Optimal Exchange Rate Policy*

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Abstract

We develop a general policy analysis framework that features nominal rigidities and financial frictions with endogenous PPP and UIP deviations. The goal of the optimal policy is to balance output gap stabilization and international risk sharing using a mix of monetary policy and FX interventions. The nominal exchange rate plays a dual role. First, it allows for the real exchange rate adjustments when prices are sticky, which are necessary to close the output gap. Monetary policy can eliminate the output gap, but this generally requires a volatile nominal exchange rate. Volatility in the nominal exchange rate, in turn, limits the extent of international risk sharing in the financial markets with limits to arbitrage. Optimal monetary policy closes the output gap, while optimal FX interventions eliminate UIP deviations. When the natural real exchange rate is stable, both goals can be achieved by a fixed exchange rate policy — an open-economy divine coincidence. Generally, this is not the case, and the optimal policy requires a managed peg by means of a combination of monetary policy and FX interventions, without requiring the use of capital controls.

We explore various constrained optimal policies, when either monetary policy or FX interventions are restricted, and characterize the possibility of central bank’s income gains and losses from FX interventions.

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1 Introduction

What is the optimal exchange rate policy? Should exchange rates be optimally pegged, managed or allowed to freely float? What defines a freely floating exchange rate? Do open economies face a trilemma constraint in choosing between inflation and exchange rate stabilization? These classic questions in international macroeconomics are generally difficult to address, as the exchange rate is neither a policy instrument, nor a direct objective of the policy, but rather an endogenous general equilibrium variable with direct equilibrium links in both product and financial markets. At the same time, equilibrium exchange rate behavior features a variety of puzzles from the point of view of conventional business cycle models, which thus casts doubt on their exchange rate policy implications.

We address these questions by developing a general policy analysis framework with nominal rigidities and financial frictions that are both central for equilibrium exchange rate determination and result in an empirically realistic model of exchange rates. We extend the framework in Itskhoki and Mukhin (2021b), where we study positive implications of a switch between floating and fixed exchange rate regimes, to allow for explicit policy analysis using both conventional monetary policy and foreign exchange interventions (FXI) in the financial market. We show that this framework is easily amenable to normative analysis and characterize the optimal exchange rate policies implied by the model.

We focus on a problem of a small open economy with tradable and non-tradable goods with a segmented international financial market resulting in endogenous uncovered interest rate parity (UIP) deviations. Productivity shocks determine the value of the frictionless real exchange rate, or departures from purchasing power parity (PPP). Nominal rigidities constitute another — frictional — source of PPP violations. The presence of both endogenous PPP and UIP violations is essential for the optimal exchange rate policy analysis, as exchange rates are key determinants of both deviations. Historically, the main frameworks for the exchange rate policy analysis features PPP deviations due to sticky prices, but assumed that UIP holds, resulting in a Trilemma constraint on feasible exchange rate policies (Fleming 1962, Mundell 1963, Dornbusch 1976, Obstfeld and Rogoff 1995, Galí and Monacelli 2005).\footnote{Trilemma states that in the absence of capital controls, the policy must choose between an independent monetary policy or a nominal exchange rate targeting. In other words, without capital controls, inward looking monetary policy uniquely determines the exchange rate outcomes, and vice versa exchange rate targeting fully ties hands of monetary policy from the perspective of inflation and output and output gap. UIP deviations can relax the Trilemma constraint on the nominal exchange rate imposed by inward-looking monetary policy.}

The nominal exchange rate plays a dual role — in the goods and asset markets. First, it allows for expenditure switching and the real exchange rate adjustment when prices (or wages) are sticky, and in the absence of such nominal exchange rate movements, the economy features an output gap resulting in welfare losses. Monetary policy can eliminate the output gap, but this generally requires a volatile nominal exchange rate that accommodates fundamental macroeconomic shocks. Volatility in the nominal exchange rate, in turn, limits the extent of international risk sharing in the financial market, as international financial flows must be intermediated by risk-averse market makers who hold the nominal exchange rate risk. This also leads to welfare losses. Financial market interventions can shift this risk away from arbitrageurs, stabilizing the resulting equilibrium UIP deviations and improving the extent of international risk sharing. Thus, the goal of the optimal policy is to balance output gap
We begin our analysis by characterizing the optimal allocation, which ensures efficient level of production and optimal risk sharing in the tradable sector. We then show how an unconstrained joint use of monetary policy and FX interventions allows to implement the optimal allocation, with monetary policy eliminating the output gap and FX interventions eliminating the intermediation wedge and the resulting UIP deviation in the international financial market. Exchange rate stabilization is not a direct goal of a welfare maximizing policy. The resulting equilibrium generally features volatile nominal exchange rate and inflation targeting, with financial interventions targeting UIP deviations as their policy goal. Such policy mix allows the exchange rate to accommodate fundamental macroeconomic shocks and optimal expenditure switching, while it neutralizes the effects of non-fundamental currency demand shocks in the financial market on exchange rate volatility. This is the sense in which economies with a segmented financial market do not feature a trilemma constraint, as market segmentation offers the financial regulator an additional instrument to stabilize market volatility, even when monetary policy focuses exclusively on domestic inflation and output gap stabilization.

Figure 1 provides an illustration to the policy tradeoff and the optimal policy choice, comparing...
our framework with endogenous UIP deviations to two alternative classes of models, namely classic Trilemma models without UIP deviations and alternative models with exogenous financial shocks resulting in UIP (or CIP) deviations. Specifically, the figure plots the policy tradeoffs in the space of output gap and nominal exchange rate volatility. The first best corresponds to a fully eliminated output gap and a nominal exchange rate that, under sticky prices, must accommodate the volatility of the first-best real exchange rate that ensures efficient expenditure switching. In trilemma models without UIP deviations, a freely floating exchange rate under inward-looking inflation-stabilizing monetary policy and no FXI achieves just that, as suggested by Friedman (1953). More generally, when UIP deviations are featured in equilibrium, a laissez-faire float results in excessive exchange rate volatility, which reflects both macro-fundamental and non-fundamental financial volatility, consistent with exchange rate disconnect. In such models, both free floats and full pegs are, generally, suboptimal, and the optimal policy requires an additional use of FXI to offset financial volatility.

Implementing the optimal allocation in the goods and asset market, in general, requires an unconstrained use of both monetary and FX instruments. There exists, however, an important special case when addressing both frictions could be done with a nominal exchange rate peg by means of monetary policy alone. We refer to this case as “divine coincidence” in an open economy, by analogy with a closed-economy divine coincidence. Indeed, if the natural real exchange rate that ensures efficient risk sharing is stable, then there is no tradeoff from the points of view of the goods and asset markets. Specifically, a fixed nominal exchange rate is consistent with efficient expenditure switching under sticky prices in the goods market, as well as eliminates risk in the international financial market allowing for frictionless intermediation. Direct nominal exchange rate targeting is favored over inflation stabilization in this case as it guarantees a unique optimal equilibrium. While our analysis is consistent with the optimal currency areas logic, it identifies not only circumstances when the costs of a fixed exchange rate are low in the goods market, but also the risk-sharing benefits associated with a fixed exchange rate. In Figure 1, the case of the Divine coincidence corresponds to the situation when $\sigma_q = 0$, and the entire blue area collapses to the origin, making the Peg and the first best (FB) coincide.

Next, we explore circumstances where either monetary policy is constrained (e.g., due to the zero lower bound) or the financial interventions are constrained (e.g., due to non-negative requirement on central bank foreign reserves or value-at-risk constraints for the central bank portfolio). In this case, there are two independent policy goals — the output gap and the risk sharing wedge — and only one unconstrained policy instrument, thus making it generally impossible to replicate the optimal allocation. Fixing the exchange rate using the monetary policy tool is generally feasible, but is also generally

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2The latter class includes models with exogenous UIP shocks (e.g., Devereux and Engel 2002, Kollmann 2005, Farhi and Werning 2012), convenience yield (e.g., Jiang, Krishnamurthy, and Lustig 2021), and financial frictions in the form of balance sheet constraints (e.g., Gabaix and Maggiori 2015, Basu, Boz, Gopinath, Roch, and Unsal 2020). In Itskhoki and Mukhin (2021a), we show that all such models can be equally successful in explaining the general exchange rate disconnect, yet unlike the model with endogenous UIP deviations due to limits to arbitrage these models cannot readily explain the Mussa facts (see Itskhoki and Mukhin 2021b), essential for the optimal exchange rate policy analysis. Mussa Puzzle point in Figure 1 illustrates the challenge for models with exogenous financial shocks, where a monetary peg counterfactually absorbs all the floating exchange rate volatility into inflation and output gap.

3This is the case both in trilemma models and in a model with endogenous UIP deviations, but not in models with exogenous financial shocks that do not disappear under a fixed nominal exchange rate.
suboptimal outside the case of divine coincidence. Similarly, targeting the output gap alone is also suboptimal, and monetary policy trades-off output gap and exchange rate stabilization (managed and crawling pegs) in the absence of FX interventions. Managed peg and dirty floats with monetary policy may emerge as the second best policy, even when divine coincidence is not satisfied, yet there are tight constraint on the balance sheet of the central bank making effective FX interventions infeasible. Using financial interventions to stabilize output gap is generally infeasible.

Lastly, we explore the monopoly power of the government in the international financial market and the ability of the central bank to earn rents without compromising the expenditure switching and risk sharing goals of optimal exchange rate policy. When the financial sector is offshore, the policymaker can compete with financial intermediaries for rents (international transfers) that emerge from exogenous shifts in currency demand. In the presence of an additional capital control instrument, it is possible to extract maximum rents without compromising the other objectives of the policy. In particular, FX interventions are used only part way in this case, without offsetting excess currency demand and eliminating the entire intermediation rent, while capital controls are used in addition to eliminate the effect of rents on the optimal risk sharing. This, however, requires a flexible state-contingent use of capital controls, which may be infeasible. Without capital controls, the policymaker can still use FX interventions to implement the frictionless allocation at no expected financial costs. This, however, is generally suboptimal as it fails to take advantage of trading off frictionless risk sharing for financial market rents from undersupplying currency reserves.

We finish our analysis with a number of extensions. In particular, we extend our small open economy model to global equilibrium with a continuum of small open economies, one of which issues a dominant funding currency that is used for international borrowing and lending against other national currencies. Unconstrained use of non-cooperative monetary policy and FX interventions eliminates all international risk sharing wedges and ensures efficient level of output in every country. When policies are constrained, however, international spillovers can no longer be internalized by non-cooperative policies. We characterize such spillovers that emerge in both dominant and non-dominant countries, and show how a cooperative policy of international FX interventions can address these spillovers. We also discuss costs and benefits associated with a global currency union and a gold standard.

**Related literature** We build on a vast literature studying the role of exchange rates in both goods and financial markets, as well as the optimal macroeconomic and financial policies in an open economy. Our model is most closely related to two strands of literature. On the one hand, our emphasis on the role of demand for and supply of currency in financial markets and the modelling of financial intermediaries follows the tradition of Kouri (1983), Driskill and McCafferty (1987), Dornbusch (1988) (Chapter 7) and a more recent work of Jeanne and Rose (2002), Blanchard, Giavazzi, and Sa (2005), Camanho, Hau, and Rey (2022), Gourinchas, Ray, and Vayanos (2019), Greenwood, Hanson, Stein, and Sunderam (2020). In contrast to these papers, we embed the financial sector into a realistic general equilibrium model, which is a prerequisite for any policy analysis. On the other hand, we share the assumption of segmented asset markets with Alvarez, Atkeson, and Kehoe (2009), Gabaix and Maggiori (2015). However, guided by the evidence form Itskhoki and Mukhin (2021b), we assume that limits to arbitrage come from the
risk aversion of intermediaries rather than the borrowing constraints or convenience yields, which considerably changes the optimal policy and distinguishes our analysis from otherwise closely related work by (see also Basu, Boz, Gopinath, Roch, and Unsal 2020, henceforth IPF for Integrated Policy Framework) and the other normative papers listed below.


Kareken and Wallace (1981)
Ilzetzki, Reinhart, and Rogoff (2018) + “fear of floating”
2 Modeling Framework

This section introduces the baseline theoretical framework and derives the optimal policy problem. Building on Itskhoki and Mukhin (2021b), we choose the ingredients of the model with an eye to the main empirical properties of exchange rates and intentionally make several strong assumptions to keep the policy problem as simple as possible. We derive a novel linear-quadratic approximation to the planner’s problem, which allows us to characterize optimal policies in Section 3. Sections 4 and 5 generalize the setup in several dimensions and consider a number of extensions.

2.1 Setup

We consider a small open economy with tradable and non-tradable goods. There are two frictions — sticky prices and a segmented financial market — that distort the equilibrium allocation, justify government interventions, and give rise to a policy tradeoff. The policymaker can choose the path of nominal interest rates and carry out FX interventions in the currency market.

**Real sector** The households have log-linear preferences over consumption of tradables $C_{Tt}$, non-tradables $C_{Nt}$ and hours worked $L_t$:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{Tt} + (1 - \gamma) \left( \log C_{Nt} - L_t \right) \right],$$  \hspace{1cm} (1)

where $\gamma$ is the expenditure share on the tradable good capturing the openness of the economy. Households receive labor income $W_t L_t$, firm profits $\Pi_t$, and transfers $T_t$, and can borrow or lend using a one-period risk-free home-currency bond $B_t$:

$$P_t C_{Tt} + P_{Nt} C_{Nt} + \frac{B_t}{R_t} = B_{t-1} + W_t L_t + \Pi_t + T_t,$$

where $R_t$ is the gross nominal interest rate.

The endowment of tradable goods $Y_{Tt}$ is exogenous and stochastic generating demand for international risk sharing. The prices of tradables are flexible and satisfy the law of one price:\footnote{Exogenous terms of trade due to a homogenous tradable good eliminate the beggar-thy-neighbour policy motive that typically complicates the normative analysis (see Corsetti and Pesenti 2001) and make the international risk sharing dependent on the structure of the asset markets despite logarithmic preferences (cf. Cole and Obstfeld 1991).}

$$P_{Tt} = \mathcal{E}_t P_{Tt}^*,$$

where $P_{Tt}^*$ is the international price of the tradable good and $\mathcal{E}_t$ is the nominal exchange rate in units of home currency for one unit of foreign currency (i.e., an increase in $\mathcal{E}_t$ corresponds to a home depreciation). We assume a stable price level in the foreign country, $P_{Tt}^* = 1$, and therefore the home-currency tradable price tracks the nominal exchange rate, $P_{Tt} = \mathcal{E}_t$. 
Output of non-tradables is endogenous and depends on the labor input and productivity shock:

\[ Y_{Nt} = A_t L_t. \]

We assume that prices are permanently sticky at an exogenous level, \( P_{Nt} = 1 \), and output is demand determined, \( C_{Nt} = Y_{Nt} \).\(^5\) Total profits in the economy are given by \( \Pi_t = P_{Tt} Y_{Tt} + P_{Nt} Y_{Nt} - W_t L_t \).

The equilibrium in the goods sector is characterized by two optimality conditions. Given that households split their consumption between tradables and non-tradables according to \( \gamma P_{Nt} C_{Nt} = (1 - \gamma) P_{Tt} C_{Tt} \), and goods prices are \( P_{Tt} = \mathcal{E}_t P^*_{Tt} = \mathcal{E}_t \) and \( P_{Nt} = 1 \), the equilibrium expenditure switching condition is given by:

\[ \frac{\gamma}{1 - \gamma} \frac{C_{Nt}}{C_{Tt}} = \frac{\mathcal{E}_t P^*_{Tt}}{P_{Nt}} = \mathcal{E}_t. \quad (2) \]

The relative demand for goods depends on their relative price, \( \mathcal{E}_t P^*_{Tt} / P_{Nt} \), which under fully sticky prices of non-tradables is equal to the nominal exchange rate \( \mathcal{E}_t \). The optimal consumption-savings decision of households is described by a standard Euler equation:

\[ \beta R_t \mathcal{E}_t \frac{C_{Nt}}{C_{Nt+1}} = 1, \quad (3) \]

and depends on the nominal interest rate \( R_t \) set by the policymaker. Finally, the optimality condition for labor supply, \( C_{Nt} = W_t / P_{Nt} = W_t \), determines the equilibrium nominal wage.

**Financial sector** While the equilibrium in the goods market is conventional to the open-economy sticky-price models, our analysis deviates from this literature by introducing segmentation in global asset markets. In particular, we assume that home households have access exclusively to local-currency bonds, and hence all international capital flows have to be intermediated by specialized financial traders.\(^6\)

Household demand for the home-currency bond \( B_t \) reflects fundamental macroeconomic forces and shapes the equilibrium path of net exports and net foreign assets. Additionally, there are three types of agents that can trade home and foreign currency bonds in the international financial market — the government, noise traders and intermediaries (arbitrageurs) — all residing in the home economy. For these agents who have access to foreign-currency (dollar) saving and borrowing, the dollar bond is in a perfectly elastic international supply at an exogenous interest rate \( R^*_t \). Section 4 considers extensions that allow for foreign intermediaries and noise traders resulting in cross-border financial income transfers, as well as an endogenize \( R^*_t \) in a multi-country global economy.

Each period, arbitrageurs choose a zero capital portfolio \( (D_t, D^*_t) \) such that \( D_t / R_t = -\mathcal{E}_t D^*_t / R^*_t \), where \( 1 / R_t \) and \( 1 / R^*_t \) are prices of the two bonds. The dollar net income of arbitrageurs from such

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\(^5\)We focus on the fully sticky price case as a limiting benchmark which simplifies the analysis by avoiding an additional dynamic equation, yet maintains all the qualitative tradeoffs of a more general environment (see Section 5.1). By having price stickiness only in the non-tradable sector we avoid the need to choose between PCP, LCP and DCP frameworks (see Section 5.2); equivalently, we could focus on sticky wages.

\(^6\)Similarly to the assumption of fully rigid prices, complete segmentation of asset markets substantially simplifies our analysis by limiting the number of dynamic equations. Yet, our main insights remain valid under a more realistic form of segmentation where households can trade foreign currency bonds subject to additional transaction costs and gradual portfolio adjustment (see Aiyagari and Gertler 1999, Bacchetta, Tieche, and Van Wincoop 2020).
a carry trade is given by \(\pi_{t+1} = D_t - D_t/E_t = \hat{R}_{t+1}^{*} - \frac{D_{t+1}}{R_{t+1}}\), where \(\hat{R}_{t+1}^{*} = R_{t}^{*} - R_t E_{t+1}\) is a one-period return on one dollar holding of a carry trade portfolio. This income is transferred lump-sum to households.

Arbitrageurs choose their portfolio to maximize min-variance preferences, \(\mathbb{E}_t\{\Theta_{t+1} \pi_{t+1}^{D^*}\} - \frac{1}{2} \omega \text{var}_t(\pi_{t+1}^{D^*})\), where \(\Theta_{t+1} = \beta \frac{C_t^{TE}}{C_{t+1}^{TE}}\) is the stochastic discount factor of home households and \(\omega\) is the risk aversion parameter of arbitrageurs. The second term in the objective function is the additional risk penalty due to intermediary frictions that create limits to arbitrage. The optimal portfolio choice satisfies:

\[
\frac{D_t}{R_t^*} = \frac{\mathbb{E}_t\{\Theta_{t+1} \hat{R}_{t+1}^{*}\}}{\omega \sigma_t^2},
\]

where \(\sigma_t^2 \equiv \text{var}_t(\hat{R}_{t+1}^{*}) = R_t^2 \cdot \text{var}_t(E_t/E_{t+1})\) is a measure of the nominal exchange rate volatility. As \(\omega \to 0\), the risk-aversion capacity of arbitrageurs increases unboundedly, and the uncovered interest rate parity (UIP) holds in equilibrium, \(\mathbb{E}_t\{\Theta_{t+1} \hat{R}_{t+1}^{*}\} = 0\).

Noise traders also hold a zero capital portfolio \((N_t, N_t^*)\), such that \(N_t/R_t = -E_t N_t^*/R_t^*\), and \(N_t^*\) is an exogenous liquidity demand shock for foreign currency that is uncorrelated with macroeconomic fundamentals. A positive \(N_t^*\) means that noise traders short home-currency bonds to buy foreign-currency bonds, and vice versa. Noise traders’ net income and losses are transferred to the households. Although difficult to measure in the data, these shocks are necessary to match the disconnect properties of the exchange rate. Importantly, our normative results do not require that \(N_t^*\) is pure noise, and go through when one assumes that currency demand is driven by household preference shock for foreign-currency bonds (see Section 5.3).

Finally, the government holds a portfolio \((F_t, F_t^*)\) of home- and foreign-currency bonds with the net value of the portfolio given by \(F_t/R_t + E_t F_t^*/R_t^*\). Changes in \(F_t\) and \(F_t^*\) correspond to open market operations of the government. The net government income and losses are also transferred to the households. Therefore, net transfers of income to the households from financial transactions of the government, noise traders and arbitrageurs are equal to:

\[
T_t = \left(\frac{F_{t-1}}{R_t} - \frac{F_t}{R_t}\right) + E_t \left(\frac{F_{t-1}^*}{R_t^*} - \frac{F_t^*}{R_t^*}\right) + E_t \hat{R}_t^* \cdot \frac{N_{t-1}^* + D_t^*}{R_t^* - R_{t-1}^*}.
\]

The financial market clearing requires that the home-currency bond positions of all four types of agents balance out, \(B_t + N_t + D_t + F_t = 0\). We define \(B_t^*\) to be the net foreign asset (NFA) position of the home country, expressed in foreign currency, such that:

\[
\frac{B_t^*}{R_t^*} = \frac{1}{E_t} \frac{B_t + F_t^*}{R_t} + \frac{F_t}{R_t^*}.
\]

Thus, home NFA is the value of the combined position of home households and the government, as the remaining agents in the financial market hold zero value portfolios, albeit exposed to currency risk.

Using this definition and the zero value portfolios of noise traders and arbitrageurs, we rewrite

\[\text{More precisely, in this limit, the household SDF } \Theta_{t+1} \text{ prices the exchange rate risk, and the expected return on the carry trade is given by } \mathbb{E}_t \hat{R}_{t+1}^{*} = R_t^* \cdot \text{cov}_t(\Theta_{t+1}, E_t/E_{t+1}), \text{ a property of the optimal international risk sharing.}\]
financial market clearing condition as:

\[ B_t^* = F_t^* + N_t^* + D_t^*. \]  \( \text{(5)} \)

In words, the NFA position of the country equals the combined foreign-currency bond position in the financial market. That is, currency market equilibrium requires that currency supply \( B_t^* \) from accumulated NFA equals aggregate currency demand, \( F_t^* + N_t^* + D_t^* \).\(^8\)

**Equilibrium** Two international conditions — the country budget constraint and international risk sharing — complete the description of the equilibrium system. To derive the country budget constraint, we substitute the expressions for profits \( \Pi_t \) and financial transfers \( T_t \) into the household budget constraint. This yields:

\[ \frac{B_t^*}{R_t^*} - B_{t-1} = Y_{Tt} - C_{Tt}, \]  \( \text{(6)} \)

where the right-hand side is net exports expressed in dollars (or in terms of tradables, since \( P_{Tt}^* = 1 \)). Intuitively, trade surpluses lead to the accumulation of net foreign assets — a macro-fundamental source of currency supply to the home market.

To derive the international risk-sharing condition, we combine household optimality (2) and (3) with the equilibrium conditions in the financial market (4) and (5). This results in:

\[ \beta R_t^* \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \]  \( \text{(7)} \)

where \( \omega \sigma_t^2 = R_t^2 \cdot \operatorname{var} \left( \frac{E_t}{E_{t+1}} \right) \).

If households had direct access to dollar bonds, then a conventional Euler equation \( \beta R_t^* \frac{C_{Tt}}{C_{Tt+1}} = 1 \) would hold. Instead, household positions need to be intermediated by the financial sector which charges a risk premium — a risk-sharing wedge. This risk premium depends both on the size of the currency exposure of arbitrageurs, \( D_t^* = B_t^* - F_t^* - N_t^* \), and the price of risk \( \omega \sigma_t^2 \) per dollar of the exposure.

Currency outflows — due to both fundamental (\( B_t^* < 0 \)) and non-fundamental (\( N_t^* > 0 \)) reasons — require intermediation (\( D_t^* < 0 \)) and expose arbitrageurs to currency depreciation risk, resulting in an equilibrium risk premium and a risk-sharing wedge.\(^10\) Greater exchange rate volatility \( \sigma_t^2 \) increases the price of risk and the resulting risk-sharing wedge for given gross currency positions. A policymaker can intervene either by reducing expected exchange rate volatility or by absorbing the currency risk into the government balance sheet with FX interventions (\( F_t^* \downarrow \)), as we study in the next section.

Finally, we define the equilibrium in this economy. Given the stochastic path of exogenous shocks \( \{A_t, Y_{Tt}, R_t^*, N_t^*, F_t^*\} \), sticky non-tradable prices \( P_{Nt} \equiv 1 \), and the path of policies \( \{R_t, F_t, F_t^*\} \), an equi-

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8Since \( B_t^*, N_t^*, D_t^* \) and possibly \( F_t^* \) can take positive and negative values, who supplies and demands currency in the market can change. Positive (negative) values of \( N_t^* \), \( D_t^* \) and \( F_t^* \) correspond to currency demand (supply), and vice versa for \( B_t^* \).

9Household Euler equation (3) together with the expenditure allocation (2) imply \( E_t \{\Theta_{t+1} | R_t, R_{t+1} / E_{t+1} \} = 1 \). Arbitrageurs’ portfolio choice (4) implies \( E_t \{\Theta_{t+1} | R_t^* - E_t R_t / E_{t+1} \} = \omega \sigma_t^2 D_t^* / R_t^* \), which is the frictional UIP deviation. Adding the two expressions together eliminates the \( R_t \) term; using financial market clearing (5) to substitute for \( D_t^* \) results in (7). Note from this derivation that the risk-sharing wedge is equal to the frictional UIP deviation, and they both disappear as \( \omega \sigma_t^2 \rightarrow 0 \).

10For given \( R_t^* \) and \( R_t \), an equilibrium risk premium results from an exchange rate depreciate on impact, \( E_t \{E_t / E_{t+1} \} > 0 \), and hence positive expected carry trade returns for arbitrageurs. Depreciated exchange rate makes tradables more expensive, depressing \( C_{Tt} \), and thus creating a risk-sharing wedge.
librium vector \( \{C_{Nt}, C_{Tt}, B_t^*, D_t^*, \xi_t\} \) and the implied \( \{\sigma_t^2\} \) solve the dynamic system (2)–(7) with the initial condition \( B^*_{t-1} \) and the transversality condition \( \lim_{T \to \infty} B^*_T / \prod_{t=0}^T R^*_t = 0 \).\(^{11}\) Note that Ricardian equivalence does not hold vis-à-vis foreign currency position \( F^*_t \), as households cannot directly hold foreign currency bonds. As a result, both the country’s NFA position \( B^*_t \) and the arbitrageurs’ currency exposure \( D^*_t = B^*_t - N^*_t - F^*_t \) are endogenous state variables of the equilibrium system. In contrast, the model features Ricardian equivalence for home-currency bonds — a change in \( F_t \) merely crowds out private \( B_t \), and hence it is not a state variable for the equilibrium allocation.

### 2.2 Policy problem

In our baseline analysis, we focus on the Ramsey problem of choosing a sequence of government policies that maximize welfare under commitment. Given the equilibrium definition above, the government chooses a feasible path of monetary policy and FX interventions, \( \{R_t, F^*_t\} \), that maximizes household welfare (1).\(^{12}\) We set up the exact non-linear policy problem in Appendix A1, which allows for characterization of the first-best allocation and the policies that decentralize it. To make progress for the main cases of interest, where the first-best allocation is not feasible given the available policy instruments, we work with a linear-quadratic approximation to the policy problem around the first-best allocation.

In this section, we derive the approximate policy problem. In doing so, we address two major challenges associated with the transition to a linear-quadratic environment. The first challenge relates to the quadratic approximation of the welfare function in an open economy, and in particular where the best possible risk sharing is not full insurance, as the international financial market is incomplete and features risk free bonds only. The second challenge arises due to the risk-sharing friction driven by time-varying risk premium in the currency market that disappears in conventional linear approximations. Our approach ensures that the risk-sharing friction remains in the linear-quadratic environment, preserving the key policy tradeoff between output gap stabilization and international risk sharing.

**First-best allocation** The first-best allocation is the path of tradable and non-tradable consumption and labor, which we denote tildes \( \{\tilde{C}_{Tt}, \tilde{C}_{Nt}, \tilde{L}_t\} \), that maximizes the household welfare in (1) subject to the country budget constraint (6) and the non-tradable production possibility frontier \( C_{Nt} = Y_t = A_tL_t \), taking as given the path of shocks \( \{Y_{Tt}, A_t, R^*_t\} \) and the initial net foreign assets \( B^*_{t-1} \), as well as the NPGC for \( B^*_\infty \). This problem abstracts from both the sticky price friction in the goods market and the intermediation friction in the financial market. Furthermore, the local planner takes as given the structure of the international financial market which provides a perfectly elastic supply of dollar risk-free.

\(^{11}\)Note that (4) is redundant given (7) and (5). Hence, we have four independent equilibrium conditions, (2)–(3) and (6)–(7), to solve for four endogenous variables \( \{C_{Nt}, C_{Tt}, B_t^*, \xi_t\} \), and a side equation (5) to solve for \( D_t^* \). The other endogenous variables \( \{W_t, L_t, Y_{Nt}, B_t\} \) are recovered from static equilibrium conditions outlined above. Specifically, from the goods market clearing and labor supply \( Y_{Nt} = C_{Nt} \) and \( W_t = C_{Nt}; \) from production function \( L_t = Y_{Nt}/A_t; \) and \( B_t \) can be backed out from \( \{B_t^*, F_t^*, F_t\} \) given the definition of NFA \( B_t^* \).

\(^{12}\)The government does not take the welfare of arbitrageurs and noise traders into account, as these agents pass on all their financial incomes and losses to the households. Yet, their behavior — namely, exogenous currency demand of noise traders and endogenous inelastic currency supply by arbitrageurs — affects the equilibrium allocation in the financial market via a wedge in the risk-sharing condition (7). This is akin to the behavior of a monopolist in the goods market that creates a markup wedge and passes on all profits back to the households.
bonds at an exogenous interest rate $R_t^*$.  

Given the log-linear utility (1), the first-best allocation features a constant labor supply $\tilde{L}_t = 1$ yielding $\tilde{C}_{Nt} = A_t$, and a path of $\tilde{C}_{Tt}$ that solves a frictionless Euler equation $\beta R_t^* E_t \{ C_{Tt} / C_{Tt+1} \} = 1$ together with the country budget constraint (6). Therefore, $\tilde{C}_{Tt}$ is a function of shocks $\{ Y_{Tt}, R_t^* \}$ and the initial net foreign assets $B_t^*$. The first-best path of NFA satisfies the country budget constraint (6), that is $\tilde{B}_t^* = R_t^* ( \tilde{B}_{t-1}^* + Y_{Tt} - \tilde{C}_{Tt} )$.

