERPT mute transmission into aggregate prices and quantities via expenditure switching

Is RER Stationary?

- RER is, in general, non-stationary (integrated) even when underlying shocks are transitory
  - assumption of LR mean reversion in q_t is not generally justified
- IBC provides theoretical discipline on the future path of q_t:
  - no-bubble condition, $\lim_{j\to\infty} \beta^j b_{t+j} = 0$, instead of an *ad hoc* assumption that $\lim_{j\to\infty} q_{t+j} = \bar{q}$
- RER is not a RW with notable departures from $\mathbb{E}_t \Delta q_{t+1} = 0$
  - e.g., due to risk premia and/or financial frictions
  - yet with virtually unbounded half lives
- Integrated RER has tendency for imperfect mean reversion
  - $\mathrm{corr}(\Delta q_t, \Delta q_{t-1}) < 0$, yet AR(1) is inadequate
  - equilibrium RER persistence is a result of GE forces, and *not* of the type of the PPP deviation (Lemma 1)
  - Proposition 1 then links q_t^W and q_t

Monetary Policy and Exchange Rates

Bringing back together e_t and q_t

- In PE, it is conventional to take e_t shocks as exogenous and study their transmission into prices and, thus, q_t
- In GE, however, a reverse approach is more fruitful:
  - combine the equilibrium behavior of q_t with monetary policy shaping p_t and p_t^* to obtain equilibrium $e_t = q_t + p_t - p_t^*$
  - in particular, if monetary policy stabilizes π_t and π_t^* independently of Δe_t volatility; then Δe_t tracks Δq_t at all horizons
  - departures from martingale q_t can predict $e_{t+h} - e_t$
- How is inflation-stabilizing monetary policy consistent with volatile and persistent e_t?
  - violates the monetarist view: Δe_t does not track $\pi_t - \pi_t^*$
  - no inconsistency in real models with inflation stabilization
  - same is true in in a monetary (sticky price) model provided:
    - (a) monetary shocks are not the key drivers of (e_t, q_t)
    - (b) home bias and incomplete ERPT limit effect of e_t on π_t
- ER regimes and the Mussa puzzle: mother of all puzzles!

Conclusion

- Some form of PPP violation is necessary for a theory of RER
  - home bias in tradables and due to non-tradables
  - amplified by PTM, distribution margin, imported intermediate inputs, foreign-currency export price stickiness

- PPP deviations are insufficient to explain properties of RER
  1. large volatility and persistence
  2. close comovement with NER and lack of correlation with macro fundamentals, nominal or real

- RER dynamics is shaped by GE forces:
  1. goods market clearing (expenditure switching mechanism)
  2. international risk sharing (expected changes in RER)
  3. country budget constraint (unexpected level jumps in RER)

Conclusion
continued

- Two types of RER driving forces (shocks):
  1. shocks to relative supply of goods (productivity or monetary)
     — result in counterfactual correlations and insufficient volatilty
  2. financial (international risk sharing) shocks
     — consistent with RER equilibrium (unconditional) properties
     — yet, require further research into their specific nature and endogeneity to policy
- Inflation-stabilizing monetary policy ties together nominal and real exchange rates at all horizons
- RER should be taken as a generally non-stationary variable:
  - not a pure random walk (partial mean reversion), but possibly contains a unit root
  - its long-run behavior is shaped by intertemporal budget constraint rather than by convergence to PPP
- Outstanding empirical question:
  — ER behavior conditional on various well-identified shocks