

# Online Supplement for Trade and Inequality: From Theory to Estimation\*

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## A Introduction

This online supplement contains the technical derivations for the theoretical results in the paper and reports additional empirical results and other information. Section B discusses reduced-form empirical findings for other countries that are consistent with our stylized facts for Brazil in Section 3 of the paper. Section C provides a full characterization of the structural model and discusses the relationship between the reduced-form coefficients and structural parameters. Section D deals with econometric inference, including the derivation of the likelihood function and the generalized method of moments (GMM) bounds analysis. Section G discusses the data sources and definitions. Section H contains additional empirical results and robustness checks referred to in the paper.

## B Reduced-form Empirical Findings for Other Countries

In this section, we discuss that our stylized facts for Brazil are consistent with empirical findings for a number of other countries, including the United States. First, our reduced-form results on wage inequality within and between sectors and occupations (subsection 3.1 of the paper) are consistent with the findings of a number of existing studies. [Katz and Murphy \(1992\)](#) find that much of the growth of wage inequality in the United States from 1963-87 occurred within industry-occupation cells. [Berman, Bound, and Griliches \(1994\)](#) and [Berman, Bound, and Machin \(1998\)](#) find substantial increases in the relative employment and relative wages of skilled workers within industries in the United States and other OECD countries respectively (see also the survey by [Katz and Autor 1999](#)).

Second, our evidence on worker observables and wage inequality (subsection 3.2 of the paper) is in line with the conclusions of the existing labor literature. [Juhn, Murphy, and Pierce \(1993\)](#) find that much of the growth in wage inequality in the United States from 1963-1989 is explained by a growth in residual wage inequality within narrowly-defined education and labor market experience groups (see also [Autor, Katz, and Kearney 2008](#), [Lemieux 2006](#)). Similarly, [Akerman, Helpman, Itskhoki, Muendler, and Redding \(2013\)](#), [Machin \(1996\)](#) and [Attanasio, Goldberg, and Pavcnik \(2004\)](#) show that a substantial component of the level and growth of wage inequality is unexplained by observed worker characteristics in countries as diverse as Sweden, the United Kingdom and Colombia respectively.

Third, our stylized facts on wage inequality between versus within firms (subsection 3.3 of the paper) are consistent with existing studies for other countries. For example, [Davis and Haltiwanger \(1991\)](#) find that between-plant wage dispersion accounts for around one half of the level and growth of wage inequality in U.S. manufacturing from 1975-86. Using data for a later time period, [Barth, Bryson, Davis, and Freeman \(2011\)](#) find that more than 70 percent of the increased dispersion of U.S. earnings among individuals from 1977-2002 occurred across establishments. Using West German data, [Card, Heining, and Kline \(2013\)](#) find that increasing plant-level heterogeneity and rising assortativeness in the assignment of workers to establishments explain a large share of the rise in wage inequality from 1985-2009. Finally, using data on U.K. manufacturing, [Faggio, Salvanes, and Van Reenen \(2010\)](#) find that most of the increase in individual wage inequality can be accounted for by an increase in inequality between firms within industries.

Finally, as discussed in the paper, our reduced-form results relating between-firm wage differences to firm size and export status are supported by a large empirical trade literature following [Bernard and Jensen \(1995, 1997\)](#). Our estimate of the employer-size wage premium using data on raw wages for Brazilian manufacturing of 0.12 compares to a value of 0.14 reported for U.S. manufacturing in [Bayard and Troske \(1999\)](#). Similarly, using raw firm wages, we estimate an exporter premium of 0.26 in Table 7 (after controlling for firm size), which compares to the value of 0.29 reported for U.S. manufacturing in Table 8 of [Bernard, Jensen, Redding, and Schott \(2007\)](#).

## C Theory Appendix

### C.1 Theoretical model

Consider the model in [Helpman, Itskhoki, and Redding \(2010\)](#), henceforth HIR with the following two extensions:

1. Screening costs are heterogenous across firms and given by

$$e^{-\eta} \frac{C}{\delta} (a_c)^\delta,$$

where  $\eta$  varies across firms, while  $C$  and  $\delta$  are common to all firms. In the original HIR model  $\eta \equiv 0$ .

2. Fixed costs of exporting are heterogenous across firms and given by

$$e^\varepsilon F_x,$$

where  $\varepsilon$  varies across firms and the fixed export cost  $F_x$  is common to all firms. In the original HIR model  $\varepsilon \equiv 0$ .

Simplifying HIR slightly, we set the fixed cost of production to zero for all firms ( $f_d = 0$ ).

Here we describe the solution to the problem of a given firm in an industry, taking industry labor market tightness as given. The details of the general equilibrium can be found in HIR. A firm with a shock triplet  $(\theta, \eta, \varepsilon)$  solves the following problem as explained in the text and HIR:

$$\Pi(\theta, \eta, \varepsilon) = \max_{N, a_c, \iota \in \{0, 1\}} \left\{ \frac{1}{1 + \beta\gamma} R(N, a_c, \iota; \theta) - bN - e^{-\eta} \frac{C}{\delta} (a_c)^\delta - \iota e^\varepsilon F_x \right\}, \quad (\text{C.1})$$

where:<sup>1</sup>

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<sup>1</sup>Revenues, given export status  $\iota \in \{0, 1\}$ , are a function of total output of the firm defined by:

$$R(Y|\iota) = \max_{Y_d + Y_x \leq Y} \left\{ A_d Y_d^\beta + \iota A_x (Y_x/\tau)^\beta \right\},$$

where  $A_d Y_d^\beta$  are revenues from domestic sales and  $A_x (Y_x/\tau)^\beta$  are revenues from exporting. The parameter  $\tau$  reflects variable trade costs (including iceberg transport costs). The demand is derived from a CES aggregator with elasticity  $1/(1 - \beta)$ , and  $A_d$  and  $A_x$  are demand shifters which depend on total industry expenditure and a price index, as detailed in HIR. The optimal allocation of sales across markets for an exporter satisfies  $Y_x/Y_d = \tau^{-\beta/(1-\beta)} (A_x/A_d)^{1/(1-\beta)}$ , and therefore we can write

$$R(N, a_c, \iota; \theta) = [1 + \iota(\Upsilon_x - 1)]^{1-\beta} A_d Y(N, a_c; \theta)^\beta,$$

$$Y(N, a_c; \theta) = \frac{k a_{\min}^{\gamma k}}{k-1} e^\theta N^\gamma (a_c)^{1-\gamma k},$$

and

$$\Upsilon_x = 1 + \tau^{\frac{-\beta}{1-\beta}} \left( \frac{A_x}{A_d} \right)^{\frac{1}{1-\beta}}.$$

As described in the text, the employment of the firm is given by

$$H(N, a_c) = N \cdot (a_{\min}/a_c)^k.$$

The bargained wage rate of the firm is given by:<sup>2</sup>

$$W(N, a_c, \iota; \theta) = \frac{\beta\gamma}{1 + \beta\gamma} \frac{R(N, a_c, \iota; \theta)}{H(N, a_c)},$$

and the remaining share  $1/(1 + \beta\gamma)$  of revenues goes to the firm.

Taking the first-order conditions in (C.1) with respect to  $N$  and  $a_c$ , and using the expression for  $H$  and  $W$  above, we arrive at expressions (9)–(11) in the text:

$$R = \kappa_r [1 + \iota(\Upsilon_x - 1)]^{\frac{1-\beta}{\Gamma}} \left( e^\theta \right)^{\frac{\beta}{\Gamma}} \left( e^\eta \right)^{\frac{\beta(1-\gamma k)}{\delta\Gamma}},$$

$$H = \kappa_h [1 + \iota(\Upsilon_x - 1)]^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} \left( e^\theta \right)^{\frac{\beta(1-k/\delta)}{\Gamma}} \left( e^\eta \right)^{\frac{\beta(1-\gamma k)(1-k/\delta)}{\delta\Gamma} - \frac{k}{\delta}},$$

$$W = \kappa_w [1 + \iota(\Upsilon_x - 1)]^{\frac{k(1-\beta)}{\delta\Gamma}} \left( e^\theta \right)^{\frac{\beta k}{\delta\Gamma}} \left( e^\eta \right)^{\frac{k}{\delta} \left( 1 + \frac{\beta(1-\gamma k)}{\delta\Gamma} \right)},$$

where the expressions for the constants can be found in HIR (in particular, see the Appendix to HIR at [http://www.econometricsociety.org/ecta/Supmat/8640\\_extensions.pdf](http://www.econometricsociety.org/ecta/Supmat/8640_extensions.pdf)). The first-order conditions further imply that the firm's profits are

$$\Pi(\theta, \eta, \varepsilon; \iota) = \frac{\Gamma}{1 + \beta\gamma} R(\theta, \eta; \iota) - \iota e^\varepsilon F_x,$$

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$Y_d = Y/\Upsilon_x$  and  $Y_x = (\Upsilon_x - 1)Y/\Upsilon_x$ , where  $\Upsilon_x$  is defined in the text. The resulting revenues from domestic sales are  $R_d = A_d(Y/\Upsilon_x)^\beta = R/\Upsilon_x$ .