With fully sticky non-tradable prices, the decentralization of the first-best allocation involves a path of nominal wages $\tilde{W}_t = A_t$ to ensure the first-best labor supply, and a path of the nominal exchange rate $\tilde{E}_t = \tilde{Q}_t = \gamma_1 - \gamma \tilde{C}_{Nt} \tilde{C}_{Tt}$ (8) to ensure the first-best relative price and expenditure allocation between tradables and non-tradables in (2). Equation (8) defines $\tilde{Q}_t$ to which we refer as the first-best, or natural, real exchange rate. It is the value of international relative prices that ensures the optimal expenditure allocation between tradables and non-tradables in our economy.\(^\text{13}\)

**Second-order approximation to the welfare function** We evaluate welfare losses due to a departure of the equilibrium allocation from the first best. To do so, we derive a second-order approximation to the objective function in (1) around a non-stochastic steady state and evaluate the welfare loss relative to the first-best allocation $\{ \tilde{C}_{Tt}, \tilde{C}_{Nt}, \tilde{L}_t \}$ characterized above.

To this end, we introduce two wedges central to our analysis — the output gap $x_t$ and the risk-sharing wedge $z_t$ — defined by:

$$x_t \equiv \log C_{Nt} - \log \tilde{C}_{Nt} \quad \text{and} \quad z_t \equiv \log C_{Tt} - \log \tilde{C}_{Tt}.$$  (9)

The output gap $x_t$ emerges as a result of sticky non-tradable prices, and it measures the gap in non-tradable consumption relative to $\tilde{C}_{Nt} = A_t$. This also corresponds to the departure of labor supply $L_t = C_{Nt} / A_t$ from $\tilde{L}_t = 1$. The risk-sharing wedge $z_t$ is the result of a violation of the first-best risk sharing. Specifically, an intermediation wedge in (7) causes a risk-sharing wedge. Note that all feasible paths of $C_{Tt}$, and hence $z_t$, must still satisfy the country budget constraint (6).

To make the analysis tractable, the innovation of our approach is to focus only on budget-feasible allocations $\{ C_{Tt}, C_{Nt}, L_t \}$ that satisfy the production possibilities frontier for non-tradables, $C_{Nt} = A_t L_t$, and the country budget constraint for tradables, that is (6) together with the NPGC for $B_t^*$ and given $B_t^*$. For every such allocation that results in wedges $x_t$ and $z_t$ defined in (9), the welfare loss relative to the

\(^{13}\)Formally, the real exchange rate is $E_t P_t^{Nt} / P_t = E_t^{1-\gamma}$ (with $P_t^* = P_t^{Tt} = 1$ and $P_t = P_t^{Nt} P_t^{1-\gamma} = E_t$), while $E_t P_t^{Nt} / P_t = E_t$ is the relative price of non-tradables; the two are proportional to each other in logs. More generally, in every economy, one can define a relevant concept for the first-best real exchange rate that, given goods market clearing condition, ensures an efficient expenditure allocation between home and foreign goods.
first best is given by (see Appendix A2):\(^{14}\)

\[
\text{Loss} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right].
\] (10)

Note that the approximation to the welfare function relative to the first best has no first order terms by construction. Intuitively, the weight on the output gap equals 1 - \(\gamma\) and the weight on the risk-sharing wedge equals \(\gamma\) — the expenditure weights on non-tradables and tradables, respectively.

The path of the risk-sharing wedge \(z_t\) must be consistent with the budget constraint (6), which in deviations from the first best is given by:

\[
\beta b_t^* - b_{t-1}^* = -z_t, \tag{11}
\]

where \(b_t^* = (B_t^* - \tilde{B}_t^*)/\tilde{Y}_T\) is the deviation of NFA from its first-best path scaled by the steady-state level of tradable output \(\tilde{Y}_T\), and \(\beta \tilde{R}^* = 1\) in steady state. The initial condition is \(b_{-1}^* = 0\) (as \(B_{-1}^* = B_{-1}^*\)), the NPGC is \(\lim_{t \to \infty} \beta^t b_t^* = 0\), and thus \(z_t = b_t^* = 0\) for all \(t \geq 0\) is a feasible allocation corresponding to the first-best risk sharing. Note that \(z_t\) acts simultaneously as the risk-sharing wedge and as the deviation of net exports from their first-best path, \(z_t = -(nx_t - \tilde{n}x_t)\) where \(\tilde{n}x_t \equiv (Y_{Tt} - \tilde{C}_{Tt})/\tilde{Y}_T\). Cumulated deviations of net exports \(z_t\) result in deviations of NFA \(b_t^*\), as summarized by (11).

**First-order approximation to the equilibrium system** Minimizing the welfare loss (10) subject to the budget constraint (11) alone poses no policy tradeoff as \(x_t = z_t = 0\) for all \(t \geq 0\) is a budget-feasible allocation. In addition to the budget constraint (11), the first-order approximation to the exact equilibrium system (2)-(7) involves two additional conditions — one that characterizes equilibrium in the goods market and the other that characterizes equilibrium in the financial market.

In the goods market, the expenditure allocation condition (2) can be written in log deviations as:

\[
e_t = \tilde{q}_t + x_t - z_t, \tag{12}
\]

where \(e_t = \log E_t\) and \(\tilde{q}_t = \log \tilde{Q}_t\) is the first-best real exchange rate defined in (8), and the two wedges \(x_t\) and \(z_t\) as defined in (9). Given sticky prices, the nominal exchange rate must accommodate movements in the first-best real exchange \(\tilde{q}_t\), otherwise one or both wedges open up. Indeed, if the relative price of non-tradables is off its first-best level, either tradable or non-tradable consumption (or both) must deviate from their first-best levels as well. Equation (12) captures the locus of possible equilibrium allocations in the goods market shaped by expenditure switching between tradables and non-tradables.\(^{15}\)

The remaining condition characterizes equilibrium in the financial (currency) market. The risk-sharing friction emphasized in (7) corresponds to the risk premium charged by arbitrageurs for inter-

\(^{14}\)All our approximations are around a steady state with \(B^* = F^* = N^* = 0\), \(R = \tilde{R}^* = 1/\beta\); with tradable endowment \(Y_T\), non-tradable productivity \(A\), and exchange rate \(E = \tilde{Y}_T\), resulting in \(L = 1\), \(C_N = A\), \(C_T = Y_T, N X = 0\).

\(^{15}\)Rewriting (12) as \(x_t = z_t + (e_t - \tilde{q}_t)\) one can interpret this condition as follows: a capital outflow shock \(z_t < 0\) must be accommodated by either a depreciation \(e_t > \tilde{q}_t\) or an output gap \(x_t < 0\) if the exchange rate fails to adjust.
mediating currency trades and holding the associated exchange rate risk. In conventional linear approximations, risk premia go to zero with second moments such as \( \sigma_t^2 \). We consider an alternative point of approximation in which risk premia remain first-order objects and, hence, affect first-order dynamics of the equilibrium system. To this end, we let the risk aversion parameter \( \omega \) to increase as \( \sigma_t^2 \) decreases, keeping the price of risk \( \omega \sigma_t^2 \) non-zero in the limit. We provide formal details in Appendix A2, where we show that our first-order approximation to (7) results in:

\[
\mathbb{E}_t \Delta z_{t+1} = \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*) \quad \text{with} \quad \bar{\sigma}_t^2 = \text{var}_t(e_{t+1}),
\]

where \( \bar{\omega} \equiv \omega \bar{Y}_T / \beta, \bar{f}_t^* \equiv F_t^* / \bar{Y}_T \) are FXI scaled by tradable output, and \( n_t^* \equiv (N_t^* - \bar{B}_t^*) / \bar{Y}_T \) is a combined exogenous currency demand shock.\(^{16}\) Like a conventional first-order approximation, our approximation scales linearly with the size of exogenous shocks that drive \( n_t^* \) in (13) and \( \bar{q}_t \) in (12). However, due to an unconventional point of approximation in which the risk-bearing capacity of the financial sector \( 1/ \omega \rightarrow 0 \), the equilibrium system is not linear in shocks (and hence state variables), and in particular features time-varying volatility that affects first-order equilibrium dynamics.

Condition (13), together with the budget constraint (11), characterizes the equilibrium path of tradable consumption relative to its first-best level, \( z_t \equiv \log(C_{T_t} / \bar{C}_{T_t}) \), from the point of view of households. Alternatively, it also determines the path of UIP deviations from the point of view of the financial market (recall conditions (4) and (5)). Indeed, we have that the UIP deviation equals:

\[
i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t \Delta z_{t+1},
\]

where \( i_t - i_t^* = \log(R_t / R_t^*) \).\(^{17}\) Therefore, the risk-sharing wedge in (13), \( \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*) \), is also the frictional UIP wedge. It is a first-order object that comoves with the intermediated demand for currency, \( n_t^* + f_t^* - b_t^* \), and with the unit price of the exchange rate risk, \( \bar{\omega} \sigma_t^2 \). Thus, variation in the conditional exchange rate volatility \( \bar{\sigma}_t^2 \) is both an equilibrium outcome and has a direct first-order feedback into equilibrium dynamics. We argue that this provides a superior point of approximation for models that focus on the joint dynamics of macroeconomic variables and risk premia.

In Appendix A2, we prove two additional results. First, the dynamic system (11)–(13) provides an accurate first-order approximation to the exact equilibrium dynamics. That is, taking the path of exogenous shocks \( \{ \bar{q}_t, n_t^* \} \) and policies \( \{ x_t, f_t^* \} \), this system characterizes the path of endogenous equilibrium outcomes \( \{ z_t, b_t^*, e_t, \bar{\sigma}_t^2 \} \). Note that we take output gap \( x_t \) as the policy variables since it is directly controlled by the monetary policy instrument \( i_t \), and we discuss the implementation below.

Second, we prove that minimizing the second-order welfare loss in (10) with respect to \( \{ x_t, f_t^*, z_t, b_t^*, e_t \} \)

\(^{16}\)Note that \( n_t^* \) features both noise trader demand for foreign currency (\( N_t^* > 0 \)) net of supply of foreign currency accumulated from the first-best path of net exports (that is, NFA \( \bar{B}_t^* > 0 \)); of course, these variables can take both positive and negative values with corresponding interpretations.

\(^{17}\)From the frictionless international Euler equation, \( i_t^* = \log(R_t / R_t^*) = \mathbb{E}_t \Delta \bar{e}_{T_t+1} \). The home Euler equation (3) together with (2), in turn, implies \( i_t = \log(R_t / \bar{R}) = \mathbb{E}_t \{ \Delta c_{T_t+1} + \Delta a_{t+1} \} \). Subtracting one from the other, and using the fact that \( z_t = c_{T_t} - \bar{e}_{T_t} \) according to the definition in (9), yields the UIP expression in the text. Note also that the equilibrium path of the local interest rate can be recovered from (3) as \( i_t = \tilde{r}_t + \mathbb{E}_t \Delta x_{t+1} \), where \( \tilde{r}_t = \mathbb{E}_t \Delta a_{t+1} = \mathbb{E}_t \Delta \bar{e}_{N_t+1} \) is the natural real interest rate and \( x_t = c_{N_t} - a_t \) is the output gap.
and subject to the linearized equilibrium system (11)–(13) results in a first-order accurate description of the optimal policies in the exact non-linear problem. While the equilibrium system is non-linear in the path of shocks and state variables due to the presence of $\sigma_t^2$ in (13), the policy problem scales proportionally with the general magnitude of shocks, and in this narrow sense one may refer to this policy problem as linear-quadratic.\footnote{Formally, we let $\nu$ scale all exogenous shocks in the exact non-linear economy \{\( A_t, Y_T, R_T^*, N_T^* \)\} with $1/\omega$ scaled by $\nu^2$ to keep the unit price of risk $\omega \sigma_t^2$ stable. Then, the linearized system (11)–(13) characterizes the first-order component of the non-linear system, which scales proportionally with $\nu$, while the welfare loss in (10) scales proportionally with $\nu^2$. The optimal policy $\delta_t$ (see below) and the equilibrium price of risk $\omega \sigma_t^2$ do not change with $\nu$, but are generally time-varying.}

## 3 Optimal Policies

Given the results in Section 2.2, we restate here the baseline Ramsey policy problem (10)–(13) of a small-open economy policymaker in deviations from the first-best allocation:

\[
\min_{\{x_t, f_t^*, z_t, e_t, b_t^*, \sigma_t^2\}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [\gamma z_t^2 + (1 - \gamma) x_t^2]
\]

subject to

\[
\beta b_t^* = b_{t-1}^* - z_t,
\]

\[
e_t = \tilde{q}_t + x_t - z_t
\]

\[
E_t \Delta z_{t+1} = \tilde{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*) \quad \text{with} \quad \sigma_t^2 = \text{var}_t(e_{t+1}),
\]

and potential constraints on FXI $f_t^* \in F_t$, the initial condition $b_{-1}^* = 0$ and the transversality condition $\lim_{t \to \infty} \beta^t b_t = 0$. The policymaker directly controls the path of output gap and FXI, $\{x_t, f_t^*\}$. The path of policies may be restricted by additional constraints $\mathcal{F}_t$, e.g. a non-negativity of FX reserves $f_t^* \geq 0$ or a value-at-risk constraint $\sigma_t \cdot |f_t^*| \leq \alpha$.

All exogenous shocks affecting equilibrium dynamics are summarized by two variables — the natural (first-best) real exchange rate $\tilde{q}_t$ defined in (12) and the exogenous net currency demand shock $n_t^*$ defined in (13). In particular, $\tilde{q}_t$ is a sufficient statistic for all macroeconomic shocks \{\( A_t, Y_T, R_T^* \)\} that shape the first-best path of tradable and non-tradable consumption. In turn, $n_t^*$ summarizes currency demand shocks of noise traders $N_t^*$ and households $B_t^*$, with the latter shaped by the path of the first-best $\tilde{N} X_{\tilde{t}} = Y_{T_t} - \tilde{C}_{T_t}$. Departures from the first-best path of tradable consumption result in risk-sharing wedges $z_t \equiv \log(C_{T_t}/\tilde{C}_{T_t})$, which via (11) lead to deviations of NFA $b_t^*$ that also feed back via (13) as an additional source of endogenous currency supply (or demand, if negative).

The goal of the policy (15) is to minimize deviations from the first-best allocation — namely, eliminate to the extent possible the output gap $x_t$ and the risk-sharing wedge $z_t$, with the relative weight on the latter given by the openness of the economy $\gamma$. Policies shape the equilibrium path of the exchange rate $e_t$, and thus its conditional volatility $\tilde{\sigma}_t^2$, which in affects the dynamics of the equilibrium system via (13). We note that a particular level or volatility of the exchange rate is not a policy goal in itself. Nonetheless, the exchange rate $e_t$ emerges as the key equilibrium variable linking the financial and the goods markets, putting it at the center of the policy tradeoff. When prices are sticky, movements in the nominal exchange rate are necessary to accommodate the adjustment of relative prices in...
the goods market (12). Yet, volatility of the exchange rate is also a source of the risk-sharing wedge in
the financial market with imperfect intermediation (13).19

**Relaxed Trilemma** An important feature of the model is that the planner can sidestep the standard
trade-off between an independent monetary policy and a managed exchange rate. Even in the absence
of capital controls, the government can choose the path of the output gap \( x_t \) with an inward-looking
interest rate policy (e.g., ensure \( x_t = 0 \)), and simultaneously manipulate the path of the exchange
rate via sterilized interventions in the currency market (by means of \( f_t^* \) in (13)). This result does not
contradict the trilemma: FX interventions have real effects because market segmentation limits capital
mobility and does not allow households to undo the open market operations of the central bank. As
a result, the policymaker can move exchange rate risk between balance sheets of arbitrageurs and
households, and thus change the equilibrium outcome in the currency market (cf. Wallace 1981, Silva
2016, Kekre and Lenel 2022).20 Similarly to how nominal rigidities allow monetary policy to affect real
outcomes, intermediation frictions give rise to an additional policy instrument \( f_t^* \) in (13). Note also
that FX reserves are not essential for interventions as the same outcomes can be achieved using FX
derivatives to absorb risk from the balance sheet of market participants. This result is consistent with a
wide use of instruments such as currency swaps in central bank interventions (Patel and Cavallino 2019)
and contrasts with the mechanism based on CIP deviations (e.g., Gabaix and Maggiori 2015, Fanelli and
Straub 2021, IPF).

By this logic, the central bank can peg the nominal exchange rate in two different ways — using
either monetary policy or FX interventions, affecting \( e_t \) in (12) by means of \( x_t \) and \( z_t \), respecitively. The
policy problem (15) identifies the costs and benefits associated with each of these implementations. On
the one hand, monetary policy has the advantage that there are no restrictions on the implementable
paths of the exchange rate. However, this comes at a cost of opening the output gap \( x_t \). Unless prices
are fully flexible, a monetary peg drives a wedge between the real exchange rate and its natural level \( \tilde{q}_t \),
resulting in suboptimal expenditure switching in the goods market (cf. “divine coincidence” below).

On the other hand, FX interventions can be used to manipulate the path of the exchange rate without
any output gap side effects. However, there are important limits on the possible paths of the exchange
rate that can be implemented with FXI. First, for a given monetary policy, FX interventions affect the
nominal exchange rate by changing the real exchange rate, net exports and net foreign asset dynamics,
via \( z_t \) in (11)–(13). Therefore, while FXI can temporarily alter the dynamics of the exchange rate, it
is, for example, impossible to use them to generate a permanent appreciation, as it would result in a
permanent trade deficit.21 Second, FXI become entirely ineffective when monetary policy fully (and

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19Formally, if monetary policy stabilizes the output gap, \( x_t = 0 \), then from (12) the nominal exchange rate must equal
\( e_t = \tilde{q}_t - z_t \). This, in general, results in \( \sigma_t^2 = \text{var}(e_t+1) > 0 \), and hence a non-zero risk-sharing wedge \( z_t \) from (13).

20It is the presence of a non-zero price of currency-holding risk, \( \tilde{\omega}\sigma_t^2 > 0 \) in (13), that relaxes the trilemma constraint on
FXI and allows FXI to affect the equilibrium currency risk premium and thus the exchange rate.

21What happens when a policymaker attempts to fix the exchange rate at a level above what is consistent with a long-
run steady state, \( \tilde{e} > \tilde{q} \) in (12)? Statically, it can either result in a negative output gap, \( x < 0 \), or a positive risk-sharing
wedge, \( z > 0 \) (i.e., excess tradable consumption). However, the latter is inconsistent with the intertemporal budget con-
credibly) stabilizes the nominal exchange rate, $e_t = \bar{e}$ and hence $\bar{\sigma}_t^2 = 0$ in (13), bringing back the classic trilemma constraint. This is the case because the currency supply by arbitrageurs becomes perfectly elastic in the absence of exchange rate risk, and arbitrageurs fully neutralize the effects of any open market operations on the exchange rate.

**Optimal monetary policy** We introduce here a general characterization of the optimal monetary policy for any given path of FX interventions, which nests as special cases the specific results that we consider in turn in Sections 3.1–3.3. We prove in Appendix A3 that the solution to the policy problem (15) involves the following optimality condition:

**Theorem 1** For any given path of FX interventions $\{f_t^*\}$, the Ramsey optimal monetary policy sets the path of the output gap to satisfy $E_t x_{t+1} = 0$ and:

$$x_{t+1} = -\delta_t \cdot (e_{t+1} - E_t e_{t+1}) \quad \text{with} \quad \delta_t = \frac{2\gamma\bar{\omega}}{1-\gamma} \mu_t(n_t^* + f_t^* - b_t^*),$$

where $\delta_t$ is the intensity of the monetary policy lean against exchange rate surprises, $e_{t+1} - E_t e_{t+1}$, and $\mu_t$ is the Lagrange multiplier on the risk sharing constraint (13).

The optimality condition (16) connects the optimal path of monetary policy, summarized by the path of the output gap $\{x_{t+1}\}$, with three properties of the exchange rate and the currency market:

(i) exchange rate surprises, $e_{t+1} - E_t e_{t+1}$;
(ii) capital outflows, or currency demand, $n_t^* + f_t^* - b_t^*$;
(iii) departures from the first-best risk sharing and UIP, $E_t \Delta z_{t+1} \neq 0$, as captured by the sequence of respective Lagrange multipliers $\mu_t$ on the risk sharing constraint (13).

It is the interaction of these three features that determines the optimal monetary policy response, emphasizing already the role of non-linearity in the optimal exchange rate analysis captured by our approximation approach.

The policy lean $\delta_t$ in (16) corresponds to a free float when $\delta_t = 0$, a partial peg (or a managed float) when $\delta_t > 0$, or a full peg in the limit of $\delta_t \to \infty$. A free float is optimal in the limits of a closed economy or a frictionless financial market, while our focus is on an open economy ($\gamma > 0$) with a frictional financial intermediation ($\bar{\omega} > 0$). In general, according to (16), the optimal monetary policy responds to exchange rate surprises and, hence, deviates from the exclusive inward-looking goal of inflation and output gap stabilization ($x_{t+1} \equiv 0$). Specifically, output gap in each period is eliminated on average, $E_t x_{t+1} = 0$, but generally not state by state. In what follows, we first focus on two cases:

1. A free float is optimal in the limits of a closed economy or a frictionless financial market, while our focus is on an open economy ($\gamma > 0$) with a frictional financial intermediation ($\bar{\omega} > 0$). In general, according to (16), the optimal monetary policy responds to exchange rate surprises and, hence, deviates from the exclusive inward-looking goal of inflation and output gap stabilization ($x_{t+1} \equiv 0$). Specifically, output gap in each period is eliminated on average, $E_t x_{t+1} = 0$, but generally not state by state. In what follows, we first focus on two cases:

22Note that a (gross) currency demand shock $n_t^* > 0$, unaccommodated with FXI, results in a (net) capital outflow, $z_t < 0$, at least when $\bar{\omega}\sigma_t^2 > 0$. Therefore, we occasionally refer to $n_t^*$ as capital outflow shocks.

23Using (12), we can rewrite (16) as $x_{t+1} = -\frac{\bar{q}_t}{1-\gamma} (\bar{q}_t - z_{t+1} - E_t (\bar{q}_{t+1} - z_{t+1}))$, and the limit of a full peg, $x_{t+1}$ offsets one-for-one all exchange rate surprises arising from $\bar{q}_{t+1}$ and $z_{t+1}$. 
where output gap is fully stabilized, \( x_{t+1} \equiv 0 \), either because capital outflows are fully accommodated with FXI, or when a fixed exchange rate is optimal by “divine coincidence”. Then we consider the general case that can be described as the optimal crawling peg (or a dirty float), whereby the optimal monetary policy compromises full output gap stabilization to smooth out exchange rate surprises.

### 3.1 Unconstrained optimal policy

When both policy instruments — monetary policy that controls the path of \( x_t \) and FXI \( f_t^* \) — are available and unconstrained, the first-best allocation is feasible and, thus, is implemented by the optimal policy. Indeed, this corresponds to the special case of Theorem 1 where FXI ensure \( n_t^* + f_t^* - b_t^* = 0 \) in (16), which both results in \( z_t = b_t^* = 0 \) from (11)–(13), and renders optimal an inward-looking monetary policy that targets \( x_t = 0 \).

**Proposition 1** If both policy instruments are available and unconstrained, the optimal policy fully stabilizes both wedges, the output gap \( x_t = 0 \) and the risk sharing wedge \( z_t = 0 \), by targeting output gap with monetary policy and demand for currency with FX interventions (\( f_t = -n_t^* \)). The nominal exchange rate varies with the natural real exchange rate, \( e_t = \tilde{q}_t \), with conditional volatility \( \tilde{\sigma}_t^2 = \text{var}_t(\Delta \tilde{q}_{t+1}) \). This solution is unique, time consistent, and its implementation requires no commitment.

Though the fact that two policy instruments are sufficient to implement the first-best allocation in the presence of two frictions is perhaps not surprising, the proposition shows that there is a one-to-one mapping between instruments and optimal targets (cf. Mundell 1962). In particular, monetary policy closes the output gap \( x_t = 0 \) and stabilizes producer prices, while optimal FX interventions eliminate frictional UIP deviations, \( E_t \Delta z_{t+1} = i_t - i_t^* - E_t \Delta e_{t+1} = 0 \), and thus close the risk-sharing wedge \( z_t = 0 \). Crucially, neither policy instrument targets the exchange rate directly, nor fully stabilizes it. Instead, the optimal policy ensures \( x_t = z_t = 0 \), which in turn implies that the nominal exchange rate tracks the natural real exchange rate, \( e_t = \tilde{q}_t \), and hence generally \( \tilde{\sigma}_t^2 = \text{var}_t(\Delta \tilde{q}_{t+1}) > 0 \).

The proposition also provides a complementary characterization of the optimal policy in terms of responses to different types of shocks. Using the language of CGG, FX interventions offset currency demand shocks \( f_t^* = b_t^* - n_t^* = -n_t^* \), as \( b_t^* \equiv 0 \) along the first best, while allowing the exchange rate to accommodate fundamental macroeconomic shocks \( \{ A_t, Y_T, R_t^* \} \) that drive the natural real exchange rate \( \tilde{q}_t \). To the extent financial intermediation is frictional and results in risk-sharing wedges, FX interventions should step in to eliminate the associated UIP deviations. In practice, this means providing FX liquidity to the market to offset currency demand shocks, alleviating the need for costly intermediation by absorbing the exchange rate risk exposure from the arbitrageurs’ and into the government balance sheet. The fact that interventions offset liquidity shocks state by state and are independent of expectations about future outcomes explains why the optimal policy is time consistent and does not require commitment on the part of the government.

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24 Proposition 1, as well as Proposition 2 below, holds without approximation in the exact policy problem, as we show in Appendix A1. This is because the exact first-best allocation is feasible when the two policy instruments are available and unconstrained; similarly, it is feasible with a single monetary policy tool under the “divine coincidence” introduced below.

25 Another notable feature of this result is that capital controls are not needed for implementation, as FX interventions are sufficient to achieve the first best allocation when combined with the optimal monetary policy (see Section 4.1).
An important feature of this setup is that it allows us to distinguish between UIP and CIP deviations, and to show that the optimal policy should target the former. This contrasts with the conclusions of the previous literature where the limits to arbitrage arise due to financially-constrained arbitrageurs and CIP wedges are the only source of UIP deviations (Fanelli and Straub 2021, IPF). This difference is important from the practical perspective given that, in the data, UIP deviations are an order of magnitude larger than CIP deviations.

More generally, FX interventions should be used to eliminate all rents in the currency market due to intermediation frictions, including the monopoly power of intermediaries. Because the policymaker is the agent on behalf of the households who cannot directly participate in FX trading, the portion of UIP deviations due to risk that is priced by the household SDF (e.g., default risk) should not be eliminated with FXI. Consistent with Friedman (1953), the policymaker should take positions in the currency market as long as they are deemed profitable from the point of view of the households. As a result, the central bank should be making money, at least on average, from its FXI activity.

Implementation  The optimal policy can be implemented using a conventional Taylor interest rate rule targeting the output gap and a similar policy rule for FXI targeting ex ante UIP deviations. For example, adopting a rule  
\[ f_t^* = -\alpha E_t \Delta z_{t+1} \]  results in  
\[ E_t \Delta z_{t+1} = \frac{\omega \sigma_z^2}{1 + \alpha \omega \sigma_z^2} (n_t^* - b_t) \]  from (13), which converges to  
\[ E_t \Delta z_{t+1} = 0 \]  as  \( \alpha \to \infty \), and  
\[ f_t^* = b_t^* - n_t^* \]  in this limit. In words, FX interventions should lean against the wind intensively enough until the UIP wedge is entirely eliminated.

Despite its simplicity, the optimal FX policy might be hard to implement in practice. The challenge is that neither the UIP wedge  
\[ E_t \Delta z_{t+1} = i_t - i_t^* - E_t \Delta e_{t+1} \]  or liquidity shocks  \( n_t^* \), nor the natural level of the real exchange rate  \( \bar{q}_t \)  are directly measurable in the data. One solution to this problem is to condition the policy rule on easily measurable variables such as the ex post UIP realization,  \( i_{t-1} - i_{t-1}^* - \Delta e_t \), or on a noisy measure of the ex ante UIP wedge,  \( E_t \Delta z_{t+1} + u_t \), where  \( u_t \)  is a measurement/expectational error. In both cases, the first-best implementation is infeasible, and there is an internal optimum for the intensity of the policy response to avoid overreaction to noise in the measurement of the target. Another possibility is to condition the policy rule on the level of the exchange rates,  \( f_t^* = -\alpha e_t \), which corresponds to a second-best implementation if currency demand shocks  \( n_t^* \)  are the dominant source of the exchange rate volatility.\(^{26}\) If instead most of the exchange rate volatility is due to macro shocks affecting  \( \bar{q}_t \), then the optimal FXI do not respond to the exchange rate and set  \( \alpha = 0 \).

It is worth noting that the challenge of unobserved targets and shocks is, of course, not unique to FXI. It is a common feature of the optimal monetary policy in a closed economy, where the policymaker needs to make a judgement call about the natural rate of interest, potential output and NAIRU, to offset shocks to aggregate demand and accommodate productivity shocks (see CGG). Even though not directly observable in the data, these concepts are useful in guiding the decisions of policymakers.

\(^{26}\)There is an upper bound on the value of  \( \alpha \) beyond which the policy becomes inconsistent with the budget constraint (11) resulting in an explosive path of net foreign assets. Intuitively, a very large  \( \alpha \)  implements a nearly constant exchange rate, which generally is inconsistent with the intertemporal budget constraint. However, if the peg is set at a sufficiently depreciated level that it guarantees persistent trade surpluses and hence, accumulation of NFA and FX reserves, such policy does not violate NPGC and is feasible in the long run, albeit generally suboptimal due to  \( z_t < 0 \)  on average.
3.2 Divine coincidence

We consider now a special case of Theorem 1, whereby it is possible to achieve both policy objectives — in the goods and in the financial market — with a single monetary instrument, without recurring to capital flow or exchange rate management using FXI. By analogy with the closed-economy New Keynesian literature, we refer to this case as divine coincidence, and we further show in Section 5.1 how it generalizes the closed economy case.

The open-economy divine coincidence obtains when the first best (natural) real exchange rate is stable at some level, \( \hat{q}_t = \hat{q} \). In this case, allowing for an arbitrary path of FXI \( \{f_t^*\} \), a monetary policy rule that targets the same level of the nominal exchange rate, \( e_t = \hat{q} \), both ensures zero output gap and eliminates the risk-sharing wedge, \( x_t = z_t = 0 \), delivering the first best outcome. Indeed, in this case, \( \bar{\sigma}^2_t = \text{var}_t(\Delta e_{t+1}) = 0 \), and thus \( z_t = 0 \) is the unique solution of (11) and (13) independently of the path of \( (n_t^*, f_t^*) \). Given \( z_t = 0 \) and the fact that \( \hat{q}_t = \hat{q} \), expenditure allocation in the goods market (12) eliminates the output gap, \( x_t = 0 \), as the unique equilibrium outcome. We summarize this result in:

**Proposition 2** If the natural real exchange rate is stable, \( \hat{q}_t = \hat{q} \), then monetary policy that fully stabilizes the nominal exchange rate, \( e_t = \hat{q} \), ensures the first best allocation with \( x_t = z_t = 0 \), for any path of FX interventions, including \( f_t^* = 0 \). An exchange rate peg is superior to inflation or output gap targeting, as it rules out multiplicity of exchange rate equilibria.

In general, our model emphasizes the tension between the need for exchange rate adjustment in the goods market with sticky prices and the risk-sharing consequences of a volatile exchange rate unaccommodated by FXI. This dual role of the exchange rate, generally, makes a single policy instrument insufficient to attain efficiency in both goods and financial markets at once, as suggested by Theorem 1.

Divine coincidence is the situation when this policy tradeoff disappears, as the natural real exchange rate — a goods-market proxy for the desirable nominal exchange rate adjustment — is stable. In turn, a pegged nominal exchange rate encourages intermediaries to supply currency more elastically and, in the limit, offset any demand shocks in the currency market. Thus, a fixed nominal exchange rate comes at no cost from the perspective of the goods market and delivers the first-best risk sharing from the perspective of the financial market. In fact, the fixed exchange rate policy is implied by Theorem 1, as \( x_{t+1} = 0 \) is consistent with \( e_{t+1} = \mathbb{E}_t e_{t+1} = \hat{q} \) in this case, making sure the optimality condition (16) holds under a monetary peg (i.e., \( \delta_t \to \infty \)).

Divine coincidence provides a rationale for pegging the exchange rate. Moreover, in this case, a nominal exchange rate peg by means of monetary policy is not only efficient, but also effective, as it immediately eliminates the possibility of multiple equilibria. Consider the alternative policy of output gap (inflation) targeting that ensures \( x_t = 0 \) independently of the path of \( z_t \). Under divine coincidence, such policy is consistent with an equilibrium with \( e_t = \hat{q} \) and \( z_t = \bar{\sigma}^2_t = 0 \). However, this is not a unique equilibrium, as there exists another equilibrium with arbitrageurs uncertain about the future exchange rate, \( \bar{\sigma}^2_t > 0 \), which makes them charge a risk premium in response to currency demand shocks \( n_t^* \), resulting in a self-fulfilling volatile exchange rate equilibrium.\(^{27}\) The positive volatility

\(^{27}\)If \( n_t^* \) follows an AR(1), then \( e_t = \hat{q} - z_t \) follows an ARMA(2,1) with innovations proportional to the innovation of \( \hat{\sigma}^2 n_t^* \), where \( \hat{\sigma}^2 = \text{var}_t(e_{t+1}) > 0 \) is a fixed point, in addition to the other fixed point with \( \hat{\sigma}^2 = 0 \).
equilibrium is suboptimal as it features $E_0 z_t^2 > 0$ in contrast to the first best with $z_t = 0$. Thus, under divine coincidence, an exchange rate peg dominates inflation targeting, even though the result of the exchange rate peg is also a zero inflation and a zero output gap (cf. Marcet and Nicolini 2003, Atkeson, Chari, and Kehoe 2010, Bianchi and Coulibaly 2023).

How special is the open-economy divine coincidence? On the one hand, this result extends immediately to various generalizations of the goods market with expenditure switching between varieties of home and foreign tradable goods. In each such model, one can define a concept of the natural real exchange rate that delivers efficient expenditure switching. A stable natural real exchange rate implies that a fixed nominal exchange rate does not come into conflict with the objectives of inflation and output gap stabilization in the goods market. At the same time, a stable natural real exchange rate is, of course, a knife-edge case which we do not expect to systematically hold in practice, yet it provides a useful benchmark and a stark illustration to the model’s mechanism.

On the other hand, the divine coincidence result is quite specific to the particular structure of the financial market that we assume in our framework. In particular, in this framework an ex post stable exchange rate, $e_{t+1} \equiv 0$, implies ex ante certainty, namely $\tilde{\sigma}_t^2 = \text{var}_t(e_{t+1}) = 0$, and this in turn guarantees that UIP holds and risk sharing is undistorted. This nests two assumptions. First, it requires that a commitment to a peg is ex ante credible. Second, it relies on the structure of the model in which a fully stabilized exchange rate eliminates UIP deviations via the endogenous response of arbitrageurs who are willing to supply currency with infinite elasticity in the absence of exchange rate risk. If either the peg is not credible, or UIP deviations may coexist with $\tilde{\sigma}_t^2 = 0$, then divine coincidence result breaks down. For example, this is the case when risk-sharing frictions are driven by balance sheet constraints rather than risk, and UIP and CIP deviations are closely linked. To the extent a credible peg eliminates a large portion of UIP deviations, while leaving CIP deviations intact, — as the data seem to suggest (see Itskhoki and Mukhin 2021b) — this result offers a useful quantitative benchmark.

**Optimal currency areas** The divine coincidence result also provides an important benchmark for common currency areas, which are optimal when the natural real exchange rate between member countries is stable. In particular, this is the case when member countries share correlated fundamental shocks (see Section 4.2), confirming the logic of Mundell (1961). What is new to our result is that it not only identifies the cases when the goods-market costs of a fixed exchange rate are low, but it also emphasizes the benefits of a fixed exchange rate from the perspective of the financial market. These benefits include reduced financial volatility and improved risk sharing between member countries.

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28 No divine coincidence emerges when monetary policy is constrained and the planner can only choose FX interventions. This is because FXI have only indirect effect on output gap with the path of $x_t$ determined by the response of monetary policy to movements in $z_t$ (and/or $e_t$). Under the zero lower bound (ZLB), the path of $x_t$ is determined by the Euler equation $E_t \{ \Delta x_{t+1} + \Delta \tilde{c}_{Nt+1} \} = 0$ and is independent from $z_t$, while in the case of a currency union, carry trade is risk free and government interventions crowd out positions of arbitrageurs without any effect on risk premia or allocations (cf. elimination of the aggregate demand externality with capital controls in Farhi and Werning 2016). In our baseline model of the goods market, an example with a stable natural real exchange rate is $i_t^* = 0$ (i.e., $R_t^* = 1/\beta$), and $y_{yt} = a_t$ and both follow the same random walk. In this case, $\tilde{q}_t = a_t - \tilde{c}_{yt} = 0$, as $\tilde{c}_{yt} = y_{yt}$. Note that Proposition 2 generalizes to any deterministic path of the natural real exchange rate without unexpected surprises, that is any $\tilde{q}_t = E_{t-1} \tilde{q}_t$, so that targeting $e_t = E_{t-1} \tilde{q}_t$ maintains $\tilde{\sigma}_t^2 = \text{var}_t(\Delta x_{t+1}) = 0$. 29
benefits are larger the more the member countries trade with each other, as captured by the openness weight $\gamma$ in the welfare loss function (10). Furthermore, we expect a fixed exchange rate — or a formation of the currency union — to dominate the alternative of an unmanaged (free) float, when the volatility of the bilateral nominal exchange rate under the float is dominated by non-fundamental currency demand shocks $n^*_t$ relative to fundamental macro-trade shocks $\tilde{q}_t$, provided that $\gamma$ is sufficiently large (i.e., the exchange rate volatility is consequential for aggregate allocations).

3.3 Crawling peg

Proposition 1 suggests that it is generally optimal to combine conventional monetary policy with FX interventions. However, in practice, it is not uncommon for countries to abstain from using FXI. This may be due to incomplete information about shocks and optimal targets in the currency market, as we discuss below in Section 3.4. It also may reflect additional constraints on the central bank’s balance sheet making it costly to intervene when FX reserves are too low or too high (Krugman 1979, Amador, Bianchi, Bocola, and Perri 2016). In both cases, the central bank is prone to negative valuation effects, which can undermine its credibility and lead to a loss of independence. Thus, we now study general implications of Theorem 1 for the optimal monetary and exchange rate policy away from the first best, when FX interventions follow an arbitrary given path $\{f^*_t\}$, including $f^*_t = 0$ as one possibility.

**Discretionary monetary interventions** Before turning to the discussion of Ramsey-optimal policy, we first consider briefly the effects of discretionary ex post monetary interventions to stabilize the exchange rate and capital flows. We find that, without commitment, the optimal policy is always inward looking and focuses exclusively on the output gap. This is the case, even though, according to Theorem 1, an inward-looking monetary policy is generally suboptimal, provided there are departures from the first-best risk sharing. Ex post discretionaty monetary interventions are allocative in the goods market and, in particular, affect the exchange rate. However, they cannot improve the allocation in the financial market — that is, they neither prevent capital outflows, nor improve international risk sharing or eliminate UIP wedges.

**Proposition 3** Without commitment, the optimal discretionary monetary policy stabilizes the output gap, $x_t = 0$. Discretionary ex post interventions that depart from $x_t = 0$, affect the exchange rate $e_t$, but do not change capital flows, UIP deviations or the risk-sharing wedge $z_t$.

Proposition 3 shows that, in general, the optimal discretionary monetary policy should not respond to the exchange rate, capital flows, or risk sharing wedges. To see the intuition, consider an ex post monetary tightening and an associated reduction in output $x_t$ done in response to a capital outflow

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30This insight is consistent with the experience of the Euro Zone, where the cost of borrowing was harmonized across countries and the cross-country financial flows increased significantly since the introduction of the euro in 1999 (see the discussion of the reduced borrowing wedge in Blanchard and Giavazzi 2002). Of course, an alternative interpretation is that these capital flows were excessive and driven by inefficient risk pricing of borrowing in Southern Europe, a case that may also arise in our model environment augmented with a possibility of default on net foreign liabilities (see also Fornaro 2021).

31Two types of constraints are a lower bound of FX reserves, $f^*_t \geq 0$, or a value-at-risk constraint on the balance sheet, $\bar{\sigma}_t^2 \cdot (f^*_t)^2 \leq M$, where $\bar{\sigma}_t^2 \cdot (f^*_t)^2$ is the variance of returns on the gross position of the central bank.
shock — namely, an increase in currency demand $n_t^*$ in (13) resulting in $z_t < 0$ and an exchange rate depreciation. Monetary tightening with $x_t < 0$ leads to an appreciation of the exchange rate $e_t$, offsetting the effect of $z_t < 0$, and an expenditure switching away from home non-tradables in the goods market, according to the equilibrium condition (12).

This might be misleading for a policy success to fend off the capital outflow shock. However, this outcome results only in costs in the goods market ($x_t < 0$) and no benefits in the financial market (as $z_t < 0$ still). Indeed, the equilibrium in the financial market, and in particular the path of the risk-sharing wedge $z_t$, is characterized by (11) and (13), which remain unaffected by a discretionary monetary tightening. Neither the size of the capital outflow, $n_t^* + f_t^* - b_t^*$, nor the unit price of risk $\bar{\omega} \bar{\sigma}_t^2$ in (13) respond to post monetary tightening. Discretionary policy affects the path of $e_t$ and $\mathbb{E}_t \Delta e_{t+1}$, but it has no affect on $e_t + 1 - \mathbb{E}_t e_{t+1}$, and thus leaves the expected conditional volatility of the exchange rate, $\bar{\sigma}_t^2$, unchanged. As a result, the size of the UIP deviation (14) at $t$ is also unaffected, emphasizing the futility of discretionary monetary interventions to manage capital flows.\footnote{The result that monetary policy has no effect on capital flows whatsoever relies on the assumption that preferences are separable in tradables and non-tradables and no foreign intermediates are used in production. Despite being a special case, this provides a benchmark that illustrates the limited capacity of conventional monetary policy in capital flow management. Note that this result also generalizes to the case where noise shocks $n_t^*$ are partially elastic to the expected UIP deviation, as we show that it remains unchanged in equilibrium.}

**Commitment to a crawling peg** We now return to Theorem 1, and consider the case when FXI do not ensure the first-best risk sharing ($n_t^* + f_t^* - b_t^* \neq 0$) and divine coincidence does not apply ($\bar{\omega}_t \neq \bar{\omega}$), but the monetary authority can commit to a policy rule to respond to exchange rate surprises conditional on the state of the economy. In other words, we step outside of the special cases considered in Propositions 1–3, and study the general implications of the Ramsey-optimal monetary policy in an open economy, $\gamma > 0$, summarized in (16).

For simplicity, we consider first a one-time deviation from the first-best FXI, and show that both a pure peg ($e_t = \bar{e}$), and a pure float (corresponding to $x_t = 0$) are, in general, suboptimal. Instead, the optimal monetary policy has a structure of a managed float, or a crawling peg, with $\delta_t \in (0, \infty)$ in (16). That is, the policymaker commits to respond with monetary interventions $x_t$ to smooth out surprise changes in the exchange rate, $e_t - \mathbb{E}_{t-1} e_t$, and in particular tighten monetary policy to accommodate depreciation shocks. We prove in Appendix A3:

**Proposition 4** Consider a case in which FXI $\{f_t^*\}$ are unconstrained in every period but $t$. Then $x_t = 0$ in every period except $t + 1$, where:

$$x_{t+1} = -\frac{2\gamma \bar{\omega}}{1 - \gamma} \frac{\bar{\omega} \bar{\sigma}_t^2}{1 + \beta + \bar{\omega} \bar{\sigma}_t^2} (n_t^* + f_t^* - b_t^*)^2 (e_{t+1} - \mathbb{E}_t e_{t+1}),$$

(17)

which corresponds to the general optimality (16) with the Lagrange multiplier on the risk-sharing constraint (13) proportional to the UIP deviation, $\mu_t = (1 + \beta + \bar{\omega} \bar{\sigma}_t^2)^{-1} \mathbb{E}_t \Delta z_{t+1}$.

To see the intuition, consider a state of the world with non-zero intermediated capital flows and a binding risk-sharing condition (13), so that $\mu_t (n_t^* + b_t^* - f_t^*) \neq 0$. As discussed above, adjusting
monetary policy in period $t$ does not affect contemporaneous capital flows. Instead, the policymaker can only indirectly mitigate the risk-sharing wedge by encouraging arbitrageurs to take larger positions and lowering the required risk premium. Monetary policy achieves this by leaning against surprise exchange rate innovations at $t + 1$ and lowering the perceived conditional variances of the exchange rate, $\hat{\sigma}_t^2 = \text{var}_t(\Delta e_{t+1})$. This makes financial intermediation less risky and relaxes the risk-sharing constraint (13). In particular, this implies that an unexpected depreciation, $e_{t+1} > \mathbb{E}_t e_{t+1}$, requires a monetary tightening that results in an output gap, $x_{t+1} < 0$. Importantly, this commitment does not depend on the source of volatility in the exchange rate at $t + 1$—namely, whether exchange rate surprises are driven by financial noise shocks $n_{t+1}^\ast$ or fundamental macro shocks $\tilde{q}_{t+1}$. Thus, optimal monetary policy is no longer inward-looking and limits the free float of the exchange rate.

Proposition 4 has several important implications. First, the optimal monetary policy always stabilizes the expected output gap, $\mathbb{E}_t x_{t+1} = 0$, irrespective of the path of the exchange rate $e_t$ and the risk-sharing wedge $z_t$. Symmetrically, any expected changes in the exchange rate, $\mathbb{E}_t \Delta e_{t+1}$, do not require accommodation with a monetary policy response. In other words, it is only exchange rate surprises, $e_{t+1} - \mathbb{E}_t e_{t+1}$, that require a policy response. Therefore, the optimal policy rule has the structure of a crawling peg—it fully allows for expected exchange rate adjustment and responds only to unexpected exchange rate movements. The implication is that any medium-run exchange rate adjustment can be accommodated with expected exchange rate changes, a managed float, without resulting in welfare costs in goods or financial markets.

Second, optimal monetary interventions in the currency market are state-contingent. Exploiting non-linearity allowed by our approximation, Proposition 4 shows that the intensity of the optimal policy lean $\delta_t$ increases with the size of the frictional UIP wedge (13) and the size of the capital (out)flow shock. A constant-intensity policy rule, $\delta_t \equiv \delta$, is feasible, and results in a constant conditional exchange rate volatility, $\sigma_t^2 = \hat{\sigma}_t^2$. However, it is suboptimal and dominated, in the context of Proposition 4, by a state-contingent policy rule with a policy lean:

$$\delta_t = \frac{2\gamma \bar{\omega} \sigma_t^2}{1 - \gamma (1 + \beta + \bar{\omega} \sigma_t^2)} (n_t^\ast + f_t^\ast - b_t^\ast)^2,$$

which is both increasing in the unit price of exchange rate risk, $\bar{\omega} \sigma_t^2$, and increasing and convex in the size of unaccommodated capital outflow shocks, $|n_t^\ast + f_t^\ast - b_t^\ast|$. Recall from (13)–(14) that the size of the frictional UIP deviation is given by $\mathbb{E}_t \Delta z_{t+1} = \bar{\omega} \sigma_t^2 (n_t^\ast + f_t^\ast - b_t^\ast)$, and thus the optimal policy lean is quadratic in the UIP deviation.

It follows that the crawling peg is more relevant for countries with a larger tradable sector $\gamma$ and, thus, higher welfare costs of capital flow shocks. Furthermore, periods with larger expected exchange

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33 Substituting (16) into (12) and using $\mathbb{E}_t x_{t+1} = 0$ yields $e_{t+1} - \mathbb{E}_t e_{t+1} = \frac{1}{1 + \bar{\omega}} [(\hat{q}_{t+1} - \mathbb{E}_t \hat{q}_{t+1}) - (z_{t+1} - \mathbb{E}_t z_{t+1})]$, which shows how the optimal policy lean $\delta_t > 0$ dampens equally the exchange rate surprises from $\hat{q}_{t+1}$ and $z_{t+1}$.

34 Some oil exporting countries, such as Saudi Arabia, follow this type of exchange rate policy, in parallel accumulating an FX sovereign wealth fund for the future when global demand for oil declines. If the present value of all future oil revenues were known, this would indeed be the first best policy.

35 In this class of policies, one can optimize over $(\delta, \sigma^2)$ to show that $\delta$ is increasing in openness $\gamma$ and in the ratio of volatilities of noise $n_t^\ast$ to fundamental $\hat{q}_t$ shocks.
rate volatility, $\bar{\sigma}_t^2$, and larger excess demand or supply of currency that requires intermediation, $|n_t^* + f_t^* - b_t^*|$, call for a commitment to a stronger future response of monetary policy, $x_{t+1}$, to unexpected exchange rate movements, $e_{t+1} = E_t e_{t+1}$. This suggests a state-contingent policy approach to financial market volatility, which can be ignored when it causes no spikes in risk premia (intermediation wedges), but should be smoothed out with monetary policy when such volatility distorts risk sharing and direct financial market interventions (FXI) are limited.

### 3.4 Optimal FXI and forward guidance

We have focused so far on cases when the first-best allocation is implementable in all but perhaps one period, which significantly simplifies the analysis and allows us to solve for the optimal policy rule in Proposition 4. More generally, Theorem 1 shows that the policy depends on the history of previous shocks as well as expectations about their future realizations as summarized by the endogenous Lagrange multiplier $\mu_t$. While no closed-form solution is available in the general case, we provide here a further characterization of the second-best optimal policies.

When FXI are unconstrained in period $t$, the risk-sharing constraint (13) is not binding, and therefore $\mu_t = 0$.\(^{36}\) From Theorem 1, this implies that monetary policy at $t+1$ can focus sole on closing the output gap, $x_{t+1} = \delta_t = 0$, irrespective of binding risk-sharing constraints in any other periods. Even in this case, however, optimal FXI at $t$ do not necessarily just offset currency demand shocks ($f_t^* = -n_t^*$) or close the UIP deviation ($E_t \Delta z_t = 0$). Indeed, facing occasionally binding FX constraints, the planner can improve the allocation at $t - j$ by offering forward guidance using future FXI $f_t^*$, or accumulate FX to alleviate future binding constraints at $t + j$, without the need to distort monetary policy.

Similarly to conventional monetary guidance, the planner exploits the fact that $z_t$ is a forward-looking variable and depends on its own future expectation, $E_t z_{t+1}$, as a result of consumption smoothing of imports.\(^{37}\) In addition, and differently from a conventional forward guidance, future FXI can also lean against the exchange rate surprises, stabilizing $z_{t+1}$ around the same mean $E_t z_{t+1}$, to reduce the exchange rate volatility $\bar{\sigma}_t^2$ and the resulting UIP deviations at $t$. Formally, we can rewrite the risk-sharing condition (13) as:

$$z_t = E_t z_{t+1} - \bar{\omega}_t \bar{\sigma}_t^2 (n_t^* + f_t^* - b_t^*) \quad \text{with} \quad \bar{\sigma}_t^2 = \text{var}_t (\bar{q}_{t+1} + x_{t+1} - z_{t+1}) .$$

When $f_t^*$ is constrained, forward guidance using $f_{t+1}^*$ impacts both $E_t z_{t+1}$ and $\bar{\sigma}_t^2$ via $z_{t+1} - E_t z_{t+1}$, thus affecting $z_t$. Commitment to future interventions that lean against the wind — especially, in periods of large shocks — mitigates the deviation from the optimal risk sharing and reduces the size of required interventions today.\(^{38}\)

We now provide a general characterization for the optimal path of FXI and UIP deviations away from

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\(^{36}\)In fact, $\mu_t = 0$ is the first order condition with respect to $f_t^*$ when it is unconstrained.

\(^{37}\)Consider a capital outflow shock, $n_t^* > 0$, that depresses consumption of tradables $z_t$. When FXI $f_t^*$ are constrained at $t$, a planner can promise not to accommodate future capital inflows, $n_{t+j}^* < 0$, resulting in increased $z_{t+j}$ and consequently also propped-up $z_t$ (cf. Werning 2011, Fanelli and Straub 2021).

\(^{38}\)This mechanism can also rationalize the signaling channel of FXI, which is often viewed by central bankers as the key channel of transmission in currency markets (see Patel and Cavallino 2019).
the first best, which complements the characterization of the optimal monetary policy in Theorem 1.

**Theorem 2** For any path of monetary policy \( \{x_t\} \), and with occasionally binding constraints on FXI, the optimal UIP deviation at \( t \) is given by:

\[
\mathbb{E}_t \Delta z_{t+1} = \left(1 + \beta + \bar{\omega} \sigma_t^2 \right) \mu_t - \beta \mathbb{E}_t \mu_{t+1} - \left[1 + 2 \bar{\omega} (n_t^* + f_t^* - b_t^*) (e_t - \mathbb{E}_{t-1} e_t) \right] \mu_{t-1},
\]

and it is supported by an FX intervention \( f_t^* \) that satisfies (13), that is \( \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*) = \mathbb{E}_t \Delta z_{t+1} \).

Optimality condition (18) can be read in two ways. First, it determines the optimal UIP deviation, \( \mathbb{E}_t \Delta z_{t+1} \), given the path of Lagrange multipliers \( \{\mu_t\} \). Conversely, it is a dynamic equation determining the path of \( \mu_t \) given equilibrium UIP deviations. Furthermore, conditions (16) and (18) in Theorems 1 and 2, together with constraints (11)–(13), fully describe the solution \( \{x_t, f_t^*, z_t, b_t^*, e_t, \mu_t\} \) to the Ramsey-optimal policy problem given constraints on the path of policies \( \{x_t, f_t^*\} \). In particular, either \( f_t^* \) is constrained at \( t \) or \( \mu_t = 0 \), with complementary slackness.\(^\text{39}\)

Even though unconstrained FXI at \( t \) implies \( \mu_t = 0 \), such interventions do not always eliminate UIP deviations at \( t \), and thus do not necessarily offset currency demand shocks, \( f_t^* \neq b_t^* - n_t^* \), unlike in the first best allocation. Indeed, UIP is optimally distorted, \( \mathbb{E}_t \Delta z_{t+1} \neq 0 \), if either \( \mu_{t-1} \neq 0 \) or \( \mathbb{E}_t \mu_{t+1} \neq 0 \). The effect of \( \mathbb{E}_t \mu_{t+1} \) in (18) is a macroprudential FXI at \( t \) in expectation of the future binding constraint at \( t + 1 \). An expected distortionary capital outflow, \( \mathbb{E}_t \{\mathbb{E}_{t+1} \Delta z_{t+2}\} > 0 \), calls for an earlier intervention that results in \( \mathbb{E}_t \Delta z_{t+1} < 0 \) and, hence, \( f_t^* < b_t^* - n_t^* \), more than offsetting the currency demand shock at \( t \).

The effect of \( \mu_{t-1} \) in (18) captures the forward guidance motive at \( t \) to alleviate the previous-period UIP deviation at \( t - 1 \), and this requires commitment. The two terms in the bracket in front of \( \mu_{t-1} \) reflect, respectively, the conventional forward guidance motive and the additional channel aimed at reducing \( \sigma_{t-1}^2 \). Conventional forward guidance requires supplying more FX to the market (i.e., selling reserves), \( f_t^* < b_t^* - n_t^* \), to induce \( \mathbb{E}_t \Delta z_{t+1} < 0 \) after a period with an unaccommodated capital outflow, \( \mathbb{E}_{t-1} \Delta z_t > 0 \). The additional channel uses FXI \( f_t^* \) to lean against exchange rate surprises, \( e_t - \mathbb{E}_{t-1} e_t \), in the same way the optimal monetary policy does it according to Theorem 1. Specifically, an unexpected devaluation, \( e_t > \mathbb{E}_{t-1} e_t \), following a capital outflow at \( t - 1 \) must be partially offset with an additional sales of reserves, \( f_t^* \downarrow \). Note that the last term in (18) is proportional to \( x_t = -\delta_{t-1} (e_t - \mathbb{E}_{t-1} e_t) \) in (16), aligning the additional UIP deviation at \( t \) with the optimal output gap \( x_t \), both leaning against exchange rate surprises at \( t \).

Finally, when FXI are constrained at \( t \), the UIP deviation is determined by the financial market equilibrium condition (13), and (18) determines the corresponding value of \( \mu_t \). For example, when FXI are unconstrained both at \( t - 1 \) and \( t + 1 \), \( \mu_{t-1} = \mu_{t+1} = 0 \), we have that

\[
\mu_t = \frac{\bar{\omega} \sigma_t^2}{1 + \beta + \bar{\omega} \sigma_t^2} (n_t^* + f_t^* - b_t^*),
\]

\(^{39}\)Note that (11), (13) and (18) are three dynamic equations for the state variable \( b_t^* \), the jump variable \( z_t \), and the co-state variable \( \mu_t \), respectively; the three boundary conditions are \( b_{t-1}^* = 0 \), \( \lim_{t \to \infty} \beta^t b_t^* = 0 \), and \( \mu_{t-1} = 0 \).
Figure 2: Optimal capital flows and output gaps

Note: The figure plots the optimal allocation \( \{x_t, z_t\} \) when FXI are unconstrained in all periods but \( t \), i.e. \( \mu_{t+j} = 0 \) for all \( j \neq 0 \), and there is an expected unaccommodated capital outflow at \( t \), \( \mathbb{E}_t \Delta z_{t+1} = \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*) > 0 \). Left panel describes the case when capital outflow is expected, \( \mathbb{E}_0 n_t^* = n_t^* > 0 \), and there are no further shocks. Right panel considers the case where \( n_t^* > \mathbb{E}_{t-j} n_t^* > 0 \) and \( \epsilon_{t+1} \neq \mathbb{E}_t \epsilon_{t+1} \), requiring the response of \( (x_{t+1}, z_{t+1}) \) to the exchange rate surprise.

and \( \mathbb{E}_t \Delta z_{t+1} = (1 + \beta + \bar{\omega} \sigma_t^2) \mu_t \), consistent with Proposition 4. The terms \( \mu_t \) and \( \beta \mu_t \) in the period \( t \) UIP deviation induce the forward guidance and the macroprudential motives at \( t+1 \) and \( t-1 \), respectively. The remaining term, \( \bar{\omega} \sigma_t^2 \mu_t \), reflects the additional incentive to reduce \( z_t \), and hence also \( z_{t-j} \) for \( j > 0 \), in order to accumulate net foreign assets \( b_t^* \) via trade surpluses in (11) and ease the binding currency demand at \( t \). We illustrate all these effects in Figure 2 using an example with a full analytical solution provided in Appendix A3.

From the discussion above, an unconstrained use of FXI at \( t \) does not aim to just eliminate UIP deviations at \( t \), but also to smooth out UIP violations in previous and future periods, according to the optimality condition (18). Nonetheless, this does not imply that the optimal policy in any given period depends on the entire sequence of past and future binding constraints. Perhaps surprisingly, we show next that the Ramsey policy features both optimal amnesia and optimal myopia. For the purpose of this result, we describe the optimal monetary policy (16) in terms of its lean \( \delta_t \) against exchange rate surprises.

**Proposition 5**

(i) If FXI are unconstrained at \( t-1 \), \( \mu_{t-1} = 0 \), then the optimal policy \( \{\delta_{t+j}, f_{t+j}^*\}_{j \geq 0} \) does not depend on the previous history ("amnesia"). (ii) If, in addition, \( \mu_t = 0 \), then the optimal policy at \( t \), \( (\delta_t, f_t^*) \), is the same under commitment and under discretion. (iii) If FXI are also expected to be unconstrained on average at \( t+1 \), \( \mathbb{E}_t \mu_{t+1} = 0 \), the optimal policy closes both the output gap and the UIP deviation, \( \delta_t = \mathbb{E}_t \Delta z_{t+1} = 0 \), irrespective of any future shocks and binding constraints ("myopia").

This proposition has a number of implications. Unlike with the output gap (Theorem 1), it is only optimal to eliminate the period \( t \) UIP deviation if FXI are unconstrained simultaneously at \( t-1, t \) and \( t+1 \). Otherwise, risk sharing at \( t \) is distorted due to either current, past or future shocks and a limited ability to offset them with FXI. However, no past shocks and frictions — summarized by \( \{\mu_{t-j}\}_{j \geq 0} \) —
matter at $t$ if FXI is unconstrained at $t - 1$ and, hence, $\mu_{t-1} = 0$. Therefore, the optimal FXI exhibit memory loss after the first unconstrained state is reached. Similarly, no future shocks and binding constraints, $\{\mu_{t+j}\}_{j>0}$, matter for the optimal intervention at $t$ if FXI are unconstrained at $t + 1$ and, hence, $\mu_{t+1} = 0$.

Why is this the case? For concreteness, consider that risk sharing was distorted in the past, $\mu_{t-j} \neq 0$ for some $j > 0$. Unconstrained FXI at $t$ does not necessarily mean that UIP holds at $t$, but instead UIP is optimally distorted at $t$ to absorb all intertemporal spillovers from earlier risk-sharing wedges, thus erasing history for future periods. Unlike with monetary guidance, full FXI guidance is achieved in one unconstrained period because the use of reserves introduces no distortions (like output gaps) and comes at no cost in the optimal allocation.\footnote{The use of reserves is costly, resulting in income losses, if the government it trying to shift the path of the exchange rate away from the equilibrium in which UIP holds, as in Jeanne (2012) or Amador, Bianchi, Bocola, and Perri (2019).}

If past shocks are sufficiently large and/or FX reserves are still insufficient at $t$, the accumulated distortions persist beyond period $t$ into the future, until an unconstrained state is reached. For example, a large and persistent currency demand shocks, $n^*_t > 0$, that cannot be accommodated with sufficient FXI, $\{f^*_{t-j}, \ldots, f^*_t\}$, results in UIP deviations, depressed imports $z_t$ and accumulation of net foreign assets $b^*_t$ until an unconstrained state is reached. A reversal in the currency demand shock to $n^*_t < 0$ may bring the timing of the unconstrained state closer.