<sup>2</sup>As detailed in HIR, at the bargaining stage the firm's costs are sunk and the firm bargains with its workers to divide revenues from production  $R(N, a_c, \iota; \theta)$ . The firm's revenues are a power function of employment  $H$  with power  $\gamma\beta$ , and the firm and the workers know only that each worker has an ability of at least  $a_c$ , and hence an expected ability of  $\bar{a}$ . The outside option of the worker is normalized to zero, and the [Stole and Zwiebel \(1996a\)](#) bargaining condition under these circumstances is

$$\frac{\partial}{\partial H} [R(H) - W(H)H] = W(H),$$

where we emphasize that at the bargaining stage the revenues and the resulting wage rate depend only on employment of the firm. The solution to this differential equation in  $H$  is  $W = \beta\gamma/(1 + \beta\gamma) \cdot R/H$ .

where

$$\Gamma = 1 - \beta\gamma - \beta(1 - \gamma k)/\delta.$$

Revenues are a function of the firm's export status  $\iota \in \{0, 1\}$ . Similarly, the optimally chosen  $N$  and  $a_c$  are a function of export status given  $(\theta, \eta)$ . The firm chooses to export when  $\Pi(\theta, \eta, \varepsilon; \iota = 1) \geq \Pi(\theta, \eta, \varepsilon; \iota = 0)$ , which can be written as condition (12) in the text:

$$\iota = \mathbb{I} \left\{ \kappa_\pi \left( \Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1 \right) \left( e^\theta \right)^{\frac{\beta}{\Gamma}} \left( e^\eta \right)^{\frac{\beta(1-\gamma k)}{\delta\Gamma}} \geq F_x e^\varepsilon \right\},$$

where  $\kappa_\pi = \Gamma\kappa_r/(1 + \beta\gamma)$ , and  $\mathbb{I}\{\cdot\}$  is the indicator function. The above statements describe the relevant equilibrium conditions of the model. The remaining general equilibrium conditions can be found in HIR.

## C.2 Derivation of the empirical model

We start by taking logs of the expressions for  $H$ ,  $W$  and inside the indicator function in the expression for  $\iota$ :

$$\begin{cases} h &= \alpha_h + (1 - \frac{k}{\delta}) \left[ \iota \cdot \frac{1-\beta}{\Gamma} \log \Upsilon_x + \frac{\beta}{\Gamma} \theta + \left( \frac{\beta(1-\gamma k)}{\delta\Gamma} - \frac{k/\delta}{1-k/\delta} \right) \eta \right], \\ w &= \alpha_w + \frac{k}{\delta} \left[ \iota \cdot \frac{1-\beta}{\Gamma} \log \Upsilon_x + \frac{\beta}{\Gamma} \theta + \left( 1 + \frac{\beta(1-\gamma k)}{\delta\Gamma} \right) \eta \right], \\ \iota &= \mathbb{I} \left\{ \frac{1}{\sigma} \left( \frac{\beta}{\Gamma} \theta + \frac{\beta(1-\gamma k)}{\delta\Gamma} \eta - \varepsilon \right) \geq f \right\}, \end{cases}$$

where

$$f = \frac{1}{\sigma} \left( -\alpha_\pi + \log F_x - \log \left[ \Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1 \right] \right),$$

$\alpha_s = \log \kappa_s$  (for  $s = h, w, \pi$ ) and  $\sigma$  is the variance of  $[(\beta/\Gamma) \cdot \theta + \beta(1 - \gamma k)/(\delta\Gamma) \cdot \eta - \varepsilon]$ , explicitly given in (C.2) below.

It is convenient to introduce the following notation to simplify the expressions in subsequent derivations:

$$\begin{aligned} \chi &= \frac{k/\delta}{1 - k/\delta}, & \lambda_1 &= \frac{\beta}{\Gamma}, & \lambda_2 &= \frac{\beta(1 - \gamma k)}{\delta\Gamma}, \\ \mu_h &= \frac{1}{1 + \chi} \frac{1 - \beta}{\Gamma} \log \Upsilon_x, & \text{and} & & \mu_w &= \frac{\chi}{1 + \chi} \frac{1 - \beta}{\Gamma} \log \Upsilon_x = \chi \mu_h. \end{aligned}$$

With this notation we can write the structural model simply as:

$$\begin{cases} h &= \alpha_h + \mu_h \iota + \frac{\lambda_1}{1 + \chi} \theta + \frac{\lambda_2 - \chi}{1 + \chi} \eta, \\ w &= \alpha_w + \mu_w \iota + \frac{\chi \lambda_1}{1 + \chi} \theta + \frac{\chi(1 + \lambda_2)}{1 + \chi} \eta, \\ \iota &= \mathbb{I} \left\{ (\lambda_1 \theta + \lambda_2 \eta - \varepsilon) / \sigma \geq f \right\}, \end{cases}$$

Using the definition of  $\Gamma$ , we show that:

$$\lambda_2 - \chi = -\frac{\chi}{\Gamma} \left( 1 - \frac{\beta}{k} \right) < 0,$$

since the model's parameter restrictions are  $\beta < 1 < k$ . This implies that the effect of  $\eta$  on  $h$  is negative.

Finally, we make the distributional assumption on the shocks:

$$(\theta, \eta, \varepsilon)' \sim \mathcal{N}(\mathbf{0}, \Sigma), \quad \Sigma = \begin{pmatrix} \sigma_\theta^2 & & \\ \sigma_{\theta\eta} & \sigma_\eta^2 & \\ \sigma_{\theta\varepsilon} & \sigma_{\eta\varepsilon} & \sigma_\varepsilon^2 \end{pmatrix}.$$

The above four expressions for  $h$ ,  $w$ ,  $\iota$  and the distribution of  $(\theta, \eta, \varepsilon)$ , together with the definitions of the parameters  $(\chi, \mu_h, \mu_w, \lambda_1, \lambda_2, f)$ , fully describe the structural model. The model implies that the following parameter restrictions:<sup>3</sup>

$$\chi, \mu_h, \mu_w, \lambda_1, \lambda_2 > 0, \quad \lambda_2 < \chi, \quad \mu_w = \chi\mu_h \quad \text{and} \quad \mu_h + \mu_w = \log \Upsilon_x^{(1-\beta)/\Gamma}.$$

We now derive the relationship between the structural parameters of the model and the reduced-form coefficients of our econometric model. Our derivation involves an orthogonalization of the shocks in the wage and employment equations. Define the first reduced-form shock

$$u = \frac{\lambda_1}{1+\chi}\theta + \frac{\lambda_2 - \chi}{1+\chi}\eta$$

so that

$$h = \alpha_h + \mu_h \iota + u.$$

Next, rewrite the wage equation as

$$\begin{aligned} w &= \alpha_w + \mu_w \iota + \chi u + \chi \eta \\ &= \alpha_w + \mu_w \iota + \chi(1 + \pi)u + \chi(\eta - \pi u), \end{aligned}$$

where  $\pi$  is the projection coefficient of  $\eta$  on  $u$ , so that  $(\eta - \pi u)$  is uncorrelated with  $u$ . Under our normality assumption, we further have  $\mathbb{E}\{\eta|u\} = \pi u$ .

We can now define the second reduced-form shock

$$v = \chi(\eta - \pi u),$$

so that  $\text{cov}(u, v) = 0$ , and we can write

$$w = \alpha_w + \mu_w \iota + \zeta u + v,$$

where

$$\zeta = \chi(1 + \pi) < \chi.$$

Note that the orthogonality of  $u$  and  $v$  is without loss of generality because it is a normalization.

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<sup>3</sup>The parameter restrictions are discussed in detail in HIR, and we require  $\Upsilon_x > 0$ ,  $\beta \in (0, 1)$ ,  $1 < k < \delta$ ,  $0 < \gamma k < 1$ , and  $\Gamma > 0$ , which leads to these restrictions on the derived parameters.