Similar logic applies to future expected shocks and binding risk sharing constraints, which trigger an early use of FXI and backward spillovers into anticipatory UIP deviations when current FX reserves are insufficient to fully offset or stop the propagation of the shock. Nevertheless, there is an important asymmetry between past and future shocks as only the former ones require commitment. Indeed, without commitment, the government cannot fulfill its past promises, and the optimal policy is generally not time consistent at $t$, unless $\mu_{t-1} = \mu_t = 0$. In contrast, the optimal policy response to future shocks is generally time consistent even when FXI are expected to be constrained in the next period, $\mu_{t+1} \neq 0$.

From a practical perspective, Proposition 5 suggests that while central banks need to use FXI and monetary policy to respond to past shocks, the history-dependence is possibly short-lived and disappears as soon as the constraint on the interventions is no longer bindings. At the same time, there is a role for macroprudential policy of accumulating FX reserves and net foreign assets when there is a possibility of a future binding constraint, and this is especially the case when the government lacks commitment to an effective forward guidance.

4 \ Extensions

4.1 \ International Transfers and Capital Controls

This section generalizes the baseline model to feature capital control taxes and international wealth transfers due to valuation effects on cross-border asset holdings. The goal of this analysis is twofold. First, we study whether capital controls can substitute for other policy instruments when the latter are constrained. Second, we explore the optimal policy mix in the presence of cross-border rents from international currency provision.
Towards these goals, we extend the financial sector to additionally feature foreign financial actors, both liquidity traders and intermediaries. Specifically, the aggregate liquidity demand for currency originates from both domestic and international noise traders, \( N^*_t = N^*_{Ht} + N^*_{Ft} \). There are also domestic and foreign intermediaries — of measure \( m_H \) and \( m_F \), respectively — that supply currency \( D^*_Ht \) and \( D^*_Ft \) according to a portfolio choice rule similar to (4). Appendix A4 contains a detailed description of the environment and derivations, while this section outlines the results.

We further allow for a rich set of taxes. In particular, assume that domestic households face a tax \( \tau^h_t \) on their home-currency deposits, so that the after-tax return on their asset position is \( R^*_h t / (1 + \tau^h_t) \). Domestic financial agents — both noise traders and intermediaries — are subject to a pair of taxes \((\tau^H_t, \tau^F_t)\) on their home-currency and foreign-currency positions, respectively. As a result, their after-tax carry-trade return is given by \( \tilde{R}^*_H t+1 = \frac{R^*_H t}{1+\tau^H_t} - \frac{R^*_H t}{1+\tau^H_t} \xi^t. \) In contrast, foreign financial agents face only a tax \( \tau^F_t \) on their home-currency position, resulting in an after-tax carry-trade return \( \tilde{R}^*_F t+1 = \frac{R^*_F t}{1+\tau^F_t} - \frac{R^*_F t}{1+\tau^F_t} \xi^t. \) No other asset holdings are in the domestic policymaker’s tax jurisdiction, and in particular she cannot tax either foreign households or the foreign-currency positions of foreign traders. Note that, without loss of generality, we interpret \((\tau^H_t, \tau^F_t)\) as taxes on cross-border asset positions, that is capital controls.

The presence of foreign financial agents results in an international wealth transfer from incomes and losses on their carry trade positions. Specifically, \( \tilde{R}^*_F t+1 \cdot \frac{N^*_F + D^*_F}{R^*_F t} \) is the income transfer from home to the rest of the world, and it is subtracted from the home country budget constraint (6).

Furthermore, asset taxes introduced above affect the equilibrium risk-sharing condition (7). We show in the appendix that the generalized risk-sharing condition with asset taxes can be written as:

\[
\beta R^*_t \xi^t \frac{C_{R_t}}{C_{R_t+1}} \frac{1 + \tau^h_t}{1 + \tau^F_t} + \frac{\omega \sigma^2_t}{(1 + \tau^F_t)^2} \frac{B^*_t - N^*_t - F^*_t}{R^*_t},
\]

where \( \tau_t \) can represent one of two capital control policies: (i) a common tax on home-currency asset holdings for both home and foreign financial agents, \( \tau^H_t = \tau^F_t = \tau_t \), or (ii) a capital control tax on inflows and a subsidy on outflows, \( \tau^F_t = \frac{-\tau^H_t}{1+\tau^H_t} = \tau_t \). Note that these capital control policies work irrespectively of the composition of home and foreign intermediaries and noise traders. The other equilibrium conditions remain unchanged.

The generalized risk-sharing condition (19) clarifies two important properties of capital controls. Theoretically, capital control tax \( \tau_t \) and quantity interventions in FX markets \( F^*_t \) are substitutes and can be used interchangeably to offset the effect of liquidity shocks \( N^*_t \) on macroeconomic allocations and risk sharing. In particular, there are three ways in which asset taxes can be used to offset the distortionary effect of a capital outflow shock, \( N^*_t > 0 \):

(i) with a savings tax on households, \( \tau^h_t > 0 \), which encourage tradable consumption despite depreciated exchange rate (see the analysis of financial repression in Itskhoki and Mukhin 2022);

Note that, while \( N^*_F \) is an exogenous asset position (liquidity shock), the position of intermediaries depends on the expected carry-trade return, \( \hat{D}^*_F t = \frac{\xi^t (\theta^t + \tilde{R}^*_F t+1)}{\omega \sigma^2_t (1 + \tau^F_t)} \). Thus, arbitrageurs invest in a portfolio with a positive expected return and make positive expected profits. Therefore, rents can be extracted systematically only from positions against noise traders.
(ii) a home-currency investment subsidy for an entire financial sector, \( \tau_{Ht} = \tau_{Ft} = \tau_t < 0 \), which allow the financial sector to collect the carry trade return and accommodate the currency demand shock without distorting the household risk sharing condition;

(iii) a capital controls policy, taxing foreign-currency positions of domestic agents, \( \tau^*_H = \frac{-\gamma}{1+\gamma} \), and subsidizing home-currency positions of foreigners, \( \tau_{Ft} = \tau_t < 0 \), again resulting in a positive carry trade return for the financial agents without distorting the household risk sharing.

The feature of all three policies is that they generate a wedge in returns between households and financial sector, decoupling carry trade returns from the risk-sharing wedge.

In practice, however, the use of capital controls is complicated by the need to set state-contingent tax rates that vary significantly over time. Furthermore, the planner may be unable to distinguish between different types of agents and capital flows to impose agent- and asset-specific capital controls (Rebucci and Ma 2020). For example, condition (19) illustrates that setting a uniform tax on home bonds for all agents, \( \tau^*_t = \tau_{Ht} = \tau_{Ft} = \tau_t \), does not have the desired effect, as it is equivalent to a shift in the home-currency nominal interest rate \( R_t \) that cannot offset capital outflow shocks (recall Proposition 3). Finally, capital control policies generally require subsidies and are not budget-balanced, unlike optimal FXI that by construction generate revenues on average.

With these caveats in mind, we proceed with the analysis of optimal policies specializing to the case of capital controls on international inflows and outflows, \( \tau_{Ft} = \frac{-\gamma}{1+\gamma} \), without the use of domestic asset taxes, \( \tau^*_t = \tau_{Ht} = 0 \). We follow the same approach as in Section 2.2 and approximate the policy problem around the planner’s allocation, which now takes into account the possible rent extraction from foreign traders. We denote with \( \psi_t \equiv i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} + \tau_t \) the expected carry trade return for both home and foreign investors, which also equals the after-tax UIP deviation. We have the following two results that characterize equilibrium risk-sharing and international transfers. First, the generalized risk-sharing condition (13) is now:

\[
\mathbb{E}_t \Delta z_{t+1} = \psi_t - \tau_t, \quad \text{where} \quad \psi_t = \bar{\omega}^2 \sigma^2_t (n^*_t + f^*_t - b^*_t) \quad \text{and} \quad \bar{\sigma}^2_t = \text{var}_t(e_{t+1}). \tag{20}
\]

Capital controls \( \tau_t \) decouple the household risk-sharing wedge \( z_t \) dynamics from the risk premium \( \psi_t \) that intermediaries charge to accommodate international capital flows. In other words, while capital controls can substitute for FXI \( f^*_t \) to eliminate the risk-sharing wedge, they do so without affecting the equilibrium risk premium. Similarly to the domestic interest rate, \( R_t \), changes in capital controls \( \tau_t \) are absorbed by expected depreciation \( \mathbb{E}_t \Delta e_{t+1} \) and do not affect the expected after-tex carry-trade returns \( \psi_t \), which are determined by the balance of supply and demand in the currency market.

Second, the expected transfer abroad of carry-trade incomes and losses is given by \( \beta \left( \frac{m_F}{\bar{\omega}^2_t} \psi_t - n^*_t \right) \psi_t \), where \( m_F \) is the share of foreign intermediaries and \( n^*_t = N^*_{Ft}/\bar{Y}_T \) is the foreign noise trader demand shock normalized by tradable output. This leads to the following second-order approximation to the welfare loss function around the planner’s allocation:

\[
\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ (1 - \gamma)x_t^2 + \gamma z_t^2 + 2\beta\gamma \left( \frac{m_F}{\bar{\omega}^2_t} \psi_t - n^*_t \right) \psi_t \right]. \tag{21}
\]
The policymaker chooses the path of policies \( \{x_t, f_t^*, \tau_t\} \) and equilibrium outcomes \( \{z_t, b_t^*, \psi_t\} \) to minimize (21) subject to (20), as well as the original constraints (11) and (12). In addition to minimizing the loss from the output gap and the risk-sharing wedge, the objective now also includes minimizing transfers abroad to foreign traders.\(^{42}\)

The policymaker now has access to three instruments — capital controls \( \tau_t \), in addition to monetary policy \( x_t \) and FXI \( f_t^* \) — matching the three policy targets. The optimal monetary policy still closes the output gap \( x_t = 0 \). Furthermore, it is still feasible to use FXI to eliminate the UIP wedge, \( \psi_t = 0 \), without recurring to capital controls, \( \tau_t = 0 \). However, this is desirable if and only if \( n_{F_t}^* = 0 \). In this case, no international rents can be generated in the financial market, and \( \psi_t = 0 \) ensures not just \( z_t = 0 \), but also no financial income loss to the foreign intermediaries.\(^{43}\)

More generally, in the presence of foreign liquidity demand for currency, \( n_{F_t}^* \neq 0 \), it is no longer optimal to fully accommodate the entire currency demand \( n_t^* \) with FX interventions, as in Proposition 1. Instead, capital controls are used to eliminate the risk sharing wedge in (20), \( \tau_t = \psi_t \), which ensures \( z_t = 0 \) independently of the equilibrium UIP deviation \( \psi_t \). The maximum government revenues from interventions are attained by ensuring that \( \psi_t = \frac{\bar{\sigma}^2}{2m_F} n_{F_t}^* \), which is the peak of the rents term in (21). In turn, this requires that FX interventions fully satisfy the demand for currency of domestic noise traders and only partially for foreign noise traders, e.g. \( f_t^* = -n_{H_t}^* - \frac{1}{2} n_{F_t}^* \) when all intermediaries are foreign, \( m_F = 1 \). Collecting rents requires leaving positive carry trade returns on the table for the intermediaries (\( \psi_t \neq 0 \)), who in turn constrain the maximum rent extraction by the government.\(^{44}\)

**Proposition 6** Assume that intermediaries and noise traders are foreign agents, \( m_F = 1 \) and \( n_t^* = n_{F_t}^* \). The optimal policy closes the output gap with monetary policy, \( x_t = 0 \), partially offsets the demand of noise traders with FXI, \( f_t^* = -n_t^*/2 \), and eliminates the remaining risk-sharing wedge, \( z_t = 0 \), with capital controls, \( \tau_t = \psi_t = \frac{\bar{\sigma}^2}{2m_F} n_{F_t}^* / 2 \), without fully eliminating after-tax UIP deviations.

In the presence of foreign liquidity demand for foreign currency — whether \( n_{F_t}^* < 0 \) or \( n_{F_t}^* > 0 \) — a country can generate rents from the rest of the world by exploiting the monopoly power it has in supplying currency. Optimal FX interventions always lean against the wind of currency demand, but stop short of fully offsetting the liquidity demand of foreign noise traders, leaving the UIP premium partially open to ensure positive equilibrium rents. Echoing the recent experience of Switzerland, this result implies that a positive demand for home currency should be addressed by issuing reserves and accumulating assets in foreign currency, while simultaneously imposing capital controls or allowing the exchange rate to partially appreciate (Bacchetta, Benhima, and Berthold 2023).

\(^{42}\)Interestingly, to the second-order approximation, international rents depend only on expected returns, while ex-post valuation effects are of a higher order. As a result, the expression for transfers is largely isomorphic to the one in a deterministic case with CIP deviations replaced by expected UIP deviations (of Fanelli and Straub 2021). Furthermore, this implies that, given the structure of international asset markets and the order of approximation, the planner does not aim to use state-contingent valuation effects to ‘complete the markets’ (as in Fanelli 2017).

\(^{43}\)Note the quadratic term in \( \psi_t \) in the loss function (21), which is non-negative when \( n_{F_t}^* = 0 \). Much of the existing literature focuses on the case of \( n_{F_t}^* = 0 \) and foreign intermediaries, where FXI can generate no expected rents and are used for other purposes at the cost of income loss (Jeannie 2012, Amador, Bianchi, Bocola, and Perri 2019, Fanelli and Straub 2021).

\(^{44}\)Note that the elasticity of currency supply by foreign intermediaries is \( m_F \bar{\sigma}^2 \), exactly as it appears in the wealth transfer term in (21) and in the expression for the rent-maximizing UIP deviation \( \psi_t \).
4.2 International Cooperation

Up until now, we focused on the optimal policy in a small open economy that takes as given global economic conditions, in particular the world interest rate. This section studies international spillovers and the optimal policy coordination in a multi-country world. Towards this end, we consider a world comprised of a continuum of small open economies index by \( i \in [0, 1] \), each one isomorphic to the country in our baseline model, with country \( i = 0 \) (the US, denoted with \( * \)) issuing the global funding currency (the dollar). We denote with \( m_0 \geq 0 \) the measure of countries \( i \in (0, m_0] \) that form a dollar currency union (or dollar pegs), which in particular nests the case of a non-infinitesimal US economy when \( m_0 > 0 \). There is a global market for the tradable endowment good and a sticky-price non-tradable production sector in each economy, as in the baseline model. The law of one price still holds for tradables, and now we write it in logs as \( \ln E_t^{it} = 0 \), where \( E_t^{it} \) denotes the US dollar tradable inflation.

We make two assumptions about the structure of the asset market. First, only nominal dollar bonds are available for international risk sharing, which generates an asymmetry between the US and other economies. Second, for each currency there is a separate market, in which agents can trade it against the dollar. This segmentation of currency markets is in line with the fact that the dollar accounts for 88% of the global FX market turnover, but it is not crucial for our results which remain largely unchanged if one assumes that arbitrageurs can invest simultaneously in a portfolio of currencies. For simplicity, we assume local financial markets, as in the baseline model, to exclude the redistributive motive in national policies discussed in the previous subsection. Appendix A4 provides detailed derivations.

The equilibrium conditions for a given economy are the same as in the baseline model described in Section 2. Instead, the main difference is that the international real interest rate, \( r_t^i = i_t^i - \mathbb{E}_t \pi^*_t T_{t+1} \), is endogenous, and it is shaped by the global market clearing condition for tradables, \( \int_0^1 c_{it} d{i} = y_{IT_t} \), where \( y_{IT_t} \equiv \int_0^1 y_{it} d{i} \) is the aggregate global endowment of tradables at \( t \). We show in the appendix that, to the first order, the global planner that cannot directly redistribute wealth across countries equalizes the expected consumption growth, \( \mathbb{E}_t \Delta \hat{c}_{it+1} = \mathbb{E}_t \Delta y_{IT_t+1} \) for all \( i \in [0, 1] \), where \( \hat{c}_{it} \) is country \( i \) tradable consumption in the global planner’s allocation. Consequently, the real interest corresponding to the planners allocation equals the expected growth rate of the aggregate endowment, \( \hat{r}_t^i \equiv \mathbb{E}_t \Delta y_{IT_t+1} \).\(^{45}\) Recall that the local country \( i \) planner chooses \( \mathbb{E}_t \Delta \hat{c}_{it+1} = r_t^i \) taking \( r_t^i \) as given, whether or not it is equal to \( \hat{r}_t^i \), and therefore \( \hat{c}_{it} \) may differ from \( \hat{c}_{it} \).

Away from the optimal allocation, we define the output gap \( x_{it} \) as before, while the global risk-sharing wedge is now \( \hat{z}_{it} \equiv c_{it} - \hat{c}_{it} \), which differs from the local wedge \( z_{it} = c_{it} - \hat{c}_{it} \) when \( r_t^i \neq \hat{r}_t^i \). Interestingly, as our next Lemma shows, the UIP deviation (14) still corresponds to the local wedge, \( \psi_{it} \equiv \mathbb{E}_t \hat{z}_{it+1} \), while the global wedge additionally reflects the aggregate interest rate wedge, \( \bar{\psi}_t \equiv - (r_t^i - \hat{r}_t^i) \), where \( r_t^i \) is the equilibrium interest rate.\(^{46}\)

\(^{45}\)Under sticky prices, the global planner’s allocation is decentralized with the US monetary policy \( i_t^* = - \mathbb{E}_t a_{ot+1} \), where \( a_{ot} \) is the log non-tradable productivity in the US, expected tradable price inflation \( \mathbb{E}_t \pi^*_t T_{t+1} = \mathbb{E}_t (\Delta a_{ot+1} - \Delta y_{IT_t+1}) \), and nominal exchange rates \( e_{it} = a_{it} - \hat{c}_{it} - p^*_t T_{t} \) for \( i \in (0, 1] \), where \( p^*_t T_{t} = a_{ot} - \hat{c}_{ot} \).

\(^{46}\)The proof takes a first-order approximation to the risk-sharing condition for all \( i \), similar to (7), resulting in
Lemma 1 In an equilibrium with sticky prices and frictional financial intermediation, the aggregate welfare loss relative to the global planner’s allocation up to second order is given by:

$$\frac{1}{2} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \int_0^1 [\gamma \hat{z}_{it}^2 + (1 - \gamma) x_{it}^2] \, di,$$

and the global risk-sharing wedges $\hat{z}_{it}$ satisfy market clearing $\int_0^1 \hat{z}_{it} \, di = 0$ and risk-sharing conditions:

$$\mathbb{E}_t \Delta \hat{z}_{it+1} = \psi_{it} - \bar{\psi}_t \quad \text{for all } i \in [0, 1],$$

where $\psi_{it} \equiv \omega_i \sigma_{it}^2 (n_{it}^* + f_{it}^* - b_{it}^*)$ with $\sigma_{it}^2 \equiv \text{var}_t (e_{it+1})$ is the currency $i$ UIP wedge, and $\bar{\psi}_t \equiv \int_0^1 \psi_{it} \, di$ is the aggregate real interest rate wedge, $r_t^* - \hat{r}_t^* = -\bar{\psi}_t$.

The risk-sharing condition (A15) is the generalization of (13) which takes into account the endogeneity of the world interest rate $r_t^*$. As before, the risk sharing for country $i$ and hence UIP for currency $i$ is distorted when excess demand for the dollar relative to currency $i$, $n_{it}^* + f_{it}^* - b_{it}^*$ needs to be absorbed by the intermediaries, provided that $\omega_i \sigma_{it}^2 \neq 0$. The risk-sharing condition for the US, $i = 0$, as well as for all countries that peg to the dollar, $i \in (0, m_0]$, is therefore $\mathbb{E}_t \Delta \hat{z}_{it+1} = -\bar{\psi}_t$ with UIP satisfied, $\psi_{it} = 0$.

In addition, (A15) now features a global interest rate wedge $\bar{\psi}_t$ common for all countries, which arises due to unaccommodated shifts in the global demand for dollars, $\bar{n}_{it}^* \equiv \int_0^1 n_{it}^* \, di$. In particular, an increase in global dollar demand, $\bar{\psi}_t = \int_0^1 \omega_i \sigma_{it}^2 (n_{it}^* + f_{it}^* - b_{it}^*) \, di > 0$, depresses the world real interest rate below its efficient level $\hat{r}_t^*$. It also results in correlated UIP premia on non-dollar-pegged currencies, $i \in (m_0, 1]$, capital outflows from these countries, and depressed tradable consumption, $\hat{z}_{it} < 0$. The resulting global savings glut and the depressed world real interest rate $r_t^*$ result in suboptimal capital inflows and, hence, current account deficits in the US and dollar-pegged countries, $\hat{z}_{it} > 0$ for $i \in [0, m_0]$ (cf. Caballero, Farhi, and Gourinchas 2008, Mendoza, Quadrini, and Rios-Rull 2009).

A cooperative optimal policy minimizes the aggregate welfare loss (22) subject to market clearing, the risk-sharing conditions (A15), as well as the expenditure switching conditions

$$e_{it} = \hat{q}_{it} - p_{it}^* + x_{it} - \hat{z}_{it},$$

where $\hat{q}_{it} \equiv a_{it} - \hat{c}_{it}$, and the country budget constraints $\beta b_{it+1} = -\hat{z}_{it}$ for all $i \in [0, 1]$. This extends the non-cooperative policy problem of a small open economy (15), with (24) generalizing (12). Recall that the local country $i$ policymaker, if unconstrained, would optimally eliminate the UIP deviation, $\psi_{it} = 0$, taking $r_t^*$ as given (Proposition 1). However, such policy may not be optimal from the perspective of a global policymaker who takes into account the endogeneity of $r_t^*$.

We prove in the appendix that the cooperatively optimal unconstrained FX interventions in country $i$ ensure $\mathbb{E}_t \Delta \hat{z}_{it+1} = \psi_{it} - \bar{\psi}_t = 0$, which has the following immediate implications:

$$\mathbb{E}_t \Delta e_{it+1} = r_t^* + \psi_{it}.$$ Integrating across $i \in [0, 1]$ yields the solution for the world interest rate, $r_t^* = \mathbb{E}_t \Delta y_{T+1} - \bar{\psi}_t$, where $\mathbb{E}_t \Delta y_{T+1} = \hat{r}_t^*$. Finally, using the Euler equation for the domestic currency $i$ bond with nominal return $R_{it}$, we show that $\psi_{it}$ defined in the lemma equals the currency $i$ UIP deviation, $\psi_{it} = \log (R_{it}/R_t^*) + \mathbb{E}_t \Delta e_{it+1}$.
**Proposition 7** (a) If all countries are unconstrained, non-cooperative optimal FXI implement the global planner’s allocation, and in particular $\psi_{it} = \psi_{t} = 0$ for all $i \in (0, 1]$, eliminating all UIP deviations and implementing $r^*_i = r^*_t$. (b) When FXI are constrained in a subset of countries, non-cooperative policy is subject to an externality. The cooperative policy does not fully eliminate UIP deviations in the unconstrained countries, $\psi_{it} = \psi_{t} \neq 0$, limiting inefficient capital outflows from/to constrained economies.

When all countries are unconstrained, the optimal non-cooperative policies from Proposition 1 that eliminate UIP deviations country-by-country, $\psi_{it} = 0$ for all $i$, translate into a globally optimal outcome with $\tilde{\psi}_{t} = 0$ and $r^*_t = \tilde{r}^*_t$. That is, the Nash equilibrium played by local policymakers results in zero output gap and optimal risk sharing between all economies. Elimination of all UIP deviations rebalances capital flows and eliminates the pressure on the global real interest rate. This result suggests the usefulness of swap lines between central banks, which can relax constraints on FXI and allow countries to achieve the optimum allocation without relying on either ex ante, or ex post international wealth transfers (Bahaj and Reis 2021). Furthermore, such swap lines are not subject to incentive compatibility or time consistency issues, as the best non-cooperative use of relaxed FXI does not lead to negative international spillovers and is beneficial cooperatively.

In contrast, when FXI of a subset of countries are constrained and shocks have a correlated component, so that $\tilde{\psi}_{t} \neq 0$, this results in international spillovers that are not internalized by national policymakers. The cooperative policy eliminates the risk sharing wedge between the group of unconstrained and constrained countries, reducing the extent of inefficient capital flows. For example, with a global demand shock for dollar $\tilde{\psi}_{t} > 0$, the cooperative policy ensures $\mathbb{E}_{t}\Delta \hat{z}_{it+1} = 0$ for the unconstrained countries, yet with a UIP deviation against the dollar, $\psi_{it} = \tilde{\psi}_{t} > 0$. For comparison, a country’s non-cooperative policy that eliminates the UIP deviation, $\psi_{it} = 0$, results in $\mathbb{E}_{t}\Delta \hat{z}_{it+1} = -\tilde{\psi}_{t} < 0$.

Therefore, the cooperative policy under-reacts to the UIP wedge in order to curb inefficient capital inflows from the constrained economies, emphasizing the complementarity in the use of FXI across countries. Specifically, unconstrained FXI $f^*_it$ respond to both domestic currency demand shocks $n^*it$ and currency demand shocks in constrained economies $n^*jt$ such that $n^*it + f^*_it$ and $n^*jt + f^*_jt$ — and, thus, $\psi_{it}$ and $\psi_{jt}$ — comove. Interestingly, this amplifies the equilibrium effect of the shock on $\tilde{\psi}_{t}$ and $r^*_t$, resulting in larger capital inflows and current account deficits in the US, yet mitigates the aggregate outflow from the constrained economies.

---

\footnote{Indeed, there are no first-order externalities in our environment when countries choose consumption of tradables subject to intertemporal budget constraint. Although international asset markets are incomplete, the fact that there is only one tradable good implies that there is no pecuniary externality (Geanakoplos and Polemarchakis 1986). Similarly, there is no aggregate demand externality for risk sharing when monetary policy closes the output gap (cf. Farhi and Werning 2016). This result contrasts with the inefficient non-cooperative equilibrium in Fanelli and Straub (2021) when countries participate in a “rat race” of reserve accumulation in a second-best world with redistributive FX interventions.}

\footnote{For concreteness, consider a measure $m_0 > 0$ of the world economy, corresponding to the US and dollar pegs combined, with $\psi_{it} = 0$ for $i \in [0, m_0]$. A measure $m_1 > 0$ of countries, $i \in (m_0, m_0 + m_1]$, are constrained and face a correlated dollar demand shock against their currencies, $\tilde{\psi}_{it} \equiv \frac{1}{m_1} \int_{m_0+m_1}^{m_0+m_1} \psi_{it} \, dt > 0$. The remaining countries, $i \in (m_0 + m_1, 1]$ are unconstrained, and adopt the cooperative policy $\psi_{it} = \tilde{\psi}_{it} \equiv \frac{m_1}{m_0 + m_1} \tilde{\psi}_{t}$, where the last equality is the equilibrium fixed point. Thus, unconstrained economies mimic the average behavior of the other countries — $\psi_{it} = 0$ of the dollarized economy with measure $m_0$ and $\tilde{\psi}_{t}$ of the constrained countries with measure $m_1$.}
**Dominant currency spillovers** We close this section by briefly considering the spillovers from the US monetary policy in a non-cooperative equilibrium characterized by Proposition 4. The multi-country setup clarifies the central role of the bilateral exchange rates against the dollar when implementing a crawling peg and explains why most countries in the world — including the ones with weak trade linkages to the US — use the dollar as an anchor currency in their monetary and FX policies (Ilzetzki, Reinhart, and Rogoff 2019).49 Indeed, pegging to other currencies or baskets of currencies is suboptimal and can potentially exacerbate the risk-sharing wedge by increasing $\sigma_{it}^2$. The asymmetric role of the US dollar exchange rate is not due to the specific form of currency market segmentation, but rather the assumption that the dollar is the international funding currency.

The immediate implication of the dollar dominance is the highly asymmetric spillovers of the US monetary policy. For example, a tightening of the US monetary stance lowers the dollar price of tradables $p_{Tt}^*$ and leads to an appreciation of the dollar, $e_{it} \uparrow$ for $i \in (m_0, 1]$. To stabilize the exchange rate against the dollar and reduce the risk-sharing wedge, other countries are required to lean against the wind and raise their interest rates, which leads to a negative output gap $x_{it} < 0$. Thus, even when the US economy is small ($m_0 = 0$), all countries import its monetary stance giving rise to the global monetary cycle (Rey 2013, Egorov and Mukhin 2023).50

5 Robustness

The baseline model makes several stark assumptions to get a sharp characterization of the optimal policy. This section relaxes some of them — in particular, allowing for sluggish price adjustment, expenditure switching in tradables, and alternative structures of UIP deviations — to evaluate robustness of our main results. Detailed derivations are relegated to Appendix A4.

5.1 Staggered prices

The assumption of fully rigid prices in our baseline analysis provides emphasis to our main focus on the trade-off between output gap and international risk sharing, yet is admittedly very stark, and in particular removes domestic inflation as a policy consideration. We now generalize our results to an environment with staggered price adjustment. In particular, we assume that there is a continuum of varieties of non-tradable goods with an elasticity of substitution equal $\varepsilon > 1$ that are produced by monopolistic competitors. Firms are subject to a Calvo (1983) friction and update prices with probability $1 - \lambda$. We allow for markup shocks $\nu_t$ and assume that a constant production subsidy is used to eliminate the steady state markup wedge. The resulting planner’s problem is largely isomorphic to the

49Similarly to Hassan, Mertens, and Zhang (2023), the goal of the peg in our model is to eliminate the UIP deviation, but the anchor status of the dollar is due to the structure of financial markets, not the size of the US economy.

50This contrasts with a more symmetric global financial system under a gold standard. If gold is used as the international funding vehicle, the risk-sharing conditions remain the same except that the relevant measure of risk $\sigma_{it}^2$ is now the volatility of exchange rates against gold. Given a zero nominal return on gold, $R_{g}^* = 1$, the price of tradables $p_{Tt}$ (in units of gold) adjusts to implement the real rate of return $r_{Tt}^*$ required to clear the goods market. This eliminates the asymmetry of a dominant currency, yet may lead to larger risk-sharing wedges due to a more volatile unit of account.
baseline (15), but features both inflation $\pi_{Nt}$ and output gap $x_t$ in the objective function:

$$\min_{\{x_t, \pi_{Nt}, z_t, b_t^*, f_t^*, \sigma_t^2\}} \left\{ \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) (x_t^2 + \alpha \pi_{Nt}^2) \right] \right\},$$

subject to

$$\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (ib_t^* - n_t^* - f_t^*),$$

$$\beta b_t^* = b_{t-1}^* - z_t,$$

$$\sigma_t^2 = \text{var}_t (\tilde{q}_{t+1} - z_{t+1} + x_{t+1} + \pi_{Nt+1}),$$

$$\pi_{Nt} = \kappa x_t + \beta \mathbb{E}_t \pi_{Nt+1} + \nu_t,$$

where $\alpha \equiv \varepsilon/\lambda$ is the relative weight on welfare losses from inflation and the set of constraints now additionally features a standard NKPC with $\kappa \equiv (1 - \lambda)(1 - \beta \lambda)/\lambda$.

The first thing to notice is that the results about the first-best policies remain largely unchanged. When two policy instruments are available, the FX interventions $f_t^* = ib_t^* - n_t^*$ eliminate the risk-sharing wedge and the interest rates implements the optimal path of inflation $\pi_{Nt}$ and output gap $x_t$, as in the closed economy, generalizing Proposition 1. Similarly, by adopting an exchange rate peg, monetary policy on its own can implement the optimal allocation with $z_t = x_t = \pi_{Nt} = 0$, if firms’ markups are constant, $\nu_t = 0$, and the natural real exchange rate is stable, $\tilde{q}_t = 0$. Hence, open economy divine coincidence requires that closed economy divine coincidence is satisfied, that is there is no conflict between output gap and inflation stabilization, and additionally that a fixed exchange rate does not interfere with efficient expenditure switching. This generalizes Proposition 2.

This isomorphism to the baseline model extends further and applies also to the second-best policies. To see this, notice that the only interaction between the two sectors comes from the nominal exchange rate via expenditure switching (2), which results in the $x_{t+1} + \pi_{Nt+1}$ term in the definition of $\sigma_t^2$ in the constraint set. This implies that the planner’s problem can be broken into two sequential steps: first, solve for the optimal path $\{x_t, \pi_{Nt}\}$ given shocks to aggregate demand $m_t \equiv x_t + \pi_{Nt}$, and second, solve for the optimal trade-off between risk sharing $z_t$ and domestic conditions summarized by $m_t$. The latter problem is the same as in the baseline model, except that the output losses $x_t^2$ are replaced with the overall welfare losses due to output gap and inflation from suboptimal monetary response to markup innovations. This implies that results about the second-best policies, including the optimal partial peg (16), extend to the setup with adjusting prices.\footnote{Interestingly, in contrast to the prescriptions of the standard New-Keynesian model (Gali 2008), the optimal Ramsey policy does not target the long-run price level, and shocks in both sectors have permanent effects on price levels.}

### 5.2 Terms of trade

Another important limitation of the baseline model are constant terms of trade and no expenditure switching in exports. Following the previous normative open-economy literature (Gali and Monacelli 2005, Devereux and Engel 2003, Benigno and Benigno 2003), this extension replaces tradables and non-tradables with a home good consumed locally $C_Ht$ and exported abroad $C_H^*$ and with an imported foreign good $C_Ft$. We keep the assumption of log-linear preferences with $C_t = C_H^{1-\gamma} C_F^\gamma$, linear
technology, and CES demand for exports:

\[ A_tL_t = C_{Ht} + C^*_{Ht}, \quad C^*_{Ht} = \gamma P^*_{Ht} - \varepsilon Ht \]

where \( P^*_{Ht} \) is the export price in foreign currency, \( \varepsilon > 1 \) is the elasticity of foreign demand, and \( C^*_{t} \) is the global demand shifter. For simplicity, all prices are fully sticky in the currency of invoicing. We assume that domestic prices are set in local currency and consider two alternatives for export prices: producer currency (PCP) with rich terms-of-trade dynamics and dollar pricing, which provides a better description of the current international price system (Gopinath, Boz, Casas, Diez, Gourinchas, and Plagborg-Møller 2020).

**Producer currency pricing**  When export prices are sticky in the currency of exporter, the monetary policy can generate expenditure switching in the market of destination and simultaneously close the output gap in domestic and export sectors. As a result, the loss function can be written in terms of the total output gap \( x_t \) and the deviations of imports from the optimal level \( z_t \):

\[
\min_{\{x_t, z_t, b^*_t, f^*_t, \sigma^2_t\}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \kappa z^2_t + x^2_t \right]
\]

s.t.

\[
E_t \Delta z_{t+1} = -\bar{\omega} \sigma^2_t (ib^*_t - n^*_t - f^*_t),
\]

\[
\beta b^*_t = b^*_{t-1} + \frac{\varepsilon - 1}{\varepsilon} x_t - z_t,
\]

\[
\sigma^2_t = \text{var}_t(\tilde{q}_{t+1} + x_{t+1} - (1 - \bar{\gamma})z_{t+1}),
\]

where \( \kappa \equiv \frac{\varepsilon^2 \gamma}{\varepsilon - \gamma} \) and \( \bar{\gamma} \equiv \frac{\gamma(\varepsilon - 1)}{\varepsilon - \gamma} \) is the steady-state share of exports in total output. The only substantial difference from the baseline problem (15) is that the monetary policy affects exports via expenditure switching channel and therefore, \( x_t \) appears in the country’s budget constraint with a multiplier that depends on the elasticity of substitution \( \varepsilon \). This additional channel does not change the main results about the first-best policies. When two instruments are available, the planner can implement efficient allocation by closing the output gap \( x_t = 0 \) with interest rate policy and eliminating the risk-sharing wedge with the FX interventions \( f^*_t = ib^*_t - n^*_t \). Moreover, the divine coincidence still holds when efficient real exchange rate is constant: by stabilizing the nominal exchange rate, monetary policy alone can close both wedges \( x_t = z_t = 0 \). The condition that \( \tilde{q}_t = 0 \) is satisfied when local productivity shocks move one-to-one with global demand shocks \( a_t = c^*_t \) and both shocks follow a random walk.

Moving to the second-best policies, because of the effect of monetary policy on country’s exports, a nominal peg \( \sigma^2_t = 0 \) is no longer sufficient to implement \( z_t = 0 \). However, for any given path of \( x_t \), it is still optimal to close the UIP deviations — either using the FX interventions or by stabilizing the nominal exchange rate. In particular, a partial peg remains optimal when FX interventions are not available. While the monetary policy can also stimulate exports to increase country’s imports, the effect is relatively weak because of consumption smoothing and is not very useful to offset financial shocks.
Dominant currency pricing  When export prices are sticky in foreign currency, the law of one price does not hold creating an additional gap in the planner’s problem:

$$\min_{\{x_t, z_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 + \gamma (\varepsilon - 1) \tilde{q}_t^2 \right]$$

s.t.  
$$\mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (\iota b_t^* - n_t^* - f_t^*),$$
$$\beta b_t^* = b_{t-1}^* - (\varepsilon - 1) \tilde{q}_t - z_t,$$
$$\sigma_t^2 = \text{var}_t \left( \tilde{q}_{t+1} + x_{t+1} - z_{t+1} \right),$$

where $x_t$ is the output gap in domestic sector and $z_t$ is the deviation of consumption of foreign goods from the optimal level. Because the export prices do not respond to shocks, the deviations from the optimal exports, $(1 - \varepsilon) \tilde{q}_t$, fluctuate together with the optimal level of the real exchange rate. As a result, the exports are exogenous to monetary policy and neither interest rates nor FX interventions can close the output gap in the export sector (see ?). Moreover, the suboptimal exports imply that it is impossible to achieve the efficient level of imports $z_t = 0$. Yet, when two policy instruments are available, the optimal targets are the same as in the baseline model: the monetary policy closes domestic output gap $x_t = 0$ and the FX interventions offset financial shocks $f_t^* = \iota b_t^* - n_t^*$.

Interestingly, the divine coincidence from the baseline model is still valid under DCP: if the real exchange rate is stable, the monetary policy alone can implement the first-best allocation. Indeed, if $\tilde{q}_t = 0$, then there is no need for export prices to adjust and exports are efficient. Pegging a nominal exchange rate encourages arbitrageurs and eliminates the risk-sharing wedge, while simultaneously closing the output gap. Away from this knife-edge case, a partial peg balances $x_t$ and $z_t$.

5.3 Financial shocks

While the analysis above focuses on noise trader shocks as the main source of volatility in financial markets, the previous literature suggests that other shocks may play an important role as well. To study robustness of the optimal policy, we augment the model with three additional financial shocks. The first one is the expectation error of arbitrageurs $\xi_t$ in the spirit of Gourinchas and Tornell (2004), which implies that the subjective beliefs are given by $\mathbb{E}_t \Delta e_{t+1} = \mathbb{E}_t \Delta e_{t+1} - \xi_t$. Second, following Brunnermeier, Nagel, and Pedersen (2009) and Gabaix and Maggiori (2015), we allow for risk-appetite shocks to $\omega_t$. Finally, assume that there is a time-varying probability of default modelled as a shock $\delta_t$ to the returns on the home currency bond. Because this latter shock applies for both households and arbitrageurs, it is absorbed by the equilibrium interest rate and does not directly affect UIP deviations. Instead, it creates an additional source of carry trade risk. Combining these pieces together, the new risk-sharing condition is:

$$\mathbb{E}_t \Delta z_{t+1} = \xi_t - \bar{\omega} \sigma_t^2 (\iota b_t^* - n_t^* - f_t^*), \quad \sigma_t^2 = \text{var}_t (e_{t+1} + \delta_{t+1}).$$

The rest of the equilibrium system and the objective function remain the same as in the baseline model.
It follows that the first-best policy remains largely unchanged in the presence of additional shocks. In particular, it remains optimal to target UIP deviations with FX interventions and offset both demand shocks of noise traders and expectational errors of arbitrageurs, \( f_t^* = \iota b_t^* - n_t^* - \xi_t/\bar{\omega}_t \sigma_t^2 \), aiming to implement \( \mathbb{E}_t \Delta z_{t+1} = 0 \). In contrast, the divine coincidence result holds with respect to the risk-appetite shocks \( \omega_t \), but does not apply more generally as stabilizing the nominal exchange rate is no longer sufficient to eliminate the UIP wedge in the presence of \( \xi_t \) and \( \delta_t \) shocks. Nonetheless, an exchange rate peg still eliminates a part of the UIP wedge associated with the noise trader shocks.

6 Conclusion

This paper studies optimal exchange rate policy in an open economy with frictional goods and asset markets. In contrast to the previous normative literature, we use a framework that is consistent with the major exchange rate puzzles, including the change in macroeconomic dynamics after a switch from a peg to a float associated with the end of the Bretton-Woods system. The model is tractable and allows for an intuitive linear-quadratic approximation of the planner’s problem, yet rich enough to accommodate interesting policy trade-offs and multiple policy instruments.

We show that the constrained optimum can be implemented with monetary policy closing the output gap under sticky prices in goods markets and FX interventions targeting UIP deviations due to intermediary frictions in asset markets. In addition, when foreign agents participate in financial intermediation, the government can collect monopoly rents in home currency markets and the optimal mix of policy tools includes capital controls. The open-economy divine coincidence holds when the natural real exchange rate is constant and allows closing the two wedges with one monetary instrument by pegging the nominal exchange rate. More generally, when FX interventions are subject to additional constraints, the planner can use a crawling peg and/or FX forward guidance to mitigate financial distortions. International cooperation is not required when countries follow the unconstrained privately optimal policies, but helps mitigate international spillovers from global liquidity shocks and US monetary shocks under constrained policies.
A Appendix

A1 Exact non-linear policy problem

As described in Sections 2.1–2.2, the Ramsey problem maximizes the household welfare in (1) over policies \{R_t, F_t^*\} and the equilibrium allocation \{C_{Nt}, C_{Tt}, B_t^*, \mathcal{E}_t\} and \{\sigma_t^2\}, subject to the equilibrium system (2)–(3) and (6)–(7) (including the definition of \sigma_t^2), given the stochastic path of exogenous variables \{A_t, Y_{Tt}, R_t^*, N_t^*\} and subject to initial and transversality conditions on \{B_t^*\}:

\[
\max_{\{R_t, F_t^*, C_{Nt}, C_{Tt}, B_t^*, \mathcal{E}_t, \sigma_t^2\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{Tt} + (1 - \gamma) \left( \log C_{Nt} - \frac{C_{Nt}}{A_t} \right) \right] \tag{A1}
\]

subject to

\[
\frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt},
\]

\[
\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*},
\]

\[
\beta R_t \mathbb{E}_t \frac{C_{Nt}}{C_{Nt+1}} = 1,
\]

\[
\mathcal{E}_t = \frac{\gamma}{1 - \gamma} \frac{C_{Nt}}{C_{Tt}}
\]

\[
\sigma_t^2 = R_t^2 \cdot \text{var}_t \left( \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right),
\]

where we used the non-tradable production function and market clearing \(C_{Nt} = Y_t = A_t L_t\) to substitute for \(L_t\) in the welfare function.\(^{52}\)

**First best** The first-best allocation maximizes (A1) with respect to \{\(C_{Nt}, C_{Tt}, B_t^*+1\)\} and subject to the budget constraint only, removing the remaining four constraints. The optimality conditions for this problem imply \(\tilde{C}_{Nt} = A_t\) and \{\(\tilde{C}_{Tt}, \tilde{B}_{t+1}\)\} such that:

\[
\beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1
\]

and the budget constraint holds, \(B_t^*/R_t^* - B_{t-1}^* = Y_{Tt} - C_{Tt}\).

When the two policy instruments — monetary policy and FXI, \{\(R_t, F_t^*\)\} — are available and unconstrained, the first best allocation is feasible. This is because the two constraints on the policy problem (A1) — namely, the two Euler equations (with \(R_t\) and \(R_t^*\), respectively), with the last two constraints being static side equations defining \(\mathcal{E}_t\) and \(\sigma_t^2\) — each feature an independent policy instrument which can ensure that the respective constraint is relaxed.

\(^{52}\)Note that one can alternatively rewrite the problem in terms of wages \(W_t\), as in an equilibrium with sticky prices \(P_{Nt} = 1\) the labor supply condition implies \(W_t = P_{Nt} C_{Nt} = C_{Nt}\), and monetary policy by controlling aggregate nominal expenditure \(P_t C_t\), controls also the path of nominal wages, \(W_t = P_{Nt} C_{Nt} = (1 - \gamma) P_t C_t\).
Specifically, decentralizing the first-best allocation requires a path of \( \{R_t, F_t^\ast\} \) such that:

\[
\beta R_t \mathbb{E}_t \frac{A_t}{A_{t+1}} = 1, \\
\omega \sigma_t^2 \frac{B_t^\ast - N_t^\ast - F_t^\ast}{R_t^\ast} = 0
\]

and the implies path of the nominal exchange rate given by \( \tilde{E}_t = \frac{\gamma}{1 - \gamma} \frac{A_t}{C_{Tt}} \). The two displayed equations characterize the necessary path of policy outcomes \( \tilde{R}_t \) and \( \tilde{F}_t^\ast \), leaving aside the conventional issue of uniqueness of the decentralized equilibrium (see Atkeson, Chari, and Kehoe 2010). Therefore, the first-best monetary policy eliminates the output gap, that is, ensures \( C_{Nt} = C_{Nt} = A_t \), while the first-best financial market policy ensures a zero risk-sharing wedge. This happens when either \( \omega \sigma_t^2 = 0 \), or when \( F_t^\ast = B_t^\ast - N_t^\ast \); the latter corresponds to the case of Proposition 1, while the former to divine coincidence of Proposition 2 (or trilemma models with \( \omega = 0 \)).

The first-best path of NFA according to the budget constraint is \( \tilde{B}_t^\ast = R_t^\ast (\tilde{B}_{t-1}^\ast + Y_{T_t} - \tilde{C}_{T_t}) \), and hence the optimal FXI is \( \tilde{F}_t^\ast = \tilde{B}_t^\ast - N_t^\ast \) when \( \sigma_t^2 = \tilde{\sigma}_t^2 = \tilde{R}_t^2 \cdot \text{var}_t(\tilde{E}_t/\tilde{E}_{t+1}) \neq 0 \).

**Optimality conditions**  We make the following substitution of variables:

\[
\Gamma_t \equiv \frac{1}{C_{N_t}}, \quad \beta R_t \mathbb{E}_t \frac{\Gamma_{t+1}}{\Gamma_t} = 1, \quad \mathcal{E}_t = \frac{\gamma}{1 - \gamma} \frac{1}{\Gamma_t C_{Tt}}, \quad (A2)
\]

where the last two conditions are implied by the constraints in (A1). We can thus recover the path of \( \{R_t, C_{Nt}, \mathcal{E}_t\} \) from the path of \( \{\Gamma_t, C_{Tt}\} \). As a result, the original policy problem (A1) is equivalent to:

\[
\max_{\{\Gamma_t, C_{Tt}, B_t^\ast, \sigma_t^2\}_{t \geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{Tt} - (1 - \gamma) \left( \log \Gamma_t + \frac{1}{\Gamma_t} \right) \right] \quad (A1')
\]

subject to

\[
R_t^\ast \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega \sigma_t^2 \frac{B_t^\ast - N_t^\ast - F_t^\ast}{R_t^\ast},
\]

\[
\sigma_t^2 \beta^2 C_{Tt}^2 (\mathbb{E}_t \Gamma_{t+1})^{-2} = \mathbb{E}_t (\Gamma_{t+1} C_{Tt+1})^{-2} - (\mathbb{E}_t \Gamma_{t+1} C_{Tt+1})^2,
\]

where we used (A2) to solve out \( \{R_t, \mathcal{E}_t\} \) from the definition of \( \sigma_t^2 \):

\[
\sigma_t^2 = \frac{R_t^2}{(\beta \mathbb{E}_t [\Gamma_{t+1}/\Gamma_t])^{-2} \cdot (\Gamma_t C_{Tt})^{-2} \cdot \text{var}_t(\mathcal{E}_t/\mathcal{E}_{t+1})}
\]

resulting in the final constraint in (A1'). Note that we characterize the planner's optimality conditions for an arbitrary path of \( \{F_t^\ast\} \), and then discuss the optimal path of \( \{F_t^\ast\} \).

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53 Divine coincidence, of course, requires that \( \tilde{E}_t = \frac{\gamma}{1 - \gamma} \frac{A_t}{C_{Tt}} = \text{const} \); otherwise, at least one of the two wedges cannot be eliminated — either \( \sigma_t^2 \neq 0 \) and hence \( C_{Tt} \neq \tilde{C}_{Tt} \) (for an arbitrary path of \( F_t^\ast \neq \tilde{F}_t^\ast \)), or \( C_{Nt} \neq \tilde{C}_{Nt} = A_t \) under the peg (with \( \sigma_t^2 = 0 \) that ensures \( C_{Tt} = \tilde{C}_{Tt} \) for any path of \( F_t^\ast \)).
We write the Lagrangian for (A1') as follows:

\[
\mathcal{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log C_{Tt} - (1 - \gamma) \left( \log \Gamma_t + \frac{1}{A_t \Gamma_t} \right) \right.
\]

\[
\left. + \Lambda_t \left( B_{t-1}^* + Y_{Tt} - C_{Tt} - \frac{B_t^*}{R_t^*} \right) + M_t \left( 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*} - \beta R_t^* \mathbb{E}_t \frac{C_{Tt}}{C_{Tt+1}} \right) + D_t \left( \beta^2 \sigma_t^2 C_T^2 \frac{E_t \Gamma_t+1)^2}{\mathbb{E}_t} - \mathbb{E}_t (\Gamma_t C_{Tt+1})^2 + (\mathbb{E}_t \Gamma_t+1 C_{Tt+1})^2 \right) \right],
\]

where \( \{ \Lambda_t, M_t, D_t \} \) is the sequence of Lagrange multipliers on the respective constraints. The first order conditions with respect to \( \{ \Gamma_t, C_{Tt}, B_t^*, \sigma_t^2 \} \) are as follows:\(^{54}\)

\[
0 = - (1 - \gamma) \frac{1}{\Gamma_t} + (1 - \gamma) \frac{1}{A_t \Gamma_t} + 2 \beta^{-1} D_{t-1} C_{Tt} \left( \beta^2 \sigma_{t-1}^2 \frac{C_{Tt-1}}{C_{Tt}} \mathbb{E}_{t-1} \Gamma_t - \Gamma_t C_{Tt} + \mathbb{E}_{t-1} (\Gamma_t C_{Tt}) \right),
\]

\[
0 = \frac{\gamma}{C_{Tt}} - \Lambda_t - M_t \beta R_t^* \mathbb{E}_t \frac{1}{C_{Tt+1}} + M_{t-1} R_{t-1}^* \frac{C_{Tt-1}}{C_{Tt}} \mathbb{E}_{t-1} \Gamma_t - 2 D_t \beta^2 \sigma_t^2 C_{Tt} \frac{E_t \Gamma_t+1)^2}{\mathbb{E}_t} - 2 \beta^{-1} D_{t-1} \Gamma_t (\Gamma_t C_{Tt} - \mathbb{E}_{t-1} (\Gamma_t C_{Tt})),
\]

\[
0 = - \frac{\Lambda_t}{R_t^*} + \beta \mathbb{E}_t \Lambda_{t+1} + M_t \frac{\omega \sigma_t^2}{R_t^*},
\]

\[
0 = M_t \omega \frac{B_t^* - N_t^* - F_t^*}{R_t^*} + D_t \beta^2 \sigma_t^2 \frac{E_t \Gamma_t+1)^2}{\mathbb{E}_t}.
\]

where we define \( D_{-1} = M_{-1} = 0 \), and we use the fact that operator \( \mathbb{E}_t \{ \cdot \} \) sums across future realizations of uncertainty using conditional probabilities \( \pi(h^{t+1})/\pi(h^t) \) for any history of exogenous states \( h^{t+1} \equiv \{ A_s, Y_{Ts}, R_s \}_{s=0}^{t+1} \).

We simplify the conditions as follows. The last two conditions allow to relate the Lagrange multipliers \( \Lambda_t \) and \( D_t \) with \( M_t \):

\[
\Lambda_t - \beta R_t^* \mathbb{E}_t \Lambda_{t+1} = M_t \omega \sigma_t^2, \tag{A3}
\]

\[
D_t' = M_t \omega (\mathbb{E}_t R_t^2)^2 \frac{N_t^* + F_t^* - B_t^*}{R_t^*}, \tag{A4}
\]

where we used definitions (A2) in the second line and substituted \( D_t' \equiv (\gamma/1-\gamma)^2 D_t \). Next we simplify the first optimality condition by substituting out \( \sigma_t^2 \) using its definition (the third constraint of the problem):

\[
\beta (1 - \gamma)(X_t - 1) = 2 D_{t-1}' \left[ \frac{1}{\mathbb{E}_t} - \frac{\mathbb{E}_{t-1} \mathbb{E}_t^{-1}}{\mathbb{E}_t} - \frac{\Gamma_t}{\mathbb{E}_{t-1} \Gamma_t} \left( \mathbb{E}_{t-1} \mathbb{E}_t^{-2} - (\mathbb{E}_{t-1} \mathbb{E}_t^{-1})^2 \right) \right], \tag{A5}
\]

where we defined \( X_t \equiv C_{Nt}/A_t = 1/(A_t \Gamma_t) \) so that \( X_t - 1 \) corresponds to the output gap. Note that (A5) implies \( \mathbb{E}_{t-1} X_t = 1 \), as the conditional expectations of the right-hand side is zero. The final optimality condition:

\(^{54}\)Note that with an optimal unconstrained choice of \( F_t^* \) at \( t \), we additionally have that \( M_t = 0 \), and therefore \( D_t = 0 \), \( C_{Nt+1} = 1/(\Gamma_{t+1}) = A_{t+1} \), \( \Lambda_t = \beta R_t^* \mathbb{E}_t \Lambda_{t+1} \) and \( \beta R_t^* \mathbb{E}_t [C_{Tt}/C_{Tt+1}] = 1 \), consistent with conditions for the best allocation.

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\[\gamma \left(1 - \frac{A_t}{\gamma/C_t} \right) - (1 - \gamma)(X_t - 1) = M_t \beta R_t^\epsilon \frac{C_{t+1}}{C_{t+1}} - M_{t-1} R_{t-1}^\epsilon \frac{C_{t-1}}{C_{t-1}} - 2 D_t^* \sigma_t^2 \left(\bar{E}_t R_t\right)^2 + 2 D_t^* \sigma_t^2 \frac{C_{t-1}}{C_{t-1}} + 2 \omega \sigma_t^2 \frac{D_{t-1}}{R_t} \frac{C_{t-1}}{C_{t-1}} \frac{\Gamma_t}{\Gamma_{t-1}} \] (A6)

where we used \(D_t^* = B_t^* - N_t^* - F_t^*\).

Conditions (A3)–(A6) together with definitions in (A2) and constraints in (A1') characterize the optimal monetary policy for a given path of FXI \(\{F_t^*\}\) and the associated allocation.

### A2 Approximations

#### A2.1 Second-order approximation to the objective function

Consider any allocation \(\{C_N, C_T, L_t, B_t^*\}\) that satisfies production possibilities frontier for non-tradables, \(C_N = A_t L_t\), and the country budget constraint (6):

\[
\frac{B_t^*}{R_t^*} = B_{t-1}^* + Y_{T_t} - C_{T_t},
\]

for a given \(B_{t-1}^*\) and a transversality condition on \(B_t^*\), and corresponding to a stochastic path of shocks \(\{A_t, Y_t, R_t^*\}\). We refer to all such allocation as resource- and budget-feasible. The first best allocation corresponding to the same path of shocks is denoted with \(\{\tilde{C}_N, \tilde{C}_T, \tilde{L}, \tilde{B}^*\}\), it is also resource- and budget-feasible, and satisfies the following optimality conditions (see Appendix A1):

\[
\tilde{C}_N = A_t, \quad \tilde{L}_t = 1, \quad \beta R_t^* \bar{E}_t \frac{\tilde{C}_{T_t}}{\tilde{C}_{T_t+1}} = 1.
\]

A non-stochastic zero-NFA steady state corresponding to \((\tilde{A}, \tilde{Y}_T, \tilde{R}^*)\) such that \(\tilde{R}^* = 1/\beta\), is given by \((\tilde{C}_N, \tilde{C}_T, \tilde{L}, \tilde{B}^*)\):

\[
\tilde{C}_N = \tilde{A}, \quad \tilde{L} = 1, \quad \tilde{C}_T = \tilde{Y}_T, \quad \tilde{B}^* = 0,
\]

which implies \(\bar{N}X = \tilde{Y}_T - \tilde{C}_T = 0\) and the steady state budget constraint is satisfied. Finally, the welfare function is given by (1).

**Lemma A1** The second order Taylor expansion around a zero-NFA steady state \((\tilde{C}_N, \tilde{C}_T, \tilde{L})\) of the welfare loss for any budget- and resource-feasible allocation \(\{C_N, C_T, L_t\}\) relative to the first-best allocation \(\{\tilde{C}_N, \tilde{C}_T, \tilde{L}_t\}\) is given by:

\[
\frac{1}{2} \bar{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) z_t^2 \right],
\]

where \(z_t = \log(C_{T_t}/\tilde{C}_{T_t})\) and \(z_t = \log(C_{N_t}/\tilde{C}_{N_t})\). Therefore, it is sufficient to know the first-order dynamics of the two wedges \(\{x_t, z_t\}\) to evaluate the second-order welfare loss.

**Proof:** We take a second order Taylor expansions of (1) for any resource- and budget-feasible allocation \(\{C_N, C_T, L_t, B_t, A_t, Y_{T_t}, R_t^*\}\) around a zero-NFA steady state \((\tilde{C}_N, \tilde{C}_T, \tilde{L}, \tilde{B}^*) = (\tilde{A}, \tilde{Y}_T, 1, 0),\)
using log deviations:

\[ c_{Nt} = \log(C_{Nt}/C_N), \quad c_{Tt} = \log(C_{Tt}/C_T), \quad a_t = \log(A_t/\bar{A}), \quad y_{Tt} = \log(Y_{Tt}/\bar{Y}_T), \quad r^*_t = \log(R^*_t/R^*), \]

and for NFA we use a proportional deviation relative to \(\bar{Y}_T\):

\[ \tilde{b}^*_t = B^*_t/\bar{Y}_T. \]

Note that the first best deviation and the wedge for non-tradables are \(\tilde{c}_{Nt} = \log(\tilde{C}_{Nt}/A) = \log(A_t/\bar{A}) = a_t \) and \(x_t = \log(c_{Nt}/\tilde{C}_{Nt}) = c_{Nt} - \tilde{c}_{Nt} = c_{Nt} - a_t\). We also use the fact that for any resource-feasible allocation \(L_t = C_{Nt}/A_t\), and hence we solve out \(L_t\) from the welfare function.

We, therefore, can rewrite the welfare function (1) in terms of deviations as:

\[
\mathbb{W}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log \bar{Y}_T + (1 - \gamma)(\log \bar{A} - 1) + \gamma c_{Tt} + (1 - \gamma) \left[ \log c_{Nt} - (e^{c_{Nt} - a_t} - 1) \right] \right],
\]

as well as the flow budget constraint (6) as:

\[
\tilde{b}^*_{t-1} + e^{y_{Tt}} - e^{c_{Tt}} - \beta e^{-r^*_t} \tilde{b}^*_t = 0,
\]

using the fact that \(\bar{C}_T = \bar{Y}_T\) and \(\bar{R}^* = 1/\beta\). We characterize the welfare loss in two steps:

1. The second-order Taylor expansion for the non-tradable terms in \(\mathbb{W}_0\) is:

\[
\mathbb{E}_0 \left[ c_{Nt} - (e^{c_{Nt} - a_t} - 1) \right] = \mathbb{E}_0 \left[ \frac{c_{Nt} - (c_{Nt} - a_t)}{a_t} - \frac{1}{2} \frac{(c_{Nt} - a_t)^2}{x_t} \right] + h.o.t.
\]

and in the first best allocation \(x_t = 0\) as \(\tilde{c}_{Nt} = a_t\).