Consider now the selection equation. Note that

$$\lambda_1 \theta + \lambda_2 \eta = (1 + \zeta)u + v.$$

Define

$$z = \frac{1}{\sigma} [(1 + \zeta)u + v - \varepsilon],$$

where

$$\sigma^2 = (1 + \zeta)^2 \sigma_u^2 + \sigma_v^2 + \sigma_\varepsilon^2 - 2(1 + \zeta)\sigma_{u\varepsilon} - 2\sigma_{v\varepsilon}, \quad (\text{C.2})$$

and

$$\begin{aligned} \sigma_u^2 &= \frac{\lambda_1^2}{(1 + \chi)^2} \sigma_\theta^2 + \frac{(\lambda_2 - \chi)^2}{(1 + \chi)^2} \sigma_\eta^2, \\ \sigma_v^2 &= \chi^2 (\sigma_\eta^2 + \pi^2 \sigma_u^2 - 2\pi \text{cov}(\eta, u)) = \chi^2 (\sigma_\eta^2 - \pi^2 \sigma_u^2), \\ \sigma_{u\varepsilon} &= \frac{\lambda_1}{1 + \chi} \sigma_{\theta\varepsilon} + \frac{\lambda_2 - \chi}{1 + \chi} \sigma_{\eta\varepsilon}, \\ \sigma_{v\varepsilon} &= \chi (\sigma_{\eta\varepsilon} - \pi \sigma_{u\varepsilon}). \end{aligned}$$

Therefore, we have  $\sigma_z^2 = \text{var}(z) = 1$ . We hence rewrite the selection equation simply as

$$\iota = \mathbb{I}\{z \geq f\}.$$

The joint distribution of the reduced-form shocks  $(u, v, z)$  is given by:

$$(u, v, z)' \sim \mathcal{N}(\mathbf{0}, \Sigma_R), \quad \Sigma_R = \begin{pmatrix} \sigma_u^2 & & \\ 0 & \sigma_v^2 & \\ \rho_u \sigma_u & \rho_v \sigma_u & 1 \end{pmatrix}, \quad (\text{C.3})$$

where

$$\begin{aligned} \rho_u \sigma_u &= \frac{1}{\sigma} ((1 + \zeta)\sigma_u^2 - \sigma_{u\varepsilon}), \\ \rho_v \sigma_v &= \frac{1}{\sigma} (\sigma_v^2 - \sigma_{v\varepsilon}). \end{aligned}$$

This completes the derivation of the reduced-form system. We now discuss the model's restrictions on the reduced-form coefficients,

$$\Theta = (\alpha_h, \alpha_w, \zeta, \sigma_u, \sigma_v, \rho_u, \rho_v, \mu_h, \mu_w, f)'. \quad (\text{C.4})$$

As derived above, we have  $\mu_h, \mu_w > 0$ ,  $\mu_w/\mu_h = \chi$  and  $\zeta = \chi(1 + \pi) < \chi$ . As discussed in the paper, we estimate the model under the structural identifying assumption of a covariance condition on the structural shocks ( $\sigma_{\theta\eta} = 0$ ) as is common in the structural econometrics literature following [Koopmans \(1949\)](#), [Fisher \(1966\)](#) and [Wolpin \(2013\)](#). Under this structural covariance restriction, the expression for



the projection coefficient  $\pi$  is:

$$\pi = \frac{\text{cov}(\eta, u)}{\text{var}(u)} = \frac{\frac{\lambda_2 - \chi}{1 + \chi} \sigma_\eta^2}{\frac{\lambda_1^2}{(1 + \chi)^2} \sigma_\theta^2 + \frac{(\lambda_2 - \chi)^2}{(1 + \chi)^2} \sigma_\eta^2} \leq 0, \quad (\text{C.4})$$

where  $\pi < 0$  follows from the model's parameter restriction  $\lambda_2 < \chi$ . The structural covariance restriction ( $\sigma_{\theta\eta} = 0$ ) helps to separately identify the market access and selection forces by placing bounds on the relative market access effects ( $\mu_h/\mu_w$ ):

**Lemma S.1** *Under the structural covariance restriction  $\sigma_{\theta\eta} = 0$ , the reduced-form coefficients must satisfy the inequality constraint:*

$$\zeta \leq \frac{\mu_w}{\mu_h} < \zeta + \frac{\sigma_v^2}{(1 + \zeta) \sigma_u^2}. \quad (\text{C.5})$$

**Proof:** Consider the definition of  $\pi$  in (C.4), which combined with the definition of  $\sigma_u^2$  can be written as:

$$\pi = \frac{(\lambda_2 - \chi) \sigma_\eta^2}{(1 + \chi) \sigma_u^2}.$$

Rearranging, we can rewrite:

$$\left(1 - \frac{\lambda_2}{\chi}\right) \frac{\chi^2 \sigma_\eta^2}{(1 + \chi) \sigma_u^2} = -(\chi\pi).$$

Since  $0 < \lambda_2 < \chi$ , we have  $0 < 1 - \lambda_2/\chi < 1$ , and therefore

$$0 \leq \left(1 - \frac{\lambda_2}{\chi}\right) \frac{\chi^2 \sigma_\eta^2}{(1 + \chi) \sigma_u^2} < \frac{\chi^2 \sigma_\eta^2}{(1 + \chi) \sigma_u^2},$$

so that

$$0 \leq -(\chi\pi) < \frac{\chi^2 \sigma_\eta^2}{(1 + \chi) \sigma_u^2}.$$

Next, we use the definition of  $\sigma_v^2$  to substitute in for  $\chi^2 \sigma_\eta^2$ :

$$0 \leq -(\chi\pi) < \frac{\sigma_v^2 + (\chi\pi)^2 \sigma_u^2}{(1 + \chi) \sigma_u^2}.$$

Note that this is equivalent to:

$$0 \leq -(\chi\pi) < \frac{\sigma_v^2}{(1 + \zeta) \sigma_u^2},$$

where we have used the fact that  $(1 + \chi + \chi\pi) = 1 + \zeta$  from the definition of  $\zeta$ . Finally, note that  $\chi\pi = \zeta - \chi$  and  $\chi = \mu_w/\mu_h$ . Hence the above inequalities are equivalent to the inequality constraint (C.5). ■

We summarize the reduced-form empirical specification by:

$$\begin{cases} h &= \alpha_h + \mu_h \iota + u, \\ w &= \alpha_w + \mu_w \iota + \zeta u + v, \\ \iota &= \mathbb{I}\{z \geq f\}, \end{cases} \quad (\text{C.6})$$

with the shocks  $(u, v, z)$  distributed according to (C.3). The data is  $x = (h, w, \iota)$  and  $\Theta = (\alpha_h, \alpha_w, \zeta, \sigma_u, \sigma_v, \rho_u, \rho_v, \mu_h, \mu_w, f)'$  is the  $10 \times 1$  vector of reduced-form coefficients. We estimate the reduced-form coefficients  $\Theta$  subject to the inequality constraint (C.5) implied by our structural covariance restriction. We check whether the resulting parameter estimates  $\hat{\Theta}$  satisfy the theoretical restriction that  $\hat{\mu}_h, \hat{\mu}_w \geq 0$  (without imposing this additional restriction on the estimation).

### C.3 Reduced-form coefficients and structural parameters

The model has 10 reduced-form coefficients  $\Theta$  and 12 structural parameters

$$\Xi = (\alpha_h, \alpha_w, \mu_h, \chi, \lambda_1, \lambda_2, f, \sigma_\theta, \sigma_\eta, \sigma_\varepsilon, \sigma_{\theta\varepsilon}, \sigma_{\eta\varepsilon}),$$

where we have taken into account our structural covariance restriction that  $\sigma_{\theta\eta} = 0$ . The coefficients  $(\mu_w, \sigma, \pi)$  are not part of the structural parameter vector  $\Xi$  because we can fully recover  $(\mu_w, \sigma, \pi)$  from  $\Xi$  using (C.4), (C.2) and the condition  $\mu_w = \chi\mu_h$ . The parameters  $(k, \delta, \beta, \gamma)$  are related to  $(\chi, \lambda_1, \lambda_2)$  as shown above and can be recovered after one of the parameters  $(k, \delta, \beta, \gamma)$  is calibrated. Some other parameters like  $b$  and  $C$  are related to the constants  $(\alpha_h, \alpha_w)$  as described in HIR, but we do not attempt to recover them and hence do not discuss them here.

We can relate the reduced-form coefficients  $\Theta$  back to the structural parameters  $\Xi$  using the derivations in subsection C.2. We summarize the according relationships below:

$$\begin{aligned} \zeta &= \chi(1 + \pi), & \chi &= \frac{k/\delta}{1-k/\delta} \\ \sigma_u^2 &= \frac{\lambda_1^2}{(1+\chi)^2} \sigma_\theta^2 + \frac{(\lambda_2-\chi)^2}{(1+\chi)^2} \sigma_\eta^2, & \lambda_1 &= \frac{\beta}{\Gamma}, \\ \sigma_v^2 &= \chi^2 (\sigma_\eta^2 - \pi^2 \sigma_u^2), & \lambda_2 &= \frac{\beta(1-\gamma k)}{\delta\Gamma}, \\ \rho_u &= \frac{1}{\sigma_{\sigma_u}} ((1 + \zeta) \sigma_u^2 - \sigma_{u\varepsilon}), & \pi &= \frac{(1+\chi)(\lambda_2-\chi) \sigma_\eta^2}{\lambda_1^2 \sigma_\theta^2 + (\lambda_2-\chi)^2 \sigma_\eta^2}, \\ \rho_v &= \frac{1}{\sigma_{\sigma_v}} (\sigma_v^2 - \sigma_{v\varepsilon}), & \sigma^2 &= (1 + \zeta)^2 \sigma_u^2 + \sigma_v^2 + \sigma_\varepsilon^2 - 2(1 + \zeta) \sigma_{u\varepsilon} - 2\sigma_{v\varepsilon}, \\ \mu_h &= \frac{1}{1+\chi} \frac{1-\beta}{\Gamma} \log \Upsilon_x, & \sigma_{u\varepsilon} &= \frac{\lambda_1}{1+\chi} \sigma_{\theta\varepsilon} + \frac{\lambda_2-\chi}{1+\chi} \sigma_{\eta\varepsilon}, \\ \mu_w &= \chi\mu_h, & \sigma_{v\varepsilon} &= \chi(\sigma_{\eta\varepsilon} - \pi\sigma_{u\varepsilon}). \\ f &= \frac{1}{\sigma} \left( -\alpha_\pi + \log F_x - \log \left[ \Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1 \right] \right), \end{aligned}$$

while  $(\alpha_h, \alpha_w)$  are a part of both the reduced-form coefficients  $\Theta$  and the structural parameters  $\Xi$ . The constants  $(\alpha_h, \alpha_w)$ , market access effects  $(\mu_h, \mu_w)$  and export threshold  $(f)$  depend on  $\Upsilon_x$ , which in turn depends on variable trade costs  $(\tau)$  and relative market demands in the export and domestic markets  $(A_x/A_d)$  that are likely to change over time.

The coefficients  $(\alpha_h, \alpha_w, \mu_h, \mu_w, f)$  can be estimated directly with the reduced-form model. We can identify  $\chi$ , and hence  $k/\delta$ , from  $\chi = \mu_w/\mu_h$ . Furthermore, we can identify the market access effect from

$$\log \Upsilon_x^{\frac{1-\beta}{\Gamma}} = \mu_h + \mu_w.$$

From the reduced-form estimate of  $\zeta$  we can recover  $\pi = \zeta/\chi - 1$ , which is itself a derived pa-

parameter and provides information about structural parameters. Estimates of the covariance matrix  $(\sigma_u, \sigma_v, \rho_u, \rho_v)$  impose four conditions on the following five parameter combinations:

$$(\lambda_1 \sigma_\theta, (\lambda_2 - \chi) \sigma_\eta, \sigma_\varepsilon, \lambda_1 \sigma_{\theta\varepsilon}, (\lambda_2 - \chi) \sigma_{\eta\varepsilon}).$$

Therefore, the structural parameters are under-identified. Nonetheless, this does not constitute a limitation for our counterfactual exercises, because the coefficients of the reduced-form model  $\Theta$  are sufficient statistics for the impact of trade on wage inequality, as discussed in detail below.

We now briefly discuss what information about the structural parameters can nonetheless be obtained from the reduced-form estimates. Instead of 12 structural parameters in  $\Xi$ , there are only 11 that can be identified in principle, since  $\lambda_1$  and  $\sigma_\theta$  always show up together multiplicatively ( $\lambda_1 \sigma_\theta$ ), even in the structural equations. We split the above relationships between the reduced-form coefficients and structural parameters into two blocks—the first one defines the second moments and the second one defines the selection correlations, given the second moments:

**Block 1 (variances)** Estimates of  $(\sigma_u, \sigma_v, \chi)$  allow to recover  $(\lambda_1 \sigma_\theta, \chi \sigma_\eta, \lambda_2 / \chi - 1)$ , where we treat  $\chi \equiv \mu_w / \mu_h$  as an estimated parameter. Given that the inequality constraint (C.5) on the reduced-form coefficients is satisfied, this block always has a solution.

**Block 2 (selection)** Estimates of  $(\rho_u, \rho_v)$ —along with the parameter estimates from the previous block and definitions of the auxiliary parameters  $(\sigma, \sigma_{u\varepsilon}, \sigma_{v\varepsilon})$ —provide information about  $(\sigma_\varepsilon, \sigma_{\theta\varepsilon}, \sigma_{\eta\varepsilon})$ . This block is under-identified as we have two relationships tying together three parameters. Therefore, we need to calibrate one of the structural parameters here, and this block imposes no additional restrictions on the reduced-form coefficients.

Despite the under-identification in the second (selection) block, we can nonetheless assess the strength of the selection effect in the estimated model. Indeed, the knowledge of  $(\rho_u, \rho_v)$  is sufficient to quantify the overall contribution of productivity and screening shocks  $(\theta, \eta)$  to variation in export status. Note, first, that the amount of information in  $(\theta, \eta)$  is the same as in  $(u, v)$ , since the latter is a linear non-degenerate transformation of the former. Further, given the joint distribution (C.3), the regression of  $z$  on  $(u, v)$  is given by

$$\mathbb{E}\{z|u, v\} = \rho_u \frac{u}{\sigma_u} + \rho_v \frac{v}{\sigma_v},$$

and its  $R$ -squared equals  $\rho_u^2 + \rho_v^2$ . Therefore,  $\sqrt{\rho_u^2 + \rho_v^2}$  is an overall measure of the selection correlation in the model, and it can be calculated based on the reduced-form model. Note, however, that a particular value of this measure does not determine whether selection is mostly due to variation in  $(\theta, \eta)$  or due to the covariance between  $(\theta, \eta)$  and  $\varepsilon$ .

## C.4 Counterfactuals

Consider an estimated reduced-form model characterized by (C.6), (C.3),  $\hat{\Theta}$ . This allows us to simulate a counterfactual dataset of  $\{(h_i, w_i, \iota_i)\}_i$  for a large number of firms  $i$  and calculate measures of

worker wage inequality in this simulated dataset. In the paper we carry out four types of counterfactuals: (i) autarky; (ii) variation in fixed exporting costs  $F_x$ ; (iii) variation in variable trade cost  $\tau$ , (iv) variation in the dispersion of the employment and wage shocks ( $\sigma_u$  and  $\sigma_v$ ). We discuss each of these counterfactuals in turn.

**Autarky counterfactual** This is the most immediate counterfactual to carry out. This counterfactual maintains the estimated coefficient vector  $\hat{\Theta}$  and the distributional assumption (C.3), but substitutes the model in (C.6) with its special case that shuts down the effects of trade on the employment and wage distributions:

$$\begin{cases} h_i &= \alpha_h + u_i, \\ w_i &= \alpha_w + \zeta u_i + v_i. \end{cases} \quad (\text{C.7})$$

Model (C.7) relies on 5 coefficients  $(\alpha_h, \alpha_w, \zeta, \sigma_u, \sigma_v)$  which form a subset of  $\Theta$ . The trade-related coefficients  $(\rho_u, \rho_v, \mu_h, \mu_w, f)$  are irrelevant for the autarky counterfactual. We use the autarky model (C.7) setting  $(\alpha_h, \alpha_w, \zeta, \sigma_u, \sigma_v)$  as in  $\hat{\Theta}$  to simulate the counterfactual dataset  $\{(h_i, w_i)\}_i$  in autarky and calculate the measures of worker wage dispersion in autarky, which are directly comparable to the inequality measures calculated for the full model (C.6) under  $\hat{\Theta}$ .