2. The second-order Taylor expansion to the flow budget constraint is:

\[
0 = \tilde{b}^*_{t-1} + y_{Tt} + \frac{1}{2} y_{Tt}^2 - c_{Tt} - \frac{1}{2} c_{Tt}^2 - \beta \tilde{b}^*_t + \beta r^*_t \tilde{b}^*_t + h.o.t.,
\]

which we use to express:

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_{Tt} = \tilde{b}^*_{t-1} + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ y_{Tt} + \frac{1}{2} y_{Tt}^2 - \frac{1}{2} c_{Tt}^2 + \beta r^*_t \tilde{b}^*_t \right] + h.o.t.,
\]

using the transversality condition for NFA deviations, \(\lim_{j \to \infty} \beta^j \tilde{b}_{t+j} = 0\). Evaluating relative
to the first-best allocation, we have:

\[ E_0 \sum_{t=0}^{\infty} \beta^t \gamma \tilde{c}_{Tt} - E_0 \sum_{t=0}^{\infty} \beta^t \gamma c_{Tt} = E_0 \sum_{t=0}^{\infty} \beta^t \gamma \left[ \frac{1}{2} c_{Tt}^2 - \frac{1}{2} c_{Tt}^2 - \beta r_t^* b_t^* \right] + h.o.t. \]

\[ = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \gamma z_t^2 + E_0 \sum_{t=0}^{\infty} \beta^t \gamma [(c_{Tt} - \tilde{c}_{Tt}) \tilde{c}_{Tt} - \beta r_t^* b_t^*] + h.o.t. \]

where we expanded \( c_{Tt} = z_t + \tilde{c}_{Tt} \) and denoted with \( b_t^* = \tilde{b}_t^* - \tilde{b}_t^* = (B_t^* - \tilde{B}_t^*)/\bar{Y}_T \) the proportional deviation of the NFA position from the first best NFA. Finally, we show that:

\[ 0 = E_0 \sum_{t=0}^{\infty} \beta^t \gamma [(c_{Tt} - \tilde{c}_{Tt}) \tilde{c}_{Tt} - \beta r_t^* b_t^*] + h.o.t. \]

\[ = E_0 \sum_{t=0}^{\infty} \beta^t [(b_{t-1}^* - \beta b_t^*) \tilde{c}_{Tt} - \beta r_t^* b_t^*] + h.o.t. \]

\[ = b_{t-1}^* \cdot \tilde{c}_{T0} + \beta \sum_{t=0}^{\infty} \beta^t E_0 [b_t^* (\Delta \tilde{c}_{Tt+1} - r_t^*)] + h.o.t. \]

The second line uses the expansion of the flow budget constraint for \( c_{Tt} \) and \( \tilde{c}_{Tt} \), which implies:

\( (c_{Tt} - \tilde{c}_{Tt}) \tilde{c}_{Tt} = (b_{t-1}^* + \frac{1}{2} c_{Tt}^2 - \frac{1}{2} c_{Tt}^2 - \beta b_t^* + \beta r_t^* b_t^* + h.o.t.) \tilde{c}_{Tt} = (b_{t-1}^* - \beta b_t^*) \tilde{c}_{Tt} + h.o.t. \)

The third lines uses the fact that \( b_{t-1}^* = \tilde{b}_{t-1}^* - \tilde{b}_{t-1}^* = 0 \) by the initial condition, and the optimality condition (Euler equation) for the first-best consumption growth, which we rewrite in log deviations as \( e^{r_t^*} E_t e^{\Delta \tilde{c}_{Tt+1}} = 1 \), and take the following second-order Taylor expansion:

\[ E_t \Delta \tilde{c}_{Tt+1} - r_t^* = \frac{1}{2} (r_t^*)^2 + \frac{1}{2} E_t (\Delta \tilde{c}_{Tt+1})^2 - r_t^* E_t \Delta \tilde{c}_{Tt+1} + h.o.t. \]

and therefore using the law of iterated expectations:

\[ E_0 [b_t^* (\Delta \tilde{c}_{Tt+1} - r_t^*)] = E_0 [b_t^* (E_t \Delta \tilde{c}_{Tt+1} - r_t^*)] = 0 + h.o.t. \]

Combining these results, we evaluate the welfare loss relative to the first-best allocation to be given by:

\[ \hat{W}_0 - \bar{W}_0 = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \gamma z_t^2 + (1 - \gamma) x_t^2. \]

### A2.2 First-order approximation to the equilibrium system

Non-linear equilibrium system (2)–(3) and (6)–(7), and non-linear wedges:

\[ X_t = C_{Nt}/\bar{C}_{Nt} = C_{Nt}/A_t \quad \text{and} \quad Z_t = C_{Tt}/\bar{C}_{Tt}. \]

**Steady state** given by:

\[ \bar{B}^* = \bar{F}^* = \bar{N}^* = 0, \quad \bar{R} = \bar{R}^* = 1/\beta, \quad \bar{C}_T = \bar{Y}_T, \quad \bar{C}_N = \bar{A}, \]

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and the associated exchange rates:

\[ \tilde{\mathcal{E}} = \tilde{Q} = \frac{\gamma}{1 - \gamma} \tilde{C}_N, \]

as well as no steady state wedges, \( \tilde{X} = \tilde{Z} = 1 \).

**Deviations**  Define for any (endogenous or exogenous) variable \( Y_t \) with a non-zero steady state value its log steady-state deviation \( y_t \) as:

\[ Y_t = \bar{Y} e^{\nu y_t} \quad \text{for} \quad \nu = 1, \]

and for net foreign assets \( B_t^* \) with a zero steady state value its deviation proportional to tradable steady state output \( b_t^* \) as:

\[ B_t^* = \bar{Y}_T \nu b_t^* \quad \text{for} \quad \nu = 1. \]

For \( \nu = 0 \), we get the steady state values of variables. We take the first order Taylor expansion of the equilibrium system in \( \nu \) around steady state \( \nu = 0 \) and evaluated at \( \nu = 1 \). The approximate system is linear (scales) in \( \nu \), but is not necessarily linear in variables (deviations \( y_t \)), as we see below.

**First best** allocation \( \{\tilde{C}_N t, \tilde{C}_T t, \tilde{B}_t^*, \tilde{Q}_t\} \) solves \( \tilde{C}_N t = A_t \) and:

\[ \tilde{Q}_t = \frac{\gamma}{1 - \gamma} \tilde{C}_N t, \]

\[ \frac{\tilde{B}_t^*}{R_t} - \tilde{B}_{t-1}^* = Y_T t - \tilde{C}_T t, \]

\[ \beta R_t e^{\nu} \tilde{C}_T t = 1. \]

The first order Taylor expansion in \( \nu \) to this system is given by \( \tilde{c}_N t = a_t \) and:

\[ \tilde{q}_t = a_t - \tilde{c}_T t, \]

\[ \beta \tilde{b}_t^* - \tilde{b}_{t-1}^* = y_T t - \tilde{c}_T t, \]

\[ e^{\nu} \tilde{r}_t = r_t^*, \]

where \( \{a_t, y_T t, r_t^*\} \) are stochastic shocks (in proportional deviations) determining the dynamics of the first-best allocation.

**Proof:** Substitute the definitions of variables in terms of \( \nu \)-deviations into the non-linear system describing the first-best allocation

\[ \tilde{Q} e^{\nu \tilde{q}_t} = \frac{\gamma}{1 - \gamma} \tilde{C}_N e^{\nu (\tilde{c}_N t - \tilde{c}_T t)}, \]

\[ \beta e^{-\nu r_t} \tilde{Y}_T \nu b_t^* - \tilde{Y}_T \nu b_{t-1}^* = \tilde{Y} e^{\nu y_T t} - \tilde{C}_T e^{\nu \tilde{c}_T t}, \]

\[ e^{\nu r_t} e^{\nu \tilde{r}_t} = 1, \]
where we used the fact that $\bar{R}^* = 1/\beta$. Using the steady state value of $\bar{Q}$, and the fact that $\bar{c}_{Nt} = a_t$ (as $\bar{C}_{Nt} = A_t$), the first equation is immediately log-linear, $\bar{q}_t = a_t - \bar{c}_{Tt}$. Dividing the second equation by $\bar{Y}_T$, using the fact that $\bar{C}_T = \bar{Y}_T$, and taking the Taylor expansion, we have:

$$ (1 - \nu r_t^* + \mathcal{O}(\nu^2)) \nu/\bar{b}_t^* - \nu \bar{b}_t^* = \nu y_{Tt} - \nu \bar{c}_{Tt} + \mathcal{O}(\nu^2), $$

where $\mathcal{O}(\nu^2)$ denotes terms of order $\nu^2$ or higher (around $\nu = 0$). Dividing by $\nu$ and eliminating remaining $\mathcal{O}(\nu)$ terms results in the first-order approximate equation. The final equation is expanded as follows:

$$ 1 = E_t\{(1 + \nu r_t^* + \mathcal{O}(\nu^2))(1 - \nu \Delta \bar{c}_{Tt+1} + \mathcal{O}(\nu^2))\} = E_t\{1 + \nu r_t^* - \nu \Delta \bar{c}_{Tt+1} + \mathcal{O}(\nu^2)\}. $$

Subtracting 1 on both sides, dividing through by $\nu$, and eliminating the remaining $\mathcal{O}(\nu)$ terms results in the first-order approximate equation.

**Equilibrium system** (2) and (6)–(7) is reproduced here as:

$$ E_t = \frac{\gamma}{1 - \gamma} \frac{C_{Nt}}{C_{Tt}}, $$

$$ \frac{B_t^*}{R_t^*} - B_{t-1}^* = Y_{Tt} - C_{Tt}, $$

$$ \beta R_t^* E_t \frac{C_{Tt}}{C_{Tt+1}} = 1 + \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \quad \sigma_t^2 = R_t^2 \cdot \text{var}_t \left( \frac{E_t}{E_{t+1}} \right). $$

Rewrite this system in deviations from the first-best system:

$$ E_t = \tilde{Q}_t X_t / Z_t, $$

$$ \frac{B_t^* - \tilde{B}_t^*}{R_t^*} - (B_{t-1}^* - \tilde{B}_{t-1}^*) = -(C_{Tt} - \tilde{C}_{Tt}), $$

$$ \beta R_t^* E_t \frac{Z_t \tilde{C}_{Tt}}{Z_{t+1} \tilde{C}_{Tt+1}} - \beta R_t^* E_t \frac{\tilde{C}_{Tt}}{\tilde{C}_{Tt+1}} = \omega \sigma_t^2 \frac{B_t^* - N_t^* - F_t^*}{R_t^*}, \quad \sigma_t^2 = R_t^2 \cdot \text{var}_t \left( \frac{E_t}{E_{t+1}} \right). $$

Define the following additional proportional deviation terms:

$$ B_t^* - \tilde{B}_t^* = \tilde{Y}_T \nu b_t^*, \quad N_t^* - \tilde{B}_t^* = \tilde{Y}_T \nu n_t^*, \quad F_t^* = \tilde{Y}_T \nu f_t^* $$

and

$$ \omega = \omega_0 / \nu^2, $$

for some $\omega_0 \geq 0$, so that $\omega$ is the value of risk-aversion at $\nu = 1$, and the value of risk-aversion increases as $\nu$ decreases towards 0 (the value of risk-aversion in steady state is irrelevant given the exact absence of risk). Given this definitions, the first-order Taylor approximation in $\nu$ to the non-linear equilibrium
system around $\nu = 0$ is given by:

$$e_t = \tilde{q}_t + x_t - z_t,$$

$$\beta b_t^* - b_{t-1}^* = -z_t,$$

$$\mathbb{E}_t \Delta z_{t+1} = \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*), \quad \sigma_t^2 = \text{var}_t(\Delta e_{t+1}).$$

where $\bar{\omega} = \omega_0 Y_T / \beta$.

**Proof:** Following similar steps as above, we substitute the definitions of variables in terms of deviations. The first line immediately results in $e_t = \tilde{q}_t + x_t - z_t$, as the non-linear equation is, in fact, log linear in variables. The second equation yields $\beta b_t^* - b_{t-1}^* = -z_t$ following similar steps as above, and additionally noting that

$$\frac{C_{Tt} - \tilde{C}_{Tt}}{Y_T} = e^{\nu c_{Tt}} - e^{\nu \tilde{c}_{Tt}} = \nu z_t + O(\nu^2),$$

where $z_t = c_{Tt} - \tilde{c}_{Tt}$ by the definition of variables. Finally, the last equilibrium condition is expressed as follows:

$$e^{\nu r_t} \mathbb{E}_t \left[ e^{-\nu (\Delta z_{t+1} + \Delta \tilde{c}_{Tt+1})} - e^{-\nu \tilde{c}_{Tt+1}} \right] = (\omega_0 Y_T / \beta) \frac{1}{\nu^2} e^{\nu (2r_t - r_t^*)} \text{var}_t \left( e^{-\nu \Delta e_{t+1}} \right) \nu (b_t^* - n_t^* - f_t^*),$$

after substituting in the expression for $\sigma_t^2$ and using $\bar{R} = \bar{R}^* = 1 / \beta$ to simplify. Dividing both sides by $e^{\nu r_t}$, substituting in $\bar{\omega} = \omega_0 Y_T / \beta$, and taking a first-order Taylor expansion in $\nu$ yields:

$$\mathbb{E}_t \left[ -\nu \Delta z_{t+1} + O(\nu^2) \right] = \bar{\omega} \left( 1 + 2 \nu (r_t - r_t^*) + O(\nu^2) \right) \frac{1}{\nu^2} \text{var}_t \left( 1 - \nu \Delta e_{t+1} + O(\nu^2) \right) \nu (b_t^* - n_t^* - f_t^*),$$

Dividing both sides by $\nu$ and simplifying:

$$-\mathbb{E}_t \Delta z_{t+1} + O(\nu) = \bar{\omega} \left( 1 + O(\nu) \right) \left( \text{var}_t (\Delta e_{t+1}) + O(\nu^2) \right) (b_t^* - n_t^* - f_t^*)$$

$$= \bar{\omega} \text{var}_t (\Delta e_{t+1}) (b_t^* - n_t^* - f_t^*) + O(\nu).$$

Eliminating the remaining $O(\nu)$ terms yields the first-order approximate equation in $\nu$, which is however not linear in variables (deviations). ■

We also note that the side equation (3) that determines the path of $R_t$ is approximated in the same way as the other equations of the equilibrium system:

$$r_t = \mathbb{E}_t \Delta c_{Nt+1} = \mathbb{E}_t \Delta x_{t+1} + \mathbb{E}_t \Delta a_{t+1}.$$
there is no inflation, \(P_{Tt}^* = P_{Nt} = 1\), we have that \(i_t = r_t\) and \(i_t^* = r_t^*\), i.e., nominal and real interest rates coincide.

**Lemma A2**  The solution to the approximate equilibrium system characterizes an \(O(\nu)\) accurate dynamics of the non-linear equilibrium system. Furthermore, \(\omega \sigma_t^2 - \omega' \sigma_t^2 = O(\nu)\), where \(\omega' = \beta^3 \omega / Y_T\).

**Proof:** We first formalize the claim. Consider an exact equilibrium path \(\{C_{Nt}, C_{Tt}, \mathcal{E}_t, B_t^*, \sigma_t^2\}\) that corresponds to policies \(\{X_t, F_t^*\}\) and shocks \(\{A_t, Y_{Tt}, R_t^*, N_t^*\}\), which also determine the first-best allocation \(\{\check{C}_{Nt}, \check{C}_{Tt}, \check{B}_t^*, \check{Q}_t\}\). Note that \(X_t = C_{Nt}/\check{C}_{Nt}\) and \(Z_t = C_{Tt}/\check{C}_{Tt}\). Define the exact deviations from the steady state \(\{\hat{x}_t, \hat{f}_t^*, \hat{z}_t, \hat{\nu}_t, \hat{\sigma}_t^2\}\) as:

\[
X_t = e^{\nu \hat{x}_t}, \quad F_t^* = \check{Y}_T \nu \hat{f}_t^*, \quad Z_t = e^{\nu \hat{z}_t}, \quad \mathcal{E}_t = \hat{\mathcal{E}} e^{\nu \hat{e}_t}, \quad B_t^* - \check{B}_t^* = \check{Y}_T \nu \hat{b}_t^*,
\]

\[
N_t^* - \check{B}_t^* = \check{Y}_T \nu \hat{n}_t^* \quad \text{and} \quad \hat{Q}_t = \check{Q} e^{\nu \hat{q}_t} \quad \text{for} \quad \nu = 1.
\]

We define similarly \(\{\check{c}_{Tt}, \check{r}_t, \check{r}_t^*, \check{y}_{Tt}, \check{a}_t\}\).

Consider now an approximate equilibrium path \(\{z_t, e_t, b_t^*, \check{\sigma}_t^2\}\) that emerges as a result of policies \(\{\hat{x}_t, \hat{f}_t^*\}\) in response to shocks \(\{\hat{n}_t^*, \hat{q}_t\}\). Then:

\[
\{z_t, e_t, b_t^*, \beta^2 \omega \hat{\sigma}_t^2\} - \{\hat{z}_t, \hat{e}_t, \hat{b}_t^*, \omega \hat{\sigma}_t^2\} = O(\nu),
\]

where \(\hat{\sigma}_t^2 = \text{var}_t(\Delta e_{t+1})\), \(\hat{\sigma}_t^2 = \beta^2 e^{\nu \hat{e}_t} \cdot \text{var}_t(e^{-\nu \Delta e_{t+1}})\), and \(\omega = \omega_0 / \nu^2\) in the exact system and \(\check{\omega} = \omega_0 \check{Y}_T / \beta\) in the approximate system for some constant \(\omega_0\).

The proof of this formal claim follows from the first-order Taylor expansion of the equilibrium system in exact deviations from the first best \(\{\hat{z}_t, \hat{e}_t, \hat{b}_t^*\}\), described above, which we rewrite here as:

\[
\nu \hat{e}_t = \nu (\check{q}_t + \hat{x}_t - \hat{z}_t), \quad \beta e^{\nu \hat{e}_t} \nu \hat{b}_t^* - \nu \hat{b}_{t-1}^* = - (e^{\nu (\hat{x}_t + \check{e}_{Tt})} - e^{\nu \check{e}_{Tt}}), \quad \text{var}_t(e^{-\nu \Delta e_{t+1}}) (\hat{b}_t^* - \hat{n}_t^* - \hat{f}_t^*).
\]

The first-order Taylor expansion of this exact system is:

\[
\hat{e}_t = \check{q}_t + \hat{x}_t - \hat{z}_t, \quad \beta \hat{b}_t^* - \hat{b}_{t-1}^* = - \hat{z}_t + O(\nu), \quad \text{var}_t(\Delta \check{e}_{t+1})(\hat{n}_t^* + \hat{f}_t^* - \hat{b}_t^*), \quad O(\nu),
\]

while the approximate system for \(\{z_t, e_t, b_t^*\}\) is:

\[
e_t = \check{q}_t + \hat{x}_t - z_t, \quad \beta b_t^* - b_{t-1}^* = - z_t, \quad \text{var}_t(\Delta e_{t+1})(\hat{n}_t^* + \hat{f}_t^* - b_t^*),
\]

Therefore, the difference between the exact solution \(\{\hat{z}_t, \hat{e}_t, \hat{b}_t^*\}\) and the approximate solution \(\{z_t, e_t, b_t^*\}\)
vanishes with $O(\nu)$. Furthermore:

$$\omega \sigma_t^2 = \frac{\beta^2 \omega_0}{\nu^2} e^{2\nu \hat{\tau} t} \text{var}_{t-1}(e^{-\nu \Delta \hat{\tau} t+1}) = \beta^2 \omega_0 \text{var}_{t-1}(\Delta \hat{\tau} t+1) + O(\nu) = \beta^2 \omega_0 \sigma_t^2 + O(\nu),$$

and the last equality holds because $\text{var}_t(\Delta \hat{\tau} t+1) - \text{var}_t(\Delta e_t+1) = O(\nu^2)$ as $\{\hat{\tau}_t\} - \{e_t\} = O(\nu)$. 

### A2.3 Optimal policies

Set up a Lagrangian for the policy problem (15) for any given path of $\{f_t^*\}$:

$$\ell_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{2} (\gamma z_t^2 + (1-\gamma)x_t^2) - \gamma \lambda_t (b_{t-1}^* - z_t - \beta b_t^*) \right. \right.$$  

$$- \gamma \mu_t \left( \mathbb{E}_t \Delta z_{t+1} - \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*) \right) \right.$$  

$$- \delta_t \left( \sigma_t^2 + \mathbb{E}_t(\bar{q}_{t+1} + x_{t+1} - z_{t+1})^2 + (\mathbb{E}_t(\bar{q}_{t+1} + x_{t+1} - z_{t+1}))^2 \right),$$

where we substituted in the expression for $e_t = \bar{q}_t + x_t - z_t$ and replaced $\sigma_t^2 = \text{var}_t(\Delta e_t+1) = \mathbb{E}_t\sigma_{t+1}^2 - (\mathbb{E}_t e_{t+1})^2$. Note the analogy with the non-linear Lagrangian $\mathcal{L}_0$ in Appendix A1; the fact that we have negatives in front of the constraints in the approximate problem reflects the fact that we are minimizing welfare loss, in contrast to maximizing welfare in the exact problem.

The optimality conditions with respect to $\{x_t, z_t, b_t^*, \sigma_t^2\}$ are:

$$0 = (1-\gamma)x_t + 2\beta^{-1}\delta_{t-1}(e_t - \mathbb{E}_{t-1}e_t),$$

$$0 = \gamma z_t + \gamma \lambda_t + (\gamma \mu_t - \beta^{-1}\gamma \mu_{t-1}) - 2\beta^{-1}\delta_{t-1}(e_t - \mathbb{E}_{t-1}e_t),$$

$$0 = \beta(\gamma \lambda_t - \mathbb{E}_t \gamma \lambda_{t+1}) - \gamma \mu_t \bar{\omega} \sigma_t^2,$$

$$0 = \gamma \mu_t \bar{\omega}(n_t^* + f_t^* - b_t^*) - \delta_t.$$

where we define $\delta_{-1} = \mu_{-1} = 0$. After simplification:

$$\beta(1-\gamma)x_t = -2\delta_{t-1}(e_t - \mathbb{E}_{t-1}e_t),$$

$$\gamma z_t + (1-\gamma)x_t = -\gamma \lambda_t - \gamma(\mu_t - \beta^{-1}\mu_{t-1}),$$

$$\lambda_t - \mathbb{E}_t \lambda_{t+1} = \beta^{-1}\mu_t \bar{\omega} \sigma_t^2,$$

$$\delta_t = \gamma \mu_t \bar{\omega}(n_t^* + f_t^* - b_t^*).$$

These optimality conditions, together with the constraints in the policy problem (A7) characterize the optimal monetary policy $\{x_t\}$ for a given path of FXI $\{f_t^*\}$ and the associated equilibrium allocation.

**Lemma A3** The optimality conditions for the approximate policy problem (A7) correspond to the first-order Taylor expansion (in $\nu$ around $\nu = 0$) of the non-linear optimality conditions for the exact policy problem (A1').

**Proof:** Given Lemma A2, it remains to show that the first-order Taylor expansion of the exact optimality conditions (A3)–(A6) results in the same system of equations as the first order conditions to the
approximate problem (A7) given above.

In addition to the definitions of ν-deviations of variables in Appendix A2, we define the deviations for multipliers \{M_t, \Lambda_t, D_t\} in the Lagrangian \(\mathcal{L}_0\) for the exact policy problem (A1'):

\[
\Lambda_t = \tilde{\Lambda} e^{\nu \lambda_t}, \quad M_t = \tilde{M} \nu \mu_t, \quad D_t' = \tilde{D}' \delta_t,
\]

where \(\tilde{\Lambda} = \gamma / \tilde{C}_T = \gamma / \tilde{Y}_T\), \(\tilde{M} = \gamma\), and \(\tilde{D}' = \tilde{E}^2\) are proportional scalers. Note that \(M_t\) and \(D_t'\) are equal to zero in a zero-NFA steady state; furthermore, there is only a zero-order component of \(D_t'\), which can be verified by generalizing \(D_t' = D' \delta_t + \nu d_t + \mathcal{O}(\nu^2)\) and showing that \(d_t \equiv 0\) using our approximation below.\(^{55}\)

Consider first the expansion of (A3)–(A6):

\[
\begin{align*}
\tilde{\Lambda}(e^{\nu \lambda_t} - e^{\nu \lambda_{t+1}}) &= \tilde{M} \nu \mu_t \frac{\bar{\omega}}{\nu^2} \beta^{-2} e^{2\nu r_t} \nu_t(e^{-\nu \Delta e_{t+1}}), \\
\tilde{D}' \delta_t &= \tilde{M} \nu \mu_t \frac{\bar{\omega}}{\nu^2} \beta^{-2} e^{2\nu r_t} (e^{\nu r_{t+1}} - \nu \bar{\omega}) \tau_t + \frac{\bar{\omega}}{\nu^2} \beta^{-2} e^{2\nu r_t} \nu_t(n_t^* + f_t^* - b_t^*), \\
\beta(1 - \gamma)(e^{\nu x_t} - 1) &= \frac{2\tilde{D}'}{\tilde{E}^2} \delta_t - \frac{\bar{\omega}}{\nu^2} \beta^{-2} e^{2\nu r_t} \nu_t(n_t^* + f_t^* - b_t^*), \\
\gamma(1 - e^{\nu (\lambda_t + c_{Tt})}) - (1 - \gamma)(e^{\nu x_t} - 1) &= \tilde{M} \nu \mu_t \left( e^{\nu \tau_t} \tilde{E}_t e^{-\nu c_{Tt+1}} + 2\omega \nu r_t \beta e^{\nu r_t} \nu_t(n_t^* + f_t^* - b_t^*) \right) \\
&\quad - \beta^{-1} \tilde{M} \nu \mu_{t-1} \left( e^{\nu \tau_t} - \nu r_t \beta e^{\nu r_t} \nu_t(n_t^* + f_t^* - b_t^*) \right),
\end{align*}
\]

where we used \(\omega = \omega_0 / \nu\) and \(\Gamma_t = 1 / C_{Nt} = e^{-\nu c_{NT}} \bar{\Lambda}\). We take a first order Taylor expansion in \(\nu\) around \(\nu = 0\):

\[
\begin{align*}
\nu \lambda_t - \nu r_t^* - \tilde{E}_t \nu \lambda_{t+1} + \mathcal{O}(\nu^2) &= \beta^{-1} \mu_t \nu (1 + 2 \nu r_t + \mathcal{O}(\nu^2)) (\nu_t + \mathcal{O}(\nu^2)), \\
\delta_t &= \gamma \mu_t \bar{\omega} \left( 1 + 2 \nu (r_t + \tau_t) - \nu r_t^* + \mathcal{O}(\nu^2) \right) (n_t^* + f_t^* - b_t^*), \\
\beta(1 - \gamma) \nu x_t &= 2 \delta_t - \nu (e_t - E_{t-1} e_t) + \mathcal{O}(\nu^2) - (1 + \mathcal{O}(\nu)) \mathcal{O}(\nu^2), \\
-\gamma \nu (\lambda_t + c_{Tt}) + \mathcal{O}(\nu) - (1 - \gamma) \nu x_t &= \gamma \mu_t (1 + \mathcal{O}(\nu)) - \beta^{-1} \gamma \nu \mu_{t-1} (1 + \mathcal{O}(\nu)),
\end{align*}
\]

where we used the definitions of \((\tilde{\Lambda}, \tilde{M}, \tilde{D}')\) and \(\tilde{\omega} = \omega_0 \bar{Y}_T / \beta\), and the result in Lemma A2 that \(\omega \sigma_t^2 - \tilde{\omega} \tilde{\sigma}_t^2 = \mathcal{O}(\nu)\) and \(\nu \tilde{\omega} \tilde{\sigma}_t^2 = \mathcal{O}(\nu)\). Dividing all equations (except for the second line) by \(\nu\) and grouping together the remaining higher order terms, we obtain:

\[
\begin{align*}
\tilde{\lambda}_t - \tilde{E}_t \lambda_{t+1} &= \beta^{-1} \mu_t \tilde{\omega} \sigma_t^2 + \mathcal{O}(\nu), \\
\delta_t &= \gamma \mu_t \bar{\omega} (n_t^* + f_t^* - b_t^*) + \mathcal{O}(\nu), \\
\beta(1 - \gamma) x_t &= -2 \tilde{\delta}_{t-1} (e_t - E_{t-1} e_t) + \mathcal{O}(\nu), \\
\gamma z_t + (1 - \gamma) x_t &= -\gamma \tilde{\lambda}_t - (\gamma \mu_t - \beta^{-1} \gamma \mu_{t-1}) + \mathcal{O}(\nu),
\end{align*}
\]

where \(\sigma_t^2 = \nu_t(\Delta e_{t+1}),\) and we used the optimality condition for the first best tradable consumption,\(^{55}\)

---

\(^{55}\)Note from the solution that \(\delta_{t-1}\) is the slope of the policy rule, \(\beta(1 - \gamma) x_t = -\beta \tilde{\delta}_{t-1} (e_t - E_{t-1} e_t)\) and, just like \(\tilde{\omega} \sigma_t^2\), it does not scale with \(\nu\), while other deviations (in particular, those of \(X_t\) and \(E_t\)) scale proportionally with \(\nu\). In other words, the risk premium and the slope of the optimal policy are zero order in \(\nu\).
\( r_t^* = E_t \Delta \hat{c}_{T+1}, \) the definition of \( z_t = c_{T+1} - \hat{c}_{T+1}, \) and additionally denoted with \( \lambda_t = \lambda_t + \hat{c}_{T+1}. \) Dropping the higher order terms \( O(\nu) \), this system corresponds to the optimality conditions of the approximate problem. ■

### A3 Derivations and Proofs for Section 3

**Proof of Theorem 1** Consider the optimality conditions for the approximate policy problem (A7) derived in Appendix A2.3. In particular, the first and the last optimality conditions (with respect to \( x_t \) and \( \sigma_t^2 \)) are given by:

\[
\beta(1 - \gamma)x_t = -2\delta_{t-1}(e_t - E_{t-1}e_t), \\
\delta_t = \gamma \mu_t \bar{\omega}(n_t^* + f_t^* - b_t). 
\]

Thus, \( \delta_t \) is the optimal monetary policy lean against exchange rate surprises at \( t + 1 \) and \( \mu_t \) is the Lagrange multiplier on \( E_t \Delta z_{t+1} = \bar{\omega} \sigma_t^2(n_t^* + f_t^* - b_t^*). \) If \( E_t \Delta z_{t+1} = 0 \) in every period, then \( \delta_t = \mu_t = 0, \) as the policy problem (A7) is effectively unconstrained and \( x_t = z_t = 0 \) is feasible.

Combining the two conditions to solve out \( \delta_t \) yields:

\[
\beta(1 - \gamma)x_{t+1} = -2\gamma \mu_t \bar{\omega}(n_t^* + f_t^* - b_t^*)(e_{t+1} - E_t e_{t+1}), 
\]

which, in particular, implies \( E_t x_{t+1} = 0. \) ■

**Proof of Proposition 1** Consider the approximate policy problem (A7) where the choice of FXI \( f_t^* \) is unconstrained, and therefore we have an additional optimality condition:

\[ \mu_t = 0 \quad \text{for all} \quad t. \]

From the other optimality conditions, we have \( x_t = \delta_t = 0 \) for all \( t \geq 0, \) as well as:

\[ E_t \Delta \lambda_{t+1} = -E_t \Delta z_{t+1} = 0, \]

which together with the budget constraint implies \( z_t = b_t^* = 0 \) for all \( t \) as the unique solution. Consequently, \( e_t = \tilde{q}_t, \) and \( \bar{\sigma}_t^2 = \text{var}_t(\Delta \tilde{q}_{t+1}). \) Finally, \( E_t \Delta z_{t+1} = 0 \) requires:

\[ \bar{\omega} \bar{\sigma}_t(n_t^* + f_t^* - b_t^*) = 0, \]

and thus generically FXI must satisfy:

\[ f_t^* = b_t^* - n_t^* = -n_t^*. \]

Note that \( f_t^* = -n_t^* \) guarantees \( z_t = b_t^* = 0 \) as the unique equilibrium, as the non-linear system:

\[
E_t \Delta z_{t+1} = -\bar{\omega} \bar{\sigma}_t^2 b_t^*, \quad \bar{\sigma}_t^2 = \text{var}_t(\tilde{q}_{t+1} - z_{t+1}), \\
\beta b_t^* - b_{t-1}^* = -z_t 
\]
has a unique stable solution \( z_t = b_t^* = 0 \).