Note that  $(\alpha_h, \alpha_w)$  depend on general equilibrium variables and in general change between the autarkic and open economy equilibria. This, however, has no effect on measures of log wage dispersion since  $\alpha_h$  and  $\alpha_w$  introduce proportional shifts to the distributions of employment and wages which are inconsequential for wage inequality. Therefore, our autarky counterfactual holds regardless of the general equilibrium changes in  $\alpha_h, \alpha_w$ .

**Variation in the fixed exporting cost  $F_x$**  Recall from Subsection (C.3) that  $F_x$  affects directly only the reduced-form export threshold  $f$  and no other coefficient in  $\Theta$ . Therefore, variation in  $F_x$  translates into variation in the export threshold  $f$  (which is a linear monotonically increasing function of  $F_x$ ). In our fixed exporting cost counterfactuals, we consider a special case in which  $\mu_h$  and  $\mu_w$  are held constant, which implicitly holds constant the relative export market demand  $A_x/A_d$ , as is the case with symmetric countries when  $\Upsilon_x = 1 + \tau^{-\beta/(1-\beta)}$ . In our variable trade cost counterfactuals below, we allow changes in  $\tau$  to affect  $\mu_h, \mu_w$  and  $f$  both directly and indirectly through changes in relative export market demand  $A_x/A_d$ . Any general equilibrium effects of changes in the fixed exporting cost ( $F_x$ ) in the domestic market are captured by  $\alpha_h$  and  $\alpha_w$  which are again inconsequential for our inequality measures as discussed above. Therefore, we maintain the model (C.6), (C.3) and all estimated coefficients  $\hat{\Theta}$  except for  $f$ , which we vary from infinity (when no firm exports) to minus infinity (when all firms export). For each counterfactual value of  $f$ , we simulate a dataset of  $\{(h_i, w_i, \iota_i)\}_i$  and calculate measures of worker wage dispersion and trade openness (the fraction of exporters and the employment share in exporting firms). We then plot measures of worker wage dispersion against measures of trade openness (see Figure 1 in the paper).

**Variation in the variable trade cost  $\tau$**  This is the most involved counterfactual that we consider. Recall that  $\tau$  affects the market access variable,  $\Upsilon_x = 1 + \tau^{-\frac{\beta}{1-\beta}} (A_x/A_d)^{\frac{1}{1-\beta}}$ , which in turn directly determines the reduced-form market access coefficients  $\mu_h = \frac{1}{1+\chi} \log \Upsilon_x^{\frac{1-\beta}{\Gamma}}$  and  $\mu_w = \frac{\chi}{1+\chi} \log \Upsilon_x^{\frac{1-\beta}{\Gamma}}$ .

Furthermore, through the market access variable,  $\tau$  also affects the reduced-form coefficient  $f$  which decreases linearly in  $\Upsilon_x^{\frac{1-\beta}{\Gamma}}$ . To summarize, movements in  $\tau$  directly affect three reduced-form coefficients  $(\mu_h, \mu_w, f)$ , but in each case the effect of  $\tau$  happens through the market access variable,  $\Upsilon_x^{\frac{1-\beta}{\Gamma}}$ , which allows us to jointly move  $(\mu_h, \mu_w, f)$  in an internally-consistent way without any further knowledge of the structural parameters. Specifically, we gradually move  $\tau$  from infinity (which results in  $\Upsilon_x = 1$ ) to 1 (at which point  $\Upsilon_x$  reaches high values), and change  $(\mu_h, \mu_w, f)$  accordingly keeping other coefficients in  $\hat{\Theta}$  unchanged. For each value of  $\tau$  (and hence  $\Upsilon_x$ ), we simulate a counterfactual dataset  $\{(h_i, w_i, \iota_i)\}_i$  and calculate measures of worker wage dispersion and trade openness (the fraction of exporters and the employment share in exporting firms). We then plot measures of worker wage dispersion against measures of trade openness (see Figure 1 in the paper).

There are two caveats that need to be discussed. First, as discussed before, movements in  $\tau$  can have indirect general equilibrium effects on the intercepts  $(\alpha_h, \alpha_w)$  and on  $A_x/A_d$ , where the latter also affects  $\Upsilon_x$ . This however does not lead to any loss of generality for our counterfactual, since we plot wage inequality against measures of trade openness such as the share of employment in exporting firms, and for both of these measures  $\Upsilon_x$  is a sufficient statistic, so that knowledge of  $\tau$  and  $A_x/A_d$  is not needed. Therefore, as long as  $\tau$  has a monotonically decreasing effect on  $\Upsilon_x$  in equilibrium, our counterfactual in the right panel of Figure 2 in the paper holds regardless of the general equilibrium effect of  $\tau$  on the relative demand shifter  $A_x/A_d$ .

Second, we reproduce the expression for

$$f = \frac{1}{\sigma} \left( -\alpha_\pi + \log F_x - \log \left[ \Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1 \right] \right).$$

Note that the effect of  $\Upsilon_x$  on  $f$  depends on  $\sigma$ , one of the derived parameters of the model that is not identified (see discussion in Subsection C.3). Therefore, in order to carry out this counterfactual, we need to calibrate  $\sigma$ . Recall that

$$\sigma^2 = (1 + \zeta)^2 \sigma_u^2 + \sigma_v^2 + \sigma_\varepsilon^2 - 2(1 + \zeta) \sigma_{u\varepsilon} - 2\sigma_{v\varepsilon}.$$

We make the natural assumption that the contribution of  $\sigma_u^2$  and  $\sigma_v^2$  to  $\sigma^2$  is equal to  $\rho_u^2 + \rho_v^2$ —the  $R^2$  in the regression of  $z$  on  $(u, v)$ . That is, we assume:

$$\sigma^2 = \frac{(1 + \zeta)^2 \sigma_u^2 + \sigma_v^2}{\rho_u^2 + \rho_v^2}.$$

While this constitutes a natural benchmark, we experiment with a wide range of smaller and larger values of  $\sigma^2$  and find largely the same outcomes of the counterfactual.

## D Econometric Inference

### D.1 Derivation of the likelihood function

The likelihood function for observation  $j$  is the probability of observing a data vector  $x_j = (h_j, w_j, \iota_j)$  given the model (C.6) and coefficient vector  $\Theta$ . Since we treat all observations in our cross-section as iid, the likelihood function for the full sample  $\mathbf{X} = \{x_j\}_j$  is a product of the conditional probabilities for individual observations:

$$\mathcal{L}(\Theta|\mathbf{X}) = \prod_j \mathbb{P}\{x_j|\Theta\}.$$

We now derive the expression for  $\mathbb{P}_\Theta\{x_j\} = \mathbb{P}\{x_j|\Theta\}$ . We omit  $\Theta$  whenever the omission causes no confusion. Consider, first, an observation for a non-exporter  $\iota = 0$ :

$$\begin{aligned} \mathbb{P}\{h, w, \iota = 0\} &= \mathbb{P}\{h, w, z < f\} \\ &= \mathbb{P}\{u = h - \alpha_h, v = (w - \alpha_w) - \zeta(h - \alpha_h), z < f\} \\ &= \int_{\tilde{z} < f} \mathbb{P}\{u = h - \alpha_h, v = (w - \alpha_w) - \zeta(h - \alpha_h), z = \tilde{z}\} d\tilde{z} \end{aligned}$$

Similarly, for an exporter with  $\iota = 1$ , we can write:

$$\mathbb{P}\{h, w, \iota = 1\} = \int_{\tilde{z} \geq f} \mathbb{P}\{u = h - \alpha_h - \mu_h, v = (w - \alpha_w - \mu_w) - \zeta(h - \alpha_h - \mu_h), z = \tilde{z}\} d\tilde{z}.$$

Consider now the joint density of  $(u, v, z)$ :<sup>4</sup>

$$\mathbb{P}\{u, v, z\} = \mathbb{P}\{z|u, v\} \cdot \mathbb{P}\{u, v\}$$

From (C.3) it follows that:

$$u \sim \mathcal{N}(0, \sigma_u^2), \quad v \sim \mathcal{N}(0, \sigma_v^2) \quad \text{and} \quad z|(u, v) \sim \mathcal{N}(\rho_u u / \sigma_u + \rho_v v / \sigma_v, 1 - \rho_u^2 - \rho_v^2),$$

with  $(u, v)$  jointly normal and independent. We can therefore write:

$$\mathbb{P}\{u, v, z\} = \frac{1}{\sigma_u} \phi\left(\frac{u}{\sigma_u}\right) \cdot \frac{1}{\sigma_v} \phi\left(\frac{v}{\sigma_v}\right) \cdot \frac{1}{\sqrt{1 - \rho_u^2 - \rho_v^2}} \phi\left(\frac{z - \rho_u u / \sigma_u - \rho_v v / \sigma_v}{\sqrt{1 - \rho_u^2 - \rho_v^2}}\right),$$

where  $\phi(\cdot)$  is the standard normal probability density function given by  $\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$ .

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<sup>4</sup>We can base an alternative derivation on the complementary decomposition  $\mathbb{P}\{u, v, z\} = \mathbb{P}\{u, v|z\} \cdot \mathbb{P}\{z\}$  to show that

$$z \sim \mathcal{N}(0, 1) \quad \text{and} \quad (u, v)|z \sim \mathcal{N}\left(\begin{pmatrix} \rho_u \sigma_u z \\ \rho_v \sigma_v z \end{pmatrix}, \begin{pmatrix} (1 - \rho_u^2) \sigma_u^2 & -\rho_u \sigma_u \rho_v \sigma_v \\ -\rho_u \sigma_u \rho_v \sigma_v & (1 - \rho_v^2) \sigma_v^2 \end{pmatrix}\right),$$

and then rearrange  $\mathbb{P}\{u, v|z\} \cdot \mathbb{P}\{z\}$  to separate out terms with  $z$ .

Integrating this expression over  $z$ , we obtain:

$$\begin{aligned}\mathbb{P}\{u, v, z < f\} &= \frac{1}{\sigma_u} \phi\left(\frac{u}{\sigma_u}\right) \cdot \frac{1}{\sigma_v} \phi\left(\frac{v}{\sigma_v}\right) \cdot \Phi\left(\frac{f - \rho_u u / \sigma_u - \rho_v v / \sigma_v}{\sqrt{1 - \rho_u^2 - \rho_v^2}}\right), \\ \mathbb{P}\{u, v, z \geq f\} &= \frac{1}{\sigma_u} \phi\left(\frac{u}{\sigma_u}\right) \cdot \frac{1}{\sigma_v} \phi\left(\frac{v}{\sigma_v}\right) \cdot \left[1 - \Phi\left(\frac{f - \rho_u u / \sigma_u - \rho_v v / \sigma_v}{\sqrt{1 - \rho_u^2 - \rho_v^2}}\right)\right],\end{aligned}$$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function  $\Phi(t) = \int_{-\infty}^t \phi(s) ds$ .

Finally, we relate these expressions to the probability of the data:

$$\begin{aligned}\mathbb{P}\{h, w, \iota = 0\} &= \mathbb{P}\{u = h - \alpha_h, v = (w - \alpha_w) - \zeta(h - \alpha_h), z < f\} \\ &= \frac{1}{\sigma_u} \phi(\hat{u}(x)) \frac{1}{\sigma_v} \phi(\hat{v}(x)) \Phi\left(\frac{f - \rho_u \hat{u}(x) - \rho_v \hat{v}(x)}{\sqrt{1 - \rho_u^2 - \rho_v^2}}\right), \\ \mathbb{P}\{h, w, \iota = 1\} &= \mathbb{P}\{u = h - \alpha_h - \mu_h, v = (w - \alpha_w - \mu_w) - \zeta(h - \alpha_h - \mu_h), z \geq f\} \\ &= \frac{1}{\sigma_u} \phi(\hat{u}(x)) \frac{1}{\sigma_v} \phi(\hat{v}(x)) \left[1 - \Phi\left(\frac{f - \rho_u \hat{u}(x) - \rho_v \hat{v}(x)}{\sqrt{1 - \rho_u^2 - \rho_v^2}}\right)\right],\end{aligned}$$

where

$$\begin{aligned}\hat{u}(x) &= \frac{h - \alpha_h - \mu_h \iota}{\sigma_u}, \\ \hat{v}(x) &= \frac{(w - \alpha_w - \mu_w \iota) - \zeta(h - \alpha_h - \mu_h \iota)}{\sigma_v}.\end{aligned}$$

Combining these two expressions into one, we obtain our result (17) for  $\mathbb{P}\{x_j | \Theta\}$  in the text of the paper.

## D.2 Maximum Likelihood estimation

Maximum likelihood (ML) estimates are obtained by numerically maximizing the log-likelihood function with respect to the coefficient vector, given the dataset:

$$\hat{\Theta}_{ML} = \arg \max_{\Theta} \log \mathcal{L}(\Theta | \mathbf{X})$$

subject to the inequality constraint (C.5) implied by our structural covariance restriction. To ease the numerical optimization routine, we make the following transformations:

$$\begin{aligned}\sigma_u, \sigma_v &\longmapsto \sigma_u^2, \sigma_v^2, \\ f, \rho_u, \rho_v &\longmapsto \frac{f}{\sqrt{1 - \rho_u^2 - \rho_v^2}}, \frac{\rho_u}{\sqrt{1 - \rho_u^2 - \rho_v^2}}, \frac{\rho_v}{\sqrt{1 - \rho_u^2 - \rho_v^2}}.\end{aligned}$$

We numerically maximize the log-likelihood function with respect to the transformed coefficient vector, and upon completion we make the reverse transformation to the original vector of coefficients. Note

that this transformation automatically ensures that  $\sigma_u$  and  $\sigma_v$  are positive and that  $(\rho_u, \rho_v)$  lies inside the unit circle, that is  $\rho_u^2 + \rho_v^2 < 1$ . As a result we do not need to impose upper and lower bounds in the estimation of the transformed coefficients.

We use ML asymptotic theory to calculate standard errors for  $\hat{\Theta}_{ML}$ . Standard asymptotics imply that under the correct specification

$$\sqrt{N}(\hat{\Theta}_{ML} - \Theta) \xrightarrow{d} \mathcal{N}(\mathbf{0}, V_{ML}),$$

where for robustness we use the sandwich form of the covariance matrix

$$V_{ML} = \left( \mathbb{E} \left\{ -\frac{\partial^2 \log \mathcal{L}(\Theta|x)}{\partial \Theta \partial \Theta'} \right\} \right)^{-1} \mathbb{E} \left\{ \frac{\partial \log \mathcal{L}(\Theta|x)}{\partial \Theta} \frac{\partial \log \mathcal{L}(\Theta|x)}{\partial \Theta'} \right\} \left( \mathbb{E} \left\{ -\frac{\partial^2 \log \mathcal{L}(\Theta|x)}{\partial \Theta \partial \Theta'} \right\} \right)^{-1},$$

which we consistently estimate with

$$\begin{aligned} \hat{V}_{ML} = & \left( -\frac{1}{N} \sum_j \frac{\partial^2 \log \mathcal{L}(\hat{\Theta}_{ML}|x_j)}{\partial \Theta \partial \Theta'} \right)^{-1} \frac{1}{N} \sum_j \frac{\partial \log \mathcal{L}(\hat{\Theta}_{ML}|x_j)}{\partial \Theta} \frac{\partial \log \mathcal{L}(\hat{\Theta}_{ML}|x_j)}{\partial \Theta'} \\ & \times \left( -\frac{1}{N} \sum_j \frac{\partial^2 \log \mathcal{L}(\hat{\Theta}_{ML}|x_j)}{\partial \Theta \partial \Theta'} \right)^{-1}. \end{aligned}$$

When a constraint binds in the estimation (e.g., when  $\zeta = \mu_w/\mu_h$ , which corresponds to the lower bound of (C.5)), we use the resulting equality to substitute out redundant parameters and thus reduce the problem to an unconstrained maximization with a smaller parameter vector. We then apply standard asymptotic theory to the reduced problem, and recover the covariance matrix for the remaining parameters using the  $\Delta$ -method.