Lastly, consider the discretionary solution with the planner choosing the optimal policy as a function of natural state variables \((b_{t-1}, \tilde{q}_t, n_t^*)\). This implies that private agents form their expectations about future policies \(z_{t+1} = z(b_{t-1}^*, \tilde{q}_{t+1}, n_{t+1}^*)\) and \(x_{t+1} = x(b_{t-1}^*, \tilde{q}_{t+1}, n_{t+1}^*)\). The only way the planner can credibly manipulate the beliefs in period \(t\) in the absence of commitment is by changing the future state \(b_t^*\). The resulting policy problem corresponds to finding the Markov perfect equilibrium:

\[
V(b^*, \tilde{q}, n^*) = \min_{\{x, f^*, b^*, \sigma^2\}} \frac{1}{2} [\gamma z^2 + (1 - \gamma)x^2] + \beta \mathbb{E}[V(b^{t'}, \tilde{q}', n^{t'})|\tilde{q}, n^*] \tag{A8}
\]

subject to \(\beta b^* = b^* - z\),

\(\mathbb{E}[z(b^{t'}, \tilde{q}', n^{t'})|\tilde{q}, n^*] = z + \tilde{\omega} \sigma^2 (n^* + f^* - b^*)\),

\(\sigma^2 = \text{var}(\tilde{q}' + x(b^{t'}, \tilde{q}', n^{t'}) - z(b^{t'}, \tilde{q}', n^{t'})|\tilde{q}, n^*)\),

where functions \(x(\cdot)\) and \(z(\cdot)\) should be consistent with the solution to this policy problem. Following the primal approach, observe that given a free choice of \(f^*\), the latter two constraints do not bind. It follows that the \(x(b^*, \tilde{q}, n^*) = 0\) and problem reduces to

\[
V(b^*, \tilde{q}, n^*) = \min_{\beta b^*} \frac{\gamma}{2} (b^* - \beta b^*)^2 + \beta \mathbb{E}V(b^{t'}, \tilde{q}', n^{t'}).
\]

A combination of the first-order and envelope conditions implies that \(z(b^*, \tilde{q}, n^*) = (1 - \beta)b^*\). It follows from \(b_{t-1}^* = 0\) that \(z_t = b_t^* = 0\) and the discretionary policy implements the same allocation as the optimal policy under commitment. This allocation is also supported by the same prices, FXI and monetary policy. ■

**Proof of Proposition 2** Consider the case with \(\tilde{q}_t \equiv \tilde{q} = 0\) for all \(t\), the latter equality without loss of generality given our notation in terms of deviations. Then the equilibrium system becomes:

\[
e_t = x_t - z_t, \\
\beta b_t^* - b_{t-1}^* = -z_t, \\
\mathbb{E}_t \Delta z_{t+1} = \tilde{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*), \\
\sigma_t^2 = \text{var}_t(\Delta e_{t+1}),
\]

and it is consistent with a \(x_t = z_t = b_t^* = \sigma_t^2 = 0\) equilibrium for all \(t\) independently of \(\{n_t^*, f_t^*\}\). Since this corresponds to the first best, achieving the global minimum of the welfare loss objective, it is the solution to the optimal policy problem \((A7)\). Indeed, with \(\lambda_t = \mu_t = \delta_t = 0\) for all \(t\), all optimality conditions of \((A7)\) are satisfied, and in particular \((16)\) in Theorem 1 holds irrespective of \(\{n_t^*, f_t^*\}\).

In general, when \(\tilde{q}_t = 0\) and \(x_t = 0\), there exist other equilibria with \(\sigma_t^2 > 0\), such that:

\[
\beta b_t^* - b_{t-1}^* = -z_t, \\
\mathbb{E}_t \Delta z_{t+1} = \tilde{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*), \\
\sigma_t^2 = \text{var}_t(z_{t+1}),
\]

and thus output gap targeting, \(x_t = 0\), does not guarantee \(z_t = b_t^* = 0\) as the unique equilibrium.
In contrast, a policy rule that targets the exchange rate, \( x_t = -\delta(e_t - \mathbb{E}_{t-1}e_t) \) with \( \delta \to \infty \), ensures \( x_t = z_t = b_t^* = \sigma_t^2 = 0 \) as the only equilibrium outcome. Indeed, such a rule implies, using \( e_t = x_t - z_t \), that \( x_t = -\frac{\delta}{1+\delta}(z_t - \mathbb{E}_{t-1}z_t) = -(z_t - \mathbb{E}_{t-1}z_t) \) as \( \delta \to \infty \). This, in turn, means \( e_t = \mathbb{E}_{t-1}e_t \) and \( \sigma_t^2 = 0 \), which ensures \( z_t = b_t^* = 0 \), and hence \( x_t = 0 \), irrespectively of \( \{n_t^*, f_t^*\} \). ■

**Proof of Proposition 3** Without commitment, the planner solves problem (A8) with an additional constraint that the path of \( f_t^* \) is exogenous. Given the values of state variables \((b^*, \tilde{q}, n^*)\), there are three endogenous variables \((b_t^*, z, \sigma_t^2)\) in the three constraints of the problem. It follows that the choice of \( x \) affects the nominal exchange rate \( e = \tilde{q} + x - z \), but not capital flows \( n^* + f^* - b^* \) or UIP deviations \( \mathbb{E}[z(b^*, \tilde{q}, n^*)|\tilde{q}, n^*] - z \). Neither does it change the risk-sharing wedge \( z \) or the continuation value \( V(b^*, \tilde{q}, n^*) \), which implies that it is optimal for the monetary policy to set \( x = 0 \). ■

**Proof of Proposition 4** A further unconstrained choice of \( f_t^* \) at any \( t \geq 0 \) additionally results in \( \mu_t = 0 \), which further implies \( \delta_t = 0 \), \( x_{t+1} = 0 \), and \( \mathbb{E}_t \lambda_{t+1} = \lambda_t \).

Let \( \mu_t := \beta^{-1}\mu_t \) for this proof.

Let \( f_t^* \) be chosen optimal at all \( t \geq 1 \), but fixed at \( f_0^* \) at \( t = 0 \). [This generalizes to any \( t \geq 0 \).]

We then have:

\[
\mu_t = 0 \quad \forall t \geq 1 \quad \Rightarrow \quad x_t = 0 \quad \forall t \neq 1, \quad \mathbb{E}_0 x_1 = 0 \quad \text{and} \quad (1 - \gamma)x_1 = 2\mu_0 \bar{\omega}(n_0^* + f_0^* - b_0^*)\bar{e}_1,
\]

as well as \( \mathbb{E}_t \Delta \lambda_{t+1} = 0 \) for all \( t \geq 1 \) and \( \beta b_0^* = -z_0 \). Furthermore, \( \gamma z_t = \lambda_t \) for all \( t \geq 2 \) (since \( x_t = \mu_t = 0 \) for all \( t \geq 1 \)), and therefore:

\[
\forall t \geq 2 : \quad \gamma \mathbb{E}_t \Delta z_{t+1} = \mathbb{E}_t \Delta \lambda_{t+1} = 0 \quad \Rightarrow \quad f_t^* = b_t^* - n_t^*. 
\]

We use \( \gamma z_t + (1 - \gamma)x_t = \lambda_t + \beta \mu_t - \mu_{t-1} \) for \( t = 0,1 \) together with \( \mathbb{E}_0 \Delta \lambda_1 = -\bar{\omega} \sigma_0^2 \mu_0 \) and \( \mathbb{E}_0 \Delta z_1 = \bar{\omega} \sigma_0^2 (n_0^* + f_0^* - b_0^*) \) to solve for:

\[
\begin{align*}
\mu_0 &= -\frac{\gamma \bar{\omega} \sigma_0^2}{1 + \beta + \bar{\omega} \sigma_0^2}(n_0^* + f_0^* - b_0^*), \\
x_1 &= -\frac{2\gamma}{1 - \gamma} \frac{\bar{\omega} \sigma_0^2}{1 + \beta + \bar{\omega} \sigma_0^2}(n_0^* + f_0^* - b_0^*)^2(e_1 - \mathbb{E}_0 e_1), \\
e_1 - \mathbb{E}_0 &= \frac{(\tilde{q}_1 - z_1) - \mathbb{E}_0 (\tilde{q}_1 - z_1)}{1 + \frac{2\gamma}{1 - \gamma} \frac{\bar{\omega} \sigma_0^2}{1 + \beta + \bar{\omega} \sigma_0^2}(n_0^* + f_0^* - b_0^*)^2},
\end{align*}
\]

where \( \sigma_0^2 = \text{var}_0(e_1) \) and we used \( e_1 - \mathbb{E}_0 e_1 = x_1 + (\tilde{q}_1 - z_1) - \mathbb{E}_0 (\tilde{q}_1 - z_1) \). ■

To complete characterization, also use \( \gamma z_t + (1 - \gamma)x_t = \lambda_t + \beta \mu_t - \mu_{t-1} \) in difference at \( t = 2 \):

\[
\gamma \Delta z_2 = (1 - \gamma)x_1 + \mu_0 = -\frac{\gamma \bar{\omega} \sigma_0^2}{1 + \beta + \bar{\omega} \sigma_0^2}(n_0^* + f_0^* - b_0^*) [1 + 2\bar{\omega}(n_0^* + f_0^* - b_0^*)(e_1 - \mathbb{E}_0 e_1)]
\]

because \( \Delta \lambda_2 = \mathbb{E}_1 \Delta \lambda_2 = 0 \) as there is no uncertainty in \( (x_t, z_t, b_t^*) \) after \( t = 1 \).
We solve for $f_1^*$ from:

$$\bar{\omega}\sigma_1^2(n_1^* + f_1^* - b_1^*) = E_1\Delta z_2 = \Delta z_2 = -\frac{\bar{\omega}\sigma_0^2}{1 + \beta + \bar{\omega}\sigma_0^2} (n_0^* + f_0^* - b_0^*) \left[1 + 2\bar{\omega}(n_0^* + f_0^* - b_0^*)(\epsilon_1 - E_0\epsilon_1)\right],$$

where $\sigma_1^2 = \text{var}(v_2) = \text{var}(\bar{q}_2) = \sigma_0^2$. 

Note that $\beta b_1^* = b_0^* - z_1 = -\beta^{-1}z_0 - z_1$, and then $\beta b_1^* = b_{t-1}^* - z_2$ for $t \geq 2$. 

Finally, we close by solving for $(z_0, z_1, z_2)$ from the intertemporal budget constraint

$$z_0 + \beta z_1 + \frac{\beta^2}{1 - \beta} z_2 = 0, \quad z_t = z_2 \quad \forall t \geq 2,$$

and given solution for $\Delta z_2$ and $E_0\Delta z_1 = \bar{\omega}\sigma_0^2(n_0^* + f_0^* - b_0^*)$, and hence we have:

$$b_0^* = -\beta^{-1}z_0 = E_0[\Delta z_1 + \beta \Delta z_2] = \frac{1 + \bar{\omega}\sigma_0^2}{1 + \beta + \bar{\omega}\sigma_0^2} \bar{\omega}\sigma_0^2(n_0^* + f_0^* - b_0^*) \Rightarrow b_0^* = \frac{(1 + \bar{\omega}\sigma_0^2)\bar{\omega}\sigma_0^2}{\beta + (1 + \bar{\omega}\sigma_0^2)^2} (n_0^* + f_0^*)$$

and

$$n_0^* + f_0^* - b_0^* = \frac{1 + \beta + \bar{\omega}\sigma_0^2}{\beta + (1 + \bar{\omega}\sigma_0^2)^2} (n_0^* + f_0^*).$$

Then we solve:

$$z_1 = (1 - \beta)b_0^* - \beta \Delta z_2 = \frac{\bar{\omega}\sigma_0^2(n_0^* + f_0^*)}{\beta + (1 + \bar{\omega}\sigma_0^2)^2} \left[(1 - \beta)(1 + \bar{\omega}\sigma_0^2) + \beta \left(1 + 2\bar{\omega}\frac{1 + \beta + \bar{\omega}\sigma_0^2}{\beta + (1 + \bar{\omega}\sigma_0^2)^2} (n_0^* + f_0^*)\right)\epsilon_1\right],$$

where $\tilde{e}_1 = e_1 - E_0e_1$, so that:

$$z_1 - E_0z_1 = \frac{2\beta \bar{\omega}^2\sigma_0^2(1 + \beta + \bar{\omega}\sigma_0^2)}{[\beta + (1 + \bar{\omega}\sigma_0^2)^2]^2} (n_0^* + f_0^*)^2 \tilde{e}_1,$$

$$x_1 = -\frac{2\gamma \bar{\omega}^2\sigma_0^2(1 + \beta + \bar{\omega}\sigma_0^2)}{1 - \gamma [\beta + (1 + \bar{\omega}\sigma_0^2)^2]} (n_0^* + f_0^*)^2 \tilde{e}_1,$$

$$\tilde{e}_1 = \frac{\bar{q}_1 - E_0\bar{q}_1}{1 + \gamma + \beta(1 - \gamma) \frac{2\omega^2\sigma_0^2(1 + \beta + \bar{\omega}\sigma_0^2)}{(1 - \gamma)[\beta + (1 + \bar{\omega}\sigma_0^2)^2]} (n_0^* + f_0^*)^2}$$

so that $\sigma_0^2$ solves the fixed point:

$$\sigma_0^2 = \left(1 + \gamma + \beta(1 - \gamma) \frac{2\omega^2\sigma_0^2(1 + \beta + \bar{\omega}\sigma_0^2)}{(1 - \gamma)[\beta + (1 + \bar{\omega}\sigma_0^2)^2]} (n_0^* + f_0^*)^2\right)^{-2} \sigma_0^2.$$

**Derivations for FXI Section 3.4** Assume $x_t = 0$ for all $t$ (either discretion or $\gamma \to 0$ limit). Further start with the artificial case where $\sigma_t^2$ is assumed exogenous:

$$\min_{\{z_t, b_t^*\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} z_t^2 - \lambda_t (b_{t-1}^* - z_t - \beta b_t^*) - \beta \mu_t \left(E_t \Delta z_{t+1} - \bar{\omega}\sigma_t^2(n_t^* + f_t^* - b_t^*)\right)\right]$$

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so that FOCs for \( z_t \) and \( b_t^* \) are:

\[
-z_t = \lambda_t + \beta \mu_t - \mu_{t-1},
\]

\[
\mathbb{E}_t \lambda_{t+1} - \lambda_t = -\mu_t \sigma_t^2,
\]

\[
\mathbb{E}_t \Delta z_{t+1} = \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*),
\]

\[
\beta b_t^* = b_{t-1}^* - z_t.
\]

First condition implies:

\[
(1 + \beta + \bar{\omega} \sigma_t^2) \mu_t - \beta \mathbb{E}_t \mu_{t+1} - \mu_{t-1} = \mathbb{E}_t \Delta z_{t+1} = \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*). \tag{A12}
\]

Thus, in general, any non-zero \( \mu_{t+j} \) for \( j \in \{-1, 0, 1\} \) implies non-zero UIP deviation at \( t \) (and thus \( f_t^* \neq b_t^* - n_t \)). Conversely, when \( \mu_{t-1} = \mu_t = \mu_{t+1} = 0 \), then \( f_t^* = b_t^* - n_t \) and UIP holds at \( t \), independently of the path of \( \{n_t^*, f_t^*, z_t, b_t^*\} \) in other periods.

When \( \mu_t \neq 0 \) and \( \mu_{t+j} = 0 \) for all \( j \neq 0 \), then:

\[
-\mu_t = \mathbb{E}_t (1 + \bar{\omega} \sigma_t^2) \Delta z_{t+2} = \bar{\omega} \sigma_{t+1}^2 (n_{t+1}^* + f_{t+1}^* - b_{t+1}^*),
\]

\[
(1 + \beta + \bar{\omega} \sigma_t^2) \mu_t = \mathbb{E}_t \Delta z_{t+1} = \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*),
\]

\[
-\beta \mathbb{E}_t \mu_{t-1} = \mathbb{E}_{t-1} \Delta z_t = \bar{\omega} \sigma_{t-1}^2 (n_{t-1}^* + f_{t-1}^* - b_{t-1}^*).
\]

and \( f_{t+j}^* = b_{t+j}^* - n_{t+j}^* \) for all \( j \neq \{-1, 0, 1\} \). We have:

\[
\mu_t = \frac{\bar{\omega} \sigma_t^2}{1 + \beta + \bar{\omega} \sigma_t^2} (n_t^* + f_t^* - b_t^*).
\]

Before info at \( t \) is realized, \( \mathbb{E}_{t-1} z_t = z_{t-1} - \beta \mathbb{E}_{t-1} \mu_t \). At \( t \) onwards:

\[
z_{t+1} - z_t = (1 + \bar{\omega} \sigma_t^2) \mu_t = \bar{\omega} \sigma_t^2 (n_t^* + f_t^* - b_t^*),
\]

\[
z_{t+2} - z_{t+1} = -\mu_t = -\frac{\bar{\omega} \sigma_t^2}{1 + \beta + \bar{\omega} \sigma_t^2} (n_t^* + f_t^* - b_t^*),
\]

and the budget constraint requires

\[
0 = b_t^* + z_t + \beta z_{t+1} + \frac{\beta^2}{1 - \beta} z_{t+2} = b_t^* + \frac{1}{1 - \beta} z_t + \frac{\beta}{1 - \beta} \Delta z_{t+1} + \frac{\beta^2}{1 - \beta} \Delta z_{t+2}.
\]

Conjecture a solution such that \( z_{t-j} = b_{t-j}^* = 0 \) for \( j < 0 \), \( b_t^* = 0 \) given \( \mathbb{E}_{t-1} \mu_t = \mathbb{E}_{t-1} (n_t^* + f_t^*) = 0 \), while \( \bar{n}_t^* = (n_t^* + f_t^*) - \mathbb{E}_{t-1} (n_t^* + f_t^*) \) is a non-zero random variable. Then, our equations characterize
the solution \( \{z_t, z_{t+1}, z_{t+2}\} \) as a function of the realized \( \tilde{n}_t^* \). Note that we can simplify:

\[
\begin{align*}
\Delta z_{t+1} &= \bar{\omega}\sigma_t^2 \tilde{n}_t^*, \\
\Delta z_{t+2} &= -\frac{1}{1 + \beta + \bar{\omega}\sigma_t^2} \bar{\omega}\sigma_t^2 \tilde{n}_t^*, \\
z_t &= -\beta \Delta z_{t+1} - \beta^2 \Delta z_{t+2} = -\frac{\beta(1 + \bar{\omega}\sigma_t^2)}{1 + \beta + \bar{\omega}\sigma_t^2} \bar{\omega}\sigma_t^2 \tilde{n}_t^*.
\end{align*}
\]

Thus, an unexpected and unaccommodated capital outflow at \( t, \tilde{n}_t^* > 0 \), results in an unexpected depreciation \( z_t < 0 \), and then an expected appreciation (and a positive UIP deviation) at \( t + 1 \) followed by an expected depreciation (and a negative UIP deviation) at \( t + 2 \), such that: \( z_t < 0 < z_{t+2} < z_{t+1} \), where for all \( j \geq 2 \): \( z_{t+j} = z_{t+2} = \frac{(1-\beta)\bar{\omega}\sigma_t^2}{1 + \bar{\omega}\sigma_t^2} \tilde{n}_t^* \), which is zero as \( \beta \to 1 \). This requires that \( f_{t+1}^* < b_{t+1}^* - n_{t+1}^* \) and \( f_{t+j}^* = b_{t+j}^* - n_{t+j}^* \) for all \( j \geq 2 \) (also for \( j \leq -1 \)).

**Optimizing wrt \( \sigma_t^2 \)**

\[
0 = z_t + \lambda_t + (\beta\mu_t - \mu_{t-1}) - 2\delta_{t-1}(e_t - \mathbb{E}_{t-1}e_t),
\]

\[
\mathbb{E}_t\lambda_{t+1} - \lambda_t = -\mu_t\bar{\omega}\sigma_t^2,
\]

\[
\delta_t = \mu_t\bar{\omega}(n_t^* + f_t^* - b_t^*).
\]

and therefore:

\[
2\mu_{t-1}\bar{\omega}(n_t^* + f_t^* - b_t^*)\tilde{e}_t - z_t = \lambda_t + (\beta\mu_t - \mu_{t-1})
\]

where \( \tilde{e}_t = e_t - \mathbb{E}_{t-1}e_t \).

\[
(1 + \beta + \bar{\omega}\sigma_t^2)\mu_t - \beta\mathbb{E}_t\mu_{t+1} - [1 + 2\bar{\omega}(n_t^* + f_t^* - b_t^*)\tilde{e}_t]\mu_{t-1} = \mathbb{E}_t\Delta z_{t+1} = \bar{\omega}\sigma_t^2 (n_t^* + f_t^* - b_t^*). \tag{A13}
\]

Therefore, if only \( f_t^* \) is constraint and only \( \mu_t \neq 0 \), then:

\[
\mathbb{E}_t\Delta z_{t+1} = (1 + \beta + \bar{\omega}\sigma_t^2)\mu_t,
\]

\[
\mathbb{E}_{t-1}\Delta z_t = -\beta\mathbb{E}_{t-1}\mu_t,
\]

\[
\mathbb{E}_{t+1}\Delta z_{t+2} = -[1 + 2\bar{\omega}(n_t^* + f_t^* - b_t^*)\tilde{e}_{t+1}]\mu_t.
\]

So similar as before, but an amplified effect at \( t + 1 \) by \( \tilde{e}_{t+1} \).
A4 Derivations and Proofs for Sections 4 and 5

A4.1 Derivations for Section 4.1

Three types of agents and their portfolio returns:

1. Domestic households save in home-currency bonds $B_t$ and earn a return $R_t/(1 + \tau_t^h)$. Their Euler equation is given by:

$$\frac{R_t}{1 + \tau_t^h} E_t \left\{ \Theta_{t+1} \frac{\xi_t}{\xi_{t+1}} \right\} = 1, \quad \Theta_{t+1} = \beta \frac{C_{Tt}}{C_{Tt+1}}.$$

2. Domestic financial agents invest $(1 + \tau_H^t) N_{Ht}^*/R_t^*$ and $(1 + \tau_H^t) D_{Ht}^*/R_t^*$ dollars in a carry trade position with a return $\tilde{R}_{Ht+1}^* = \frac{R_t^*}{1 + \tau_H^t} - \frac{R_t}{1 + \tau_H^t} \xi_{t+1}$. While $N_{Ht}^*$ is exogenous, intermediaries' portfolio choice satisfies:

$$E_t \Theta_{t+1} \tilde{R}_{Ht+1}^* = \omega_H \sigma_H^2 \frac{(1 + \tau_{Ht}^*) D_{Ht}^*}{R_t^*}, \quad \sigma_H^2 = \left( \frac{R_t}{1 + \tau_{Ht}} \right)^2 \text{var}_t \left( \frac{\xi_t}{\xi_{t+1}} \right) = \frac{\sigma_t^2}{(1 + \tau_{Ht})^2}.$$

Note that $(1 + \tau_{Ht}) D_{Ht}^*/R_t = -(1 + \tau_{Ht}) \xi_t D_{Ht}^*/R_t^*$ is the home-currency position, where $D_{Ht}^*$ and $D_{Ht}$ are units of the zero-coupon bonds purchased in each currency.

3. Foreign financial agents invest $N_{Ft}^*/R_t^*$ and $D_{Ft}^*/R_t^*$ in a carry trade with a return $\tilde{R}_{Ft+1}^* = R_t^* - \frac{R_t}{1 + \tau_{Ft}} \xi_{t+1}$. While $N_{Ft}^*$ is exogenous, intermediaries' portfolio choice satisfies:

$$E_t \Theta_{t+1} \tilde{R}_{Ft+1}^* = \omega_F \sigma_F^2 \frac{D_{Ft}^*}{R_t^*}, \quad \sigma_F^2 = \left( \frac{R_t}{1 + \tau_{Ft}} \right)^2 \text{var}_t \left( \frac{\xi_t}{\xi_{t+1}} \right) = \frac{\sigma_t^2}{(1 + \tau_{Ft})^2}.$$

We further assume that $\omega$ is a common risk aversion parameter and $m_H$ and $m_F$ are masses of home and foreign arbitrageurs, $m_H + m_F = 1$, and therefore aggregate $\omega_H = \omega/m_H$ and $\omega_F = \omega/m_F$. Combining the household Euler equation with the two portfolio choice conditions:

$$\beta R_t^* E_t \frac{C_{Tt}}{C_{Tt+1}} = \frac{1 + \tau_{Ht}^*}{1 + \tau_{Ht}} + \omega_H \sigma_t^2 \frac{1 + \tau_{Ht}^*}{(1 + \tau_{Ht})^2} \frac{D_{Ht}^*}{R_t^*},$$

$$\beta R_t^* E_t \frac{C_{Tt}}{C_{Tt+1}} = \frac{1 + \tau_{Ft}^*}{1 + \tau_{Ft}} + \omega_F \sigma_t^2 \frac{1}{(1 + \tau_{Ft})^2} \frac{D_{Ft}^*}{R_t^*}.$$

Express out $\sigma_t^2 D_{Ht}^*$ and $\sigma_t^2 D_{Ft}^*$ and add the two resulting equations to solve out $D_{Ht}^* + D_{Ft}^*$ using market clearing

$$D_{Ft}^* + D_{Ht}^* = B_{t}^* - F_{t}^* - N_{Ft}^* - N_{Ht}^*$$

to obtain:

$$\beta R_t^* E_t \frac{C_{Tt}}{C_{Tt+1}} = (1 + \tau_{Ft}^* \frac{m_H (1 + \tau_{Ht})}{m_F (1 + \tau_{Ft})^2} + \frac{\omega F \sigma_t^2}{m_H (1 + \tau_{Ht})^2} + \frac{B_{t}^* - F_{t}^* - N_{t}^*}{R_t^*}.$$
Assuming either (i) \( \tau_{Ht} = \tau_{Ft} = \tau_t \) and \( \tau_{Ht}^* = 0 \) or (ii) \( \tau_{Ft} = \tau_t, \tau_{Ht}^* = \frac{-\tau_t}{1-\tau_t} \) and \( \tau_{Ht} = 0 \) results in (19) in the text.

We next derive the country’s budget constraint. To this end, notice that the net revenues of the government combine returns on FXI and taxes imposed on arbitrageurs and noise traders:

\[
T_t^g = \left( F_{t-1} - \frac{F_{t-1}^*}{R_t^*} \right) + \mathcal{E}_t \left( \frac{R_{t-1}^*}{R_t^*} \right) + B_t \left( \frac{R_{t-1}^*}{R_t^*} \right) + \frac{\tau^h}{R_t} + \tau_{Ht}^* \frac{D_{Ht}^* + N_{Ht}^*}{R_t} + \tau_{Ft}^* \frac{D_{Ft}^* + N_{Ft}^*}{R_t} + \tau_{Ht}^* \frac{E_t D_{Ht}^* + N_{Ht}^*}{R_t}
\]

\[
= \left[ R_{t-1}^* - R_{t-1} \frac{E_{t-1}}{E_t} \right] \frac{R_{t-1}^*}{E_t} + \tau^h_{Ht} B_t + \tau_{Ht}^* \frac{D_{Ht}^* + N_{Ht}^*}{R_t} + \tau_{Ft}^* \frac{D_{Ft}^* + N_{Ft}^*}{R_t} - \tau_{Ft} \frac{E_t D_{Ft}^* + N_{Ft}^*}{R_t},
\]

where we used the zero net position of arbitrageurs, noise traders and the central bank. The budget constraint of households after excluding expenditures on non-tradables is

\[
1 + \tau^h_{Ht} B_t - B_{t-1} = \mathcal{E}_t (Y_{Tt} - C_{Tt}) + T_t^g + T_t^f,
\]

where the profits of local traders are given by

\[
T_t^f = (D_{Ht-1} + N_{Ht-1}) + \mathcal{E}_t (D_{Ht-1}^* + N_{Ht-1}^*) = \left[ R_{t-1}^* - \frac{1 + \tau_{Ht-1}^* \mathcal{E}_{t-1} \frac{R_{t-1}}{R_t}}{1 + \tau_{Ht-1}^* \mathcal{E}_t} \right] \mathcal{E}_t \frac{D_{Ht-1}^* + N_{Ht-1}^*}{R_{t-1}^*}.
\]

Combine the latter three equations to obtain the country’s budget constraint:

\[
B_t \frac{R_t}{R_t} = B_{t-1} + \mathcal{E}_t (Y_{Tt} - C_{Tt}) + \left[ R_{t-1}^* - \frac{1 + \tau_{Ht-1}^* \mathcal{E}_{t-1} \frac{R_{t-1}}{R_t}}{1 + \tau_{Ht-1}^* \mathcal{E}_t} \right] \mathcal{E}_t \frac{D_{Ht-1}^* + N_{Ht-1}^*}{R_{t-1}^*} + \tau^h_{Ht} \frac{D_{Ht}^* + N_{Ht}^*}{R_t} + \tau_{Ft}^* \frac{D_{Ft}^* + N_{Ft}^*}{R_t} - \tau_{Ft} \frac{E_t D_{Ft}^* + N_{Ft}^*}{R_t},
\]

Rewrite the market clearing condition for home bonds

\[
B_t + D_{Ht} + D_{Ft} + N_{Ht} + N_{Ft} + F_t = 0
\]

using the zero net positions of traders as

\[
B_t \frac{R_t}{R_t} = 1 + \tau_{Ht}^* \frac{D_{Ht}^* + N_{Ht}^*}{R_t} + \frac{1 + \tau_{Ft} \mathcal{E}_t}{1 + \tau_{Ft} \mathcal{E}_{t-1}} \mathcal{E}_t \frac{D_{Ft-1}^* + N_{Ft-1}^*}{R_{t-1}^*} + \tau_{Ft} \frac{D_{Ft}^* + N_{Ft}^*}{R_t} + \tau_{Ft} \frac{D_{Ft}^* + N_{Ft}^*}{R_t} + \mathcal{E}_t \frac{R_{t-1}^*}{E_t} \frac{R_{t-1}}{R_t}.
\]

Using this expression, substitute \( B_t \) and \( B_{t-1} \) out of the country’s budget constraint and simplify:

\[
B_t^* = Y_{Tt} - C_{Tt} + \frac{1}{1 + \tau_{Ft-1} \mathcal{E}_t \frac{R_{t-1}}{R_t}} \mathcal{E}_t \frac{D_{Ft-1}^* + N_{Ft-1}^*}{R_{t-1}^*} + (D_{Ht-1}^* + N_{Ht-1}^*) + F_{t-1}^*.
\]

The market clearing condition for foreign bonds implies that this condition is equivalent to

\[
B_t^* = B_{t-1}^* + Y_{Tt} - C_{Tt} - \left[ R_{t-1}^* - \frac{R_{t-1}}{1 + \tau_{Ft-1} \mathcal{E}_t} \right] \frac{D_{Ft-1}^* + N_{Ft-1}^*}{R_{t-1}^*}.
\]

The last term in this budget constraint corresponds to the international transfer of income and can be
rewritten using the definition of $\tilde{R}_{t+1}$ and the optimal portfolio choice of foreign arbitrageurs:

$$\frac{B^*_t}{R^*_t} = B^*_{t-1} + Y_{Tt} - C_{Tt} - \tilde{R}^*_t \left( \frac{\mathbb{E}_{t-1}\Theta_t \tilde{R}^*_{t}}{\omega_F \sigma^2_{t-1}/(1 + \tau_{t-1})^2} + \frac{N^*_{t-1}}{R^*_{t-1}} \right).$$

For the rest of the analysis, assume that at least one of these conditions is satisfied and define net returns on carry trade $\tilde{R}^*_{t+1} = R^*_{t} - \frac{R_t}{1 + \tau_t} \mathcal{E}_{t+1}$. The gross positions of home and foreign arbitrageurs are given by $D^*_H/R^*_t$ and $D^*_F/R^*_t$, the positions of noise traders are $N^*_H/R^*_t$ and $N^*_F/R^*_t$.