### D.3 Overidentified GMM estimation

As discussed in subsection 4.3 of the paper, we also consider an overidentified generalized method of moments (GMM) estimator using eleven conditional first and second moments reported in Table 5 of the paper. In the context of GMM estimation, it is convenient to make the following substitutions:

$$\begin{aligned} \omega &= \zeta u + v, \\ \sigma_\omega^2 &= \zeta^2 \sigma_u^2 + \sigma_v^2, \\ \rho_\omega \sigma_\omega &= \zeta \rho_u \sigma_u + \rho_v \sigma_v, \end{aligned}$$

so that the new triplet of shocks is distributed

$$(u, \omega, z)' \sim \mathcal{N}(\mathbf{0}, \Sigma'_R), \quad \Sigma'_R = \begin{pmatrix} \sigma_u^2 & & \\ \zeta \sigma_u^2 & \sigma_\omega^2 & \\ \rho_u \sigma_u & \rho_\omega \sigma_\omega & 1 \end{pmatrix}.$$



Given the structure of the model (C.6) and the distributional assumption above, we can calculate in closed form the first and second moments of employment and wages conditional on export status, as well as the fraction of exporters:

$$\begin{aligned}
\mathbb{E}\iota_j &= 1 - \Phi(f), \\
m_{h0}(\Theta) &= \mathbb{E}\{h_j|\iota_j = 0\} = \alpha_h - \rho_u\sigma_u\lambda(-f), \\
m_{h1}(\Theta) &= \mathbb{E}\{h_j|\iota_j = 1\} = \alpha_h + \mu_h + \rho_u\sigma_u\lambda(f), \\
m_{w0}(\Theta) &= \mathbb{E}\{w_j|\iota_j = 0\} = \alpha_w - \rho_\omega\sigma_\omega\lambda(-f), \\
m_{w1}(\Theta) &= \mathbb{E}\{w_j|\iota_j = 1\} = \alpha_w + \mu_w + \rho_\omega\sigma_\omega\lambda(f), \\
s_{h0}^2(\Theta) &= \mathbb{V}\{h_j|\iota_j = 0\} = \sigma_u^2 - \rho_u^2\sigma_u^2\Lambda(-f), \\
s_{h1}^2(\Theta) &= \mathbb{V}\{h_j|\iota_j = 1\} = \sigma_u^2 - \rho_u^2\sigma_u^2\Lambda(f), \\
s_{w0}^2(\Theta) &= \mathbb{V}\{w_j|\iota_j = 0\} = \sigma_\omega^2 - \rho_\omega^2\sigma_\omega^2\Lambda(-f), \\
s_{w1}^2(\Theta) &= \mathbb{V}\{w_j|\iota_j = 1\} = \sigma_\omega^2 - \rho_\omega^2\sigma_\omega^2\Lambda(f), \\
c_0(\Theta) &= \mathbb{C}\{w_j, h_j|\iota_j = 0\} = \mathbb{C}(\omega_j - \rho_\omega\sigma_\omega z_j, u_j - \rho_u\sigma_u z_j) + \rho_\omega\sigma_\omega\rho_u\sigma_u\mathbb{V}\{z_j|z_j < f\} \\
&= (\zeta\sigma_u^2 - \rho_\omega\sigma_\omega\rho_u\sigma_u) + \rho_\omega\sigma_\omega\rho_u\sigma_u[1 - \Lambda(-f)] \\
&= \zeta\sigma_u^2 - \rho_\omega\sigma_\omega\rho_u\sigma_u\Lambda(-f), \\
c_1(\Theta) &= \mathbb{C}\{w_j, h_j|\iota_j = 1\} = \zeta\sigma_u^2 - \rho_\omega\sigma_\omega\rho_u\sigma_u\Lambda(f),
\end{aligned} \tag{D.1}$$

where

$$\begin{aligned}
\lambda(f) &= \phi(f)/[1 - \Phi(f)], \\
\lambda(-f) &= \phi(-f)/[1 - \Phi(-f)] = \phi(f)/\Phi(f), \\
\Lambda(f) &= \lambda(f)[\lambda(f) - f], \\
\Lambda(-f) &= \lambda(-f)[\lambda(-f) + f],
\end{aligned}$$

and  $\Theta$  is our  $10 \times 1$  coefficient vector. In the derivation we have used the expressions for the first and second moments of a truncated standard normal variable:

$$\begin{aligned}
\mathbb{E}\{z_j|z_j < f\} &= -\lambda(-f), \\
\mathbb{E}\{z_j|z_j \geq f\} &= \lambda(f), \\
\mathbb{V}\{z_j|z_j < f\} &= 1 - \lambda(-f)[\lambda(-f) + f], \\
\mathbb{V}\{z_j|z_j \geq f\} &= 1 - \lambda(f)[\lambda(f) - f]
\end{aligned}$$

for a standard normal variable  $z_j$ . In the derivation of conditional covariances we used the fact that we can write

$$\begin{aligned}
w_j &= \alpha_w + \mu_w\iota_j + \omega_j \\
&= \alpha_w + \mu_w\iota_j + \rho_\omega\sigma_\omega z_j + (\omega_j - \rho_\omega\sigma_\omega z_j) \\
h_j &= \alpha_h + \mu_h\iota_j + \rho_u\sigma_u z_j + (u_j - \rho_u\sigma_u z_j),
\end{aligned}$$

where  $(\omega_j - \rho_\omega \sigma_\omega z_j)$  and  $(u_j - \rho_u \sigma_u z_j)$  are both independent of  $z_j$ .

**Efficient Overidentified GMM estimator** For numerical GMM estimation it proves convenient to use the unconditional versions of these moments, which are given by the following moment function:

$$m(x_j|\Theta) = \begin{pmatrix} \iota_j - [1 - \Phi(f)] \\ h_j(1 - \iota_j) - m_{h0}(\Theta)\Phi(f) \\ h_j\iota_j - m_{h1}(\Theta)[1 - \Phi(f)] \\ h_j^2(1 - \iota_j) - (s_{h0}^2(\Theta) + m_{h0}^2(\Theta))\Phi(f) \\ h_j^2\iota_j - (s_{h1}^2(\Theta) + m_{h1}^2(\Theta))[1 - \Phi(f)] \\ w_j(1 - \iota_j) - m_{w0}(\Theta)\Phi(f) \\ w_j\iota_j - m_{w1}(\Theta)[1 - \Phi(f)] \\ w_j^2(1 - \iota_j) - (s_{w0}^2(\Theta) + m_{w0}^2(\Theta))\Phi(f) \\ w_j^2\iota_j - (s_{w1}^2(\Theta) + m_{w1}^2(\Theta))[1 - \Phi(f)] \\ h_j w_j(1 - \iota_j) - (c_0(\Theta) + m_{h0}(\Theta)m_{w0}(\Theta))\Phi(f) \\ h_j w_j\iota_j - (c_1(\Theta) + m_{h1}(\Theta)m_{w1}(\Theta))[1 - \Phi(f)] \end{pmatrix},$$

where we have an overidentified system of 11 moments and 10 parameters.

The efficient GMM estimator solves

$$\hat{\Theta}_{GMM} = \arg \max_{\Theta} \left( \frac{1}{N} \sum_j m'(x_j|\Theta) \right) W \left( \frac{1}{N} \sum_j m(x_j|\Theta) \right)$$

subject to the reduced-form inequality (C.5) implied by our structural covariance restriction, where at the first stage  $W = \mathbb{I}_{11}$  is an  $11 \times 11$  identity matrix, and at the second stage the optimal weighting matrix is

$$W_E = (\mathbb{E}\{m(\Theta)m'(\Theta)\})^{-1},$$

which we consistently estimate by

$$\hat{W}_E(\hat{\Theta}) = \left( \frac{1}{N} \sum_j m(x_j|\hat{\Theta})m'(x_j|\hat{\Theta}) \right)^{-1}$$

using  $\hat{\Theta}$  from the first stage. In a Monte Carlo study, the efficient overidentified GMM estimator has similar properties to the ML estimator, but is slightly inferior. In the sample, the overidentified GMM estimates are quantitatively very close to the ML estimates, and we do not report them for brevity.