### A4.2 Derivations for Section 4.2

Given exogenous shocks as well as monetary and FX policies in each economy, the global equilibrium is determined by household optimality conditions (2) and (3)

$$\frac{\gamma}{1 - \gamma} \frac{C_{Nit}}{C_{Tit}} = \frac{\mathcal{E}_t P^*_{Tt}}{P_{Nit}} \quad \text{and} \quad \beta R^*_{it} \mathcal{E}_{it} \frac{C_{Nit}}{C_{Nit+1} + 1} = 1,$$

the international risk-sharing conditions (7)

$$\beta R^*_{it} \mathcal{E}_{it} \frac{C_{Tit}}{C_{Tit+1}} = 1 + \omega_i \sigma^2_{it} \frac{B^*_{it} - N^*_{it} - F^*_{it}}{R^*_{it}}$$

where $\sigma^2_{it} = R^2_{it} \cdot \text{var}_{t} \left( \frac{\mathcal{E}_{it}}{\mathcal{E}_{it+1}} \right)$

and $\mathcal{E}_{it} = 1$ for currencies $i \in [0, m_0]$ pegged to the dollar, the countries’ budget constraints (6)

$$\frac{B^*_{it}}{R^*_{it}} - B^*_{it-1} = P^*_{Tt}(Y_{Tit} - C_{Tit}),$$

and the global market clearing condition for tradables and bonds

$$\int_0^1 C_{Tit} \text{d}i = \int_0^1 Y_{Tit} \text{d}i \equiv Y_{Tt} \quad \text{and} \quad \int_0^1 B^*_{it} \text{d}i = 0.$$

The problem of a global planner that takes as given the structure of international asset markets is

$$\max_{\{C_{Nit}, C_{Tit}, B^*_it, R^*_it, P^*_Tt, \}_{Tt+1}} \mathbb{E}_0 \sum_{t=0}^\infty \beta^t \int_0^1 \left[ \gamma \log C_{Tit} + (1 - \gamma) \left( \log C_{Nit} - \frac{C_{Nit}}{A_{it}} \right) \right] \text{d}i$$

subject to

$$\frac{B^*_{it}}{R^*_{it}} - B^*_{it-1} = P^*_{Tt}(Y_{Tit} - C_{Tit}),$$

$$\int_0^1 B^*_{it} \text{d}i = 0.$$

We denote the globally efficient allocation with hat’s and contrast it with the non-cooperative efficient allocation denoted with tilde’s.

**Lemma A4** Assume that the global real interest rate satisfies $\beta R^*_t \mathcal{E}_t \frac{Y_{Tt}}{Y_{Tt+1}} \frac{P^*_T}{P^*_{Tt+1}} = 1$. Then to the first order approximation, the local first-best allocation from Section 2.2 coincides with the global efficient allocation.
Proof: Taking the optimality conditions in the planner’s problem, we get that the optimal output in
the non-tradable sector is the same with and without cooperation \( \tilde{C}_{Nit} = \check{C}_{Nit} = A_{it} \). The optimality
conditions with respect to \( C_{Tit} \), \( B^*_{it} \), \( P^*_T \) and \( R^*_t \) are given by

\[
\begin{align*}
\lambda_{it} &= \frac{1}{P^*_T C_{Tit}}, \\
\lambda_{it} &= \beta \tilde{R}^*_t \mathbb{E}_t \lambda_{it+1} + \tilde{R}^*_t \mu_t, \\
0 &= \int_0^1 \lambda_{it} (Y_{Tt} - \check{C}_{T, it}) \, di, \\
0 &= \int_0^1 \lambda_{it} \check{B}_{it} \, di.
\end{align*}
\]

Notice that given the countries’ budget constraints and market clearing for bonds, the latter two opti-
mality conditions are isomorphic and one of them can be dropped as a redundant. Consider a steady
state with \( \bar{B}_i = 0 \) as a point of approximation. We allow for an arbitrary small cross-country variation
in \( \bar{Y}_{Ti} \), but then take the limit \( \bar{Y}_{Ti} \to \bar{Y}_T \). The last optimality condition is then automatically satisfied.
It follows from the budget constraint that \( \bar{C}_{Ti} = \bar{Y}_{Ti} \) and hence, the former two optimality conditions
require that \( 1 - \beta \tilde{R}^* = \mu \tilde{R}^* P^*_T \bar{Y}_T \). Given that the right-hand side of the equation varies with \( i \), the
only possible solution is \( \mu = 0 \), \( \beta R^* = 1 \).

Taking the first-order approximation around this steady state, we get a linearized Euler equation

\[
\mathbb{E}_t \Delta \check{c}_{Tit} = \bar{r}_t^* + \frac{\bar{P}_T \bar{Y}_T}{\beta} \mu_t,
\]

where with a slight abuse of the notation, \( \mu_t \) is the first-order deviation from zero. Integrate across all
\( i \) and impose the market clearing condition \( \int_0^1 \check{c}_{Tit} \, di = y_{Tt} \) to obtain

\[
\mathbb{E}_t \Delta \check{c}_{Tit} = \mathbb{E}_t \Delta y_{Tt+1}.
\]

This equation together with the linearized budget constraint

\[
\beta \check{b}^*_{it} = \check{b}^*_{it} + y_{Tit} - \check{c}_{Tit}, \quad \text{where} \quad \check{b}^*_{it} \equiv B^*_{it}/\bar{Y}_T
\]

and the transversality condition uniquely pin down the globally efficient allocation of tradables. These
conditions coincide with the equilibrium system describing the non-cooperative first best in Section A2.2
when the global real interest rate is equal \( r^*_t = \mathbb{E}_t \Delta y_{Tt+1} \). Thus, under this condition, consumption of
tradables is the same in the optimal cooperative and non-cooperative allocations \( \check{c}_{Tit} = \tilde{c}_{Tit} \). ■

Proof of Lemma 1 Define the global risk-sharing wedge \( \check{z}_{it} \equiv c_{Tit} - \check{c}_{Tit} \), where \( c_{Tit} \) is an arbitrary
path of tradable consumption that satisfies the feasibility constraints and \( \check{c}_{Tit} \) is the globally efficient
level. As before, we focus on a steady state with \( \bar{B}_i = 0 \) and \( \bar{C}_{Ti} = \bar{Y}_{Ti} = \bar{Y}_T \). The first-order
approximation to the market clearing condition implies that

\[ \int_0^1 c_{it} \, di = y_{Tt} = \int_0^1 \hat{c}_{it} \, di \]

and therefore, \( \int_0^1 \hat{z}_{it} \, di = 0 \). Following the results from Section A2.2, linearize the risk-sharing condition to get

\[ \mathbb{E}_t \Delta c_{iTt+1} = \hat{r}_t^* + \psi_{it}, \quad \text{where} \quad \psi_{it} \equiv \bar{\omega}_i \bar{\sigma}_{it}^2 (n_{it}^* + f_{it}^* - b_{it}^*), \quad \bar{\sigma}_{it}^2 \equiv \text{var}(e_{it+1}). \]

The proof of Lemma A4 above shows that the efficient allocation satisfies \( \mathbb{E}_t \Delta \hat{c}_{iTt+1} = \mathbb{E}_t \Delta y_{Tt+1} \equiv \hat{r}_t^* \), where without loss of generality \( \mu_t \) is set to zero. Subtracting this expression from the previous one allows rewriting the risk-sharing condition in terms of the deviations \( \hat{z}_{it} \):

\[ \mathbb{E}_t \Delta \hat{z}_{it+1} = r_t^* - \hat{r}_t^* + \psi_{it}. \]

Integrating across \( i \) and using the market clearing condition, we get that

\[ \mathbb{E}_t \Delta z_{it+1} = \psi_{it} - \bar{\psi}_t \quad \text{and} \quad r_t^* - \hat{r}_t^* = -\bar{\psi}_t, \quad \text{where} \quad \bar{\psi}_t \equiv \int_0^1 \psi_{it} \, di. \]

**Lemma A5** In an equilibrium with sticky prices and frictional financial intermediation, the aggregate welfare loss relative to the global planner’s allocation up to second order is given by:

\[
\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 \left[ \gamma \hat{z}_{it}^2 + (1 - \gamma) x_{it}^2 \right] \, di, \tag{A14}
\]

and the global risk-sharing wedges \( \hat{z}_{it} \) satisfy market clearing \( \int_0^1 \hat{z}_{it} \, di = 0 \) and risk-sharing conditions:

\[ \mathbb{E}_t \Delta \hat{z}_{it+1} = \psi_{it} - \bar{\psi}_t \quad \text{for all} \quad i \in [0, 1], \tag{A15} \]

where \( \psi_{it} \equiv \bar{\omega}_i \bar{\sigma}_{it}^2 (n_{it}^* + f_{it}^* - b_{it}^*) \) with \( \bar{\sigma}_{it}^2 \equiv \text{var}(e_{it+1}) \) is the currency i UIP wedge, and \( \bar{\psi}_t \equiv \int_0^1 \psi_{it} \, di \) is the aggregate real interest rate wedge, \( r_t^* - \hat{r}_t^* = -\bar{\psi}_t \).

**Proof of Proposition 7**

\[
\min_{\{z_{it}, b_{it}^*, \psi_{it}, \bar{\psi}_t\}} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 z_{it}^2 \, di
\]

subject to

\[
\beta b_{it}^* - b_{it-1}^* = -z_{it}, \quad \lambda_{it},
\]

\[ \mathbb{E}_t \Delta z_{it+1} = \psi_{it} - \bar{\psi}_t \quad \mu_{it}, \]

\[ \int_0^1 z_{it} \, di = 0 \quad \eta_t, \]

\[ \bar{\psi}_t = \int_0^1 \psi_{it} \, di \quad \nu_t. \]
FOCs:
\[ z_{it} = \mu_{it} - \beta^{-1} \mu_{i-1} + \lambda_{it} + \eta_t, \]
\[ \lambda_{it} = E_t \lambda_{it+1}, \]
\[ \nu_t = \bar{\mu}_t = \int_0^1 \mu_{it} di, \]
\[ \mu_{it} = \nu_t = \bar{\mu}_t, \]

where the last FOC holds only for the subset of unconstrained countries \( i \in (m_0 + m_1, 1) \), while countries \( i \in [0, m_0] \) are pegged to the dollar (and, hence, have \( \psi_{it} = 0 \)), and countries \( i \in (m_0, m_0 + m_1) \) are non-pegged and constraint with some exogenous \( \psi_{it} \). The unconstrained countries choose \( \psi_{it} \) to ensure \( \mu_{it} = \bar{\mu}_t \), while the constrained countries face an exogenous \( \psi_{it} \) and a corresponding \( \mu_{it} \).

Writing the first FOC in expected differences and using the second FOC to eliminate \( E_t \Delta \lambda_{it+1} = 0 \):
\[ E_t \Delta \mu_{it+1} - \beta^{-1} \Delta \mu_{it} + E_t \Delta \eta_{t+1} = E_t \Delta z_{it+1} = \psi_{it} - \bar{\psi}_t, \]
where the second equality is the risk-sharing constraint. Integrating over \( i \), we have:
\[ (1 + \beta) \bar{\mu}_t - \beta E_t \bar{\mu}_{t+1} - \mu_{t-1} = \beta E_t \Delta \eta_{t+1}. \]
The initial condition is \( \bar{\mu}_{-1} = \mu_{i,-1} = 0 \) for all \( i \) by construction. Conjecture \( E_t \Delta \eta_{t+1} = 0 \), so that \( E_0 \eta_t = \eta_0 \) for all \( t \). This is a reasonable conjecture given that risk-sharing conditions imply expected market clearing at all \( t > 0 \) provided market clearing at \( t = 0 \). Under this conjecture, the solution is \( \bar{\mu}_t = 0 \) for all \( t \geq 0 \). By consequence, \( \mu_{it} = \bar{\mu}_t = 0 \) for all unconstrained countries \( i \), and hence
\[ \psi_{it} = \bar{\psi}_t = m_1 \bar{\psi}_t^C + (1 - m_1 - m_0) \bar{\psi}_t = \frac{m_1}{m_0 + m_1} \bar{\psi}_t^C, \quad \bar{\psi}_t^C = \frac{1}{m_1} \int_{m_0}^{m_0 + m_1} \psi_{it} di, \]
where \( \bar{\psi}_t^C \) is the average UIP deviation for constrained countries. We have:
\[ E_t \Delta z_{it+1} = \begin{cases} - \frac{m_1}{m_0 + m_1} \bar{\psi}_t^C, & i \in [0, m_0], \\ \frac{m_0}{m_0 + m_1} \bar{\psi}_t^C, & \text{on average for } i \in (m_0, m_0 + m_1), \\ 0, & i \in (m_0 + m_1, 1], \end{cases} \]
and, therefore, the conjectured solution implies \( E_0 \int_0^1 z_{it} di = 0 \) for all \( t \geq 0 \), which is necessary for market clearing.

\[ \blacksquare \]

### A4.3 Staggered prices

The derivation of the NKPC and the loss function in the presence of inflation follows the standard steps. Using the property of the model that monetary policy affects exchange rates only via \( \sigma_t^2 \), the planner’s problem can be partitioned in two steps. The first one solves for the optimal trade-off between output
gap and inflation. Because of the certainty equivalence and only first-period innovations affecting $\sigma_t^2$, it is sufficient to focus on the following problem:

$$\min_{\{x_t, \pi_{Nt}\}} \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left( x_t^2 + \alpha \pi_{Nt}^2 \right)$$

s.t. $\pi_{Nt} = \kappa x_t + \beta \pi_{Nt+1} + \nu_t,$

$x_0 + \pi_{N0} = m_t.$

Taking the first-order conditions, we get

$$\beta^t x_t = \kappa \lambda_t + \mu_t,$$

$$\beta^t \alpha \pi_{Nt} = -\lambda_t + \lambda_{t-1} \beta + \mu_t,$$

where $\mu_t = 0$ for $t > 0$ and $\lambda_{-1} = 0$. It follows that the optimality conditions are

$$\alpha \kappa \pi_{Nt} = -x_t + x_{t-1}$$

for $t \geq 1$ and

$$\alpha \kappa \pi_{Nt} = -x_t + (1 + \kappa) \mu_t,$$

for $t = 0$. Substitute the optimality condition into the NKPC, so that dynamics for $t > 0$ is given by

$$\beta x_{t+1} - (1 + \beta + \alpha \kappa^2) x_t + x_{t-1} = \alpha \kappa \nu_t.$$ 

This difference equation has two roots $\lambda_1 > 1$ and $\lambda_2 < 1$

$$\lambda_{1,2} = \frac{1}{2 \beta} \left[ 1 + \beta + \alpha \kappa^2 \pm \sqrt{(1 + \beta + \alpha \kappa^2)^2 - 4 \beta} \right],$$

and assuming for simplicity that $\nu_t$ follows an AR(1) process, we get

$$x_t = \lambda_2 x_{t-1} - \frac{\alpha \kappa}{\beta} \frac{1}{\lambda_1 - \rho} \nu_t.$$

This means that one initial condition $x_0$ is required. At the same time, the NKPC for the first period together with the initial condition imply that

$$\alpha \kappa (m_t - x_0) = \alpha \kappa^2 x_0 - \beta \Delta x_1 + \alpha \kappa \nu_0.$$

Substitute in expression for $x_1$ and solve for

$$x_0 = \frac{\alpha \kappa}{\alpha \kappa^2 + \alpha \kappa + \beta - \beta \lambda_2} \left[ m_t - \frac{\lambda_1}{\lambda_1 - \rho} \nu_0 \right].$$

Substituting this result into equation for $x_t$, we get

$$x_t = k_{x_0} m_t - k_{\nu_0} \nu_0,$$

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\[ \pi_{Nt} = k_{x\pi} m_t - k_{\nu\pi} \varepsilon_{\nu0} \]

for some coefficients \( k \). Substitute this back into the objective function:

\[
\sum_{t=0}^\infty \beta^t (x_t^2 + \alpha \pi_{Nt}^2) = \sum_{t=0}^\infty \beta^t \left[ (k_{x\pi} m_t - k_{\nu\pi} \varepsilon_{\nu0})^2 + \alpha (k_{x\pi} m_t - k_{\nu\pi} \varepsilon_{\nu0})^2 \right]
\]

\[
= K_{x} m_t^2 + K_{\nu} \varepsilon_{\nu0}^2 + K_{x\nu} m_t \varepsilon_{\nu0} = k_1 (m_t - k_2 \varepsilon_{\nu0})^2 + k_3 \varepsilon_{\nu0}^2.
\]

Substitute solution from the first step keeping in mind that it holds for every innovation \( \varepsilon_{\nu0} \) to get the second-stage problem, which is largely isomorphic to the baseline model:

\[
\min_{\{z_t, m_t, b^*_t, f^*_t, \sigma_t^2\}} \quad \frac{1}{2} \mathbb{E} \sum_{t=0}^\infty \beta^t \left[ \gamma z_t^2 + (1 - \gamma) k_1 (m_t - k_2 \varepsilon_{\nu0})^2 \right]
\]

\[
\text{s.t.} \quad \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (ib^*_t - n^*_t - f^*_t),
\]

\[
\beta b^*_t = b^*_{t-1} - z_t,
\]

\[
\sigma_t^2 = \text{var}_t (\hat{q}_{t+1} - z_{t+1} + m_{t+1}).
\]

Going back to the policy in the non-tradable sector, consider whether the price level converges to the initial level in the long run. The optimal policy implements \( \alpha \pi_{Nt} = -\Delta x_t \) for \( t \geq 1 \), just as in a closed economy. However, in the latter case, this condition holds also for \( t = 0 \) (under timeless perspective), which implies that \( \alpha \pi_{Nt} = -x_t \) in all periods and given that \( x_t \) is stationary, the price level converges in the long run to the initial level. In contrast, in our model \( \alpha \pi_{Nt} = -(x_t + \alpha \pi_0) \) and given that \( x_t \to 0 \) in the long run, we get \( p_{Nt} \to \frac{1}{\alpha \pi_0} x_0 + \pi_0 \), which is generically not equal zero.

### A4.4 Terms of trade

To derive the loss function, follow the same steps as in the baseline model. Write down the Lagrangian of the relaxed problem without nominal or financial frictions:

\[
\mathcal{L} = \mathbb{E} \sum_{t=0}^\infty \beta^t \left\{ (1 - \gamma) \log C_{Ht} + \gamma \log C_{Ft} - L_t \right. \\
- \lambda_t \left( A_t L_t - C_{Ht} - \gamma P_{Ht}^{\pi \varepsilon_{t} \gamma} C_t^* + \mu_t \left[ B_{t-1}^* + \gamma P_{Ht}^{\pi \varepsilon_{t} \gamma} C_t^* - C_{Ft} - \frac{B_t^*}{R_t^*} \right] \right. \\
+ \lambda_t \left( A_t L_t - C_{Ht} - \gamma P_{Ht}^{\pi \varepsilon_{t} \gamma} C_t^* \right) + \mu_t \left[ B_{t-1}^* + \gamma P_{Ht}^{\pi \varepsilon_{t} \gamma} C_t^* - C_{Ft} - \frac{B_t^*}{R_t^*} \right].
\]

Notice that the planner is allowed to set optimal price in foreign market and, in equilibrium, charges a constant markup \( \frac{-\varepsilon}{\varepsilon - 1} \) over domestic price for the same goods. Take the first-order conditions and solve for the steady-state values of the Lagrange multipliers: \( \lambda = 1/A, \mu = \left( \frac{\varepsilon}{\varepsilon - 1} \frac{C_t^*}{A} \right)^{\frac{\varepsilon - 1}{\varepsilon}} C_t^* \). Using these values and expression (??), derive quadratic loss function:

\[
\mathcal{L} \propto \frac{1}{2} \mathbb{E} \sum_{t=0}^\infty \beta^t \left\{ (1 - \gamma) c_{Ht}^2 + \gamma c_{Ft}^2 + \gamma (\varepsilon - 1) p_{Ht}^{\pi \varepsilon_{t} \gamma} \right\}, \quad (A16)
\]

where as before, the small letters denote the deviations from the first-best allocation.
When sticky in producer currency, the export price in the currency of destination is equal

\[ P^*_Ht = \frac{\varepsilon}{\varepsilon - 1} \frac{P_{Ht}}{E_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{1 - \gamma}{\gamma} \frac{C_{Ft}}{C_{Ht}}, \]

where the latter equality follows from household demand for goods \( \gamma \). It follows that

\[ p^*_Ht = c_{Ft} - c_{Ht} \]

and it is sufficient to close two gaps in the loss function (A16) to implement efficient allocation. Linearizing the market clearing condition, we get

\[ l_t = (1 - \bar{\gamma}) c_{Ht} - \bar{\gamma} \varepsilon p^*_Ht, \]

where \( \bar{\gamma} \equiv \frac{\gamma(\varepsilon - 1)}{\varepsilon - \gamma} \) is the steady-state share of exports in total output. The last two equations can be solved to express \( c_{Ht} \) and \( p^*_Ht \) in terms of the normalized output gap \( x_t \equiv \frac{1}{1 + \gamma(\varepsilon - 1)} l_t \) and the risk-sharing gap \( z_t \equiv \frac{1}{1 + \gamma(\varepsilon - 1)} c_{Ft}^*: \)

\[ c_{Ht} = \varepsilon \bar{\gamma}z_t + x_t, \quad p^*_Ht = (1 - \bar{\gamma})z_t - x_t. \]

Substitute these expressions into the loss function to obtain

\[ \frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t [\kappa z_t^2 + x_t^2], \]

where \( \kappa \equiv \frac{\varepsilon^2}{\varepsilon - \gamma}. \)

Linearizing the budget constraint and substituting in expression for \( p^*_Ht \), we get

\[ \beta b^*_t = b^*_t - 1 + \frac{\varepsilon - 1}{\varepsilon} x_t - z_t, \]

where \( b^*_t \equiv B^*_t - \tilde{B}^*_t \). Normalizing noise trader shocks \( N^*_t \) and FX interventions \( F^*_t \) by \( \frac{1}{\varepsilon C_F} \), we get the risk-sharing condition

\[ \mathbb{E}_t \Delta z_{t+1} = -\tilde{\omega}\sigma^2 (tb^*_t - n^*_t - f^*_t), \]

where \( \tilde{\omega} \equiv \frac{\omega \varepsilon C_F}{\beta (1 + \gamma(\varepsilon - 1))} \). As before, the nominal exchange rate is given by

\[ e_t = (c_{Ht} + \tilde{c}_{Ht}) - (c_{Ft} + \tilde{c}_{Ft}) = \tilde{q}_t + x_t - (1 - \bar{\gamma})z_t. \]

Combining these conditions, we get the planner’s problem:

\[
\min_{\{x_t, z_t, b^*_t, f^*_t, \sigma^2_t\}} \quad \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\kappa z_t^2 + x_t^2] \\
\text{s.t.} \quad \mathbb{E}_t \Delta z_{t+1} = -\tilde{\omega}\sigma^2 (tb^*_t - n^*_t - f^*_t), \\
\quad \beta b^*_t = b^*_t - 1 + \frac{\varepsilon - 1}{\varepsilon} x_t - z_t, \\
\quad \sigma^2_t = \text{var}(\tilde{q}_{t+1} + x_{t+1} - (1 - \bar{\gamma})z_{t+1}).
\]

Because \( x_t \) drops from the budget constraint in the first-best allocation, the latter can be implemented under the same conditions as in the baseline model. A sufficient condition for \( \tilde{q}_t = 0 \) is that \( r^*_t = 0 \) and \( a_t = c_t^* \) follow a random walk. Indeed, in this case \( \tilde{c}_{Ft} \) is also a random walk and moves one-to-one
with \( a_t \), which given \( \tilde{c}_{Ht} = a_t \) implies that \( \tilde{q}_t = \tilde{c}_{Ht} - \tilde{c}_{Ft} = 0 \).

**DCP** The dollar pricing implies that \( P_{Ht}^* \) is fixed and therefore,

\[
\tilde{p}_{Ht}^* = -\tilde{p}_{Ht} = \tilde{c}_{Ht} - \tilde{c}_{Ft} = \tilde{q}_t.
\]

Define output gap as deviations from the optimal production of locally consumed goods \( x_t = c_{Ht} \) and the risk-sharing wedge as the deviation from the optimal consumption of foreign goods \( z_t = c_{Ft} \) and write the loss function (A16) as

\[
\frac{1}{2} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[ (1 - \gamma) x_t^2 + \gamma z_t^2 + \gamma (\varepsilon - 1) \tilde{q}_t^2 \right].
\]

The first-order approximation to the budget constraint is

\[
\beta b_t^* = b_{t-1}^* - (\varepsilon - 1) \tilde{q}_t - z_t,
\]

where \( b_t^* = \frac{B_t^* - \tilde{B}_t^*}{C_F} \). Intuitively, when the natural real exchange rate depreciates, the export price become too high reducing exports relative to the efficient allocation. Normalizing \( N_t^* \) and \( F_t^* \) by \( C_F \) and defining \( \bar{\omega} = \omega C_F / \beta \), the planner’s problem can be written as

\[
\min_{\{x_t, z_t, b_t^*, f_t^*, \sigma_t^2\}} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 + \gamma (\varepsilon - 1) \tilde{q}_t^2 \right]
\]

s.t. \( \mathbb{E}_t \Delta z_{t+1} = -\bar{\omega} \sigma_t^2 (ib_t^* - n_t^* - f_t^*) \),

\[
\beta b_t^* = b_{t-1}^* - (\varepsilon - 1) \tilde{q}_t - z_t,
\]

\[
\sigma_t^2 = \text{var}_t(\tilde{q}_{t+1} + x_{t+1} - z_{t+1}).
\]

It follows that when \( \tilde{q}_t = 0 \), the first-best allocation with zero losses and \( x_t = z_t = 0 \) is implementable with monetary policy that pegs the nominal exchange rate \( \sigma_t^2 = 0 \). When two policy instruments are available, the risk-sharing condition is not binding and the problem reduces to minimizing the losses subject to the intertemporal budget constraint:

\[
\min_{\{x_t, z_t\}} \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma z_t^2 + (1 - \gamma) x_t^2 \right]
\]

s.t. \( \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (z_t + (\varepsilon - 1) \tilde{q}_t) = 0 \).

Denote the Lagrange multiplier on the budget constraint with \( \mu \) and take the first-order conditions:

\[
\beta^t \gamma z_t = \beta^t \mu, \quad \beta^t (1 - \gamma) x_t = 0.
\]

Therefore, the monetary policy closes the output gap \( x_t = 0 \) and the FX interventions close the UIP gap \( \mathbb{E}_t \Delta z_{t+1} = 0 \) by setting \( f_t^* = ib_t^* - n_t^* \).
**Budget constraint with transfers** Substitute firm profits $\Pi_t = P_{N_t} Y_{N_t} - W_t L_t$ and household consumption expenditure $P t C_t = P_{N_t} C_{N_t} + P_{T_t} C_{T_t}$ into the household budget constraint and use market clearing $C_{N_t} = Y_{N_t}$ to obtain:

$$\frac{B_t}{R_t} - B_{t-1} = NX_t + T_t,$$

where $NX_t = P_{T_t} Y_{T_t} - P_{T_t} C_{T_t} = E_t (Y_{T_t} - C_{T_t})$. Next combine the household and government budget constraints to obtain:

$$\frac{B_t + F_t}{R_t} + \frac{\epsilon_t F_t^*}{R_t^*} - B_{t-1} - F_{t-1} - \epsilon_t F_{t-1}^* = NX_t + \tau \epsilon_t \pi_t^*.$$

Define $B_t^*$ such that $\frac{B_t^*}{R_t^*} = \frac{F_t^*}{R_t^*} + \frac{B_t + F_t}{\epsilon_t R_t}$ and use the market clearing $B_t + D_t + N_t + F_t = 0$ and Lemma ?? that $B_t^* = D_t^* + N_t^* + F_t^*$ to rewrite:

$$\frac{\epsilon_t B_t^*}{R_t^*} - \epsilon_t B_{t-1}^* + \epsilon_t (D_{t-1}^* + N_{t-1}^*) + (D_{t-1} + N_{t-1}) = NX_t + \tau \epsilon_t \pi_t^*.$$

Finally, recall that $\pi_t^* = \tilde{R}_t^* D_{t-1}^* + N_{t-1}^* = \left[1 - \frac{R_{t-1}}{R_t^*} \frac{\epsilon_{t-1}}{\epsilon_t} \right] (D_{t-1}^* + N_{t-1}^*)$. Subtract $\epsilon_t \pi_t^*$ on both sides of the budget of the budget constraint to obtain:

$$\frac{\epsilon_t B_t^*}{R_t^*} - \epsilon_t B_{t-1}^* + \left(\frac{R_{t-1}}{R_t^*} \epsilon_{t-1} (D_{t-1}^* + N_{t-1}^*)\right) = NX_t - (1 - \tau) \tilde{R}_t^* \frac{\epsilon_t (D_{t-1}^* + N_{t-1}^*)}{R_{t-1}^*},$$

$$= 0 \text{ as zero capital portfolio at } t - 1$$

Divide through by $\epsilon_t$, use the fact that $NX_t/\epsilon_t = Y_{T_t} - C_{T_t}$, and Lemma ?? that $D_{t-1}^* + N_{t-1}^* = B_{t-1}^* - F_{t-1}^*$ to rewrite:

$$\frac{B_t^*}{R_t^*} - B_{t-1}^* = (Y_{T_t} - C_{T_t}) - (1 - \tau) \tilde{R}_t^* \frac{\epsilon_t (B_{t-1}^* - F_{t-1}^*)}{R_{t-1}^*},$$

completing the proof of the lemma. □
References