In section 5.1 of the paper, we report the square root of the overidentified GMM objective function,

where as the weighting matrix we use the diagonal (matrix) of  $\hat{W}_E(\hat{\Theta})$ :

$$\left[ \left( \frac{1}{N} \sum_j m'(x_j|\hat{\Theta}) \right) \hat{W}_D(\hat{\Theta}) \left( \frac{1}{N} \sum_j m(x_j|\hat{\Theta}) \right) \right]^{1/2} = \left[ \sum_{k=1}^{11} \frac{\left( \frac{1}{N} \sum_j m_k(x_j|\hat{\Theta}) \right)^2}{\hat{\sigma}_{m,k}^2(\hat{\Theta})} \right]^{1/2}.$$

#### D.4 GMM Bounds

We provide here the derivations for the GMM Bounds in section 5.3 of the paper, which were omitted from the main text. The underidentified system of moments used in the GMM bounds analysis includes a number of the same moments as the overidentified GMM estimation discussed above (in particular the fraction of exporters and the conditional first moments of wages and employment). We derive here the unconditional second moments of employment and wages:

$$\begin{aligned} \sigma_h^2 &= \text{var}(\alpha_h + \mu_h \iota + u) \\ &= \sigma_u^2 + \mu_h^2 \text{var}(\iota) + 2\mu_h \text{cov}(\iota, u) \\ &= \sigma_u^2 + \mu_h^2 \Phi(f)[1 - \Phi(f)] + 2\mu_h \rho_u \sigma_u \phi(f), \end{aligned}$$

where we used the fact that  $\text{var}(\iota) = \Phi(f)[1 - \Phi(f)]$  since  $\iota = \mathbb{I}\{z \geq f\}$  is a Bernoulli zero-one random variable with  $\mathbb{P}\{\iota = 1\} = \mathbb{P}\{z \geq f\} = 1 - \Phi(f)$  and that:

$$\begin{aligned} \text{cov}(\iota, u) &= \mathbb{E}\{\iota u\} - \mathbb{E}\iota \cdot \mathbb{E}u = \mathbb{E}\{\iota u\} = [1 - \Phi(f)] \cdot \mathbb{E}\{u|\iota = 1\} \\ &= [1 - \Phi(f)] \cdot \rho_u \sigma_u \frac{\phi(f)}{1 - \Phi(f)} = \rho_u \sigma_u \phi(f), \end{aligned}$$

where the expression for  $\mathbb{E}\{u|\iota = 1\} = \mathbb{E}\{u|z \geq f\}$  was derived above.

Following similar steps we derive the expressions for:

$$\begin{aligned} \sigma_w^2 &= \text{var}(w) = \sigma_\omega^2 + \mu_w^2 \Phi(f)[1 - \Phi(f)] + 2\mu_w \rho_\omega \sigma_\omega \phi(f), \\ \sigma_{hw} &= \text{cov}(h, w) = \zeta \sigma_u^2 + \mu_h \mu_w \Phi(f)[1 - \Phi(f)] + [\mu_h \rho_\omega \sigma_\omega + \mu_w \rho_u \sigma_u] \phi(f), \end{aligned}$$

where  $w = \alpha_w + \mu_w \iota + \omega$  and we used the fact that  $\text{cov}(u, \omega) = \text{cov}(u, \zeta u + v) = \zeta \sigma_u^2$ . We also make use of the following result:

$$\text{cov}(\iota, u + \omega) = [\rho_u \sigma_u + \rho_\omega \sigma_\omega] \phi(f) = [(1 + \zeta) \rho_u \sigma_u + \rho_v \sigma_v] \phi(f),$$

which parallels the derivations for  $\text{cov}(\iota, u)$  above.

We next discuss the coefficients in the regression of wages on employment and export status.

We have:

$$\begin{aligned}
\mathbb{E}\{w|h, \iota = 1\} &= \alpha_h + \mu_h + \int_{z \geq f} \mathbb{E}\{\omega|u, z\} \frac{d\Phi(z)}{1 - \hat{\Phi}} \\
&= \alpha_h + \mu_h + \int_{z \geq f} \mathbb{E}\{\rho_\omega \sigma_\omega z + (\omega - \rho_\omega \sigma_\omega z) | (u - \rho_u \sigma_u z), z\} \frac{d\Phi(z)}{1 - \hat{\Phi}} \\
&= [\alpha_h + \mu_h + \rho_\omega \sigma_\omega \lambda(f)] + b[h - \alpha_h - \mu_h - \rho_u \sigma_u \lambda(f)], \\
\mathbb{E}\{w|h, \iota = 0\} &= \alpha_h + \int_{z < f} \mathbb{E}\{\omega|u, z\} \frac{d\Phi(z)}{\hat{\Phi}} \\
&= [\alpha_h - \rho_\omega \sigma_\omega \lambda(-f)] + b[h - \alpha_h + \rho_u \sigma_u \lambda(-f)],
\end{aligned}$$

where  $b$  is the regression coefficient in:

$$\mathbb{E}\{\omega - \rho_\omega \sigma_\omega z | u - \rho_u \sigma_u z\} = b(u - \rho_u \sigma_u z),$$

which is independent from  $z$ , as  $(u - \rho_u \sigma_u z)$  and  $(\omega - \rho_\omega \sigma_\omega z)$  are jointly normal and independent from  $z$ , and the regression coefficient  $b$  is given by:

$$b = \frac{\text{cov}(u - \rho_u \sigma_u z, \omega - \rho_\omega \sigma_\omega z)}{\text{var}(u - \rho_u \sigma_u z)} = \frac{\sigma_{u\omega} - \rho_u \sigma_u \rho_\omega \sigma_\omega}{\sigma_u^2(1 - \rho_u^2)} = \zeta - \frac{\rho_u \sigma_u \rho_v \sigma_v}{\sigma_u^2(1 - \rho_u^2)}.$$

Combining the above expressions together, we can write:

$$\mathbb{E}\{w|h, \iota\} = \lambda^o + \lambda^s h + \lambda^x \iota,$$

where

$$\begin{aligned}
\lambda^o &= [\alpha_h - \rho_\omega \sigma_\omega \hat{\lambda}(-f)] - b[\alpha_h - \rho_u \sigma_u \hat{\lambda}(-f)], \\
\lambda^s &= b, \\
\lambda^x &= (\mu_w - b\mu_h) + [\hat{\lambda}(f) - \hat{\lambda}(-f)](\rho_\omega \sigma_\omega - b\rho_u \sigma_u).
\end{aligned}$$

Additionally, we calculate the  $R^2$  in this regression:

$$R^2 = 1 - \frac{\text{var}(w - \mathbb{E}\{w|h, \iota\})}{\sigma_w^2} = \frac{2\text{cov}(w, \mathbb{E}\{w|h, \iota\}) - \text{var}(\mathbb{E}\{w|h, \iota\})}{\sigma_w^2}$$

$$\begin{aligned}
&= 2 \frac{b\sigma_{uw} + \mu_w(b\mu_h + \lambda^x)\hat{\Phi}(1 - \hat{\Phi}) + [b\mu_w\rho_u\sigma_u + (b\mu_h + \lambda^x)\rho_w\sigma_w]\hat{\phi}}{\sigma_w^2 + \mu_w^2\hat{\Phi}(1 - \hat{\Phi}) + 2\mu_w\rho_w\sigma_w\hat{\phi}} \\
&- \frac{b^2\sigma_u^2 + (b\mu_h + \lambda^x)^2\hat{\Phi}(1 - \hat{\Phi}) + 2b[b\mu_h + \lambda^x]\rho_u\sigma_u\hat{\phi}}{\sigma_w^2 + \mu_w^2\hat{\Phi}(1 - \hat{\Phi}) + 2\mu_w\rho_w\sigma_w\hat{\phi}} \\
&= \frac{(2\zeta - b)b\sigma_u^2 + (b\mu_h + \lambda^x)[2\mu_w - b\mu_h - \lambda^x]\hat{\Phi}(1 - \hat{\Phi}) + 2 \left[ \frac{b(\mu_w - b\mu_h - \lambda^x)\rho_u\sigma_u}{+(b\mu_h + \lambda^x)\rho_w\sigma_w} \right] \hat{\phi}}{\sigma_w^2 + \mu_w^2\hat{\Phi}(1 - \hat{\Phi}) + 2\mu_w\rho_w\sigma_w\hat{\phi}}.
\end{aligned}$$

With this, we can prove the following result:

**Lemma S.2**  $\lambda^s$  contains additional information beyond what is already known from the moments in (A7) in the appendix at the end of the paper, while  $\lambda^o$ ,  $\lambda^x$  and  $R^2$  provide no additional information beyond what is contained in  $\lambda^s$ .

**Proof:** The value of  $b$  cannot be reconstructed from the unconditional second moments because it depends on the covariance between  $h$  and  $w$  conditional on export status. However, given  $b$ , we can reconstruct the values of  $\lambda^o$ ,  $\lambda^x$  and  $R^2$  from the first conditional and second unconditional moments of the data  $(h, w, \iota)$ . For example,

$$\begin{aligned}
\lambda^x &= \mathbb{E}\{w - bh|h, \iota = 1\} - \mathbb{E}\{w - bh|h, \iota = 0\} \\
&= \mathbb{E}\{w - bh|\iota = 1\} - \mathbb{E}\{w - bh|\iota = 0\} \\
&= [\bar{w}_1 - b\bar{h}_1] - [\bar{w}_0 - b\bar{h}_0] \\
\lambda^o &= \mathbb{E}w - b\mathbb{E}h - \lambda_x\mathbb{E}\iota,
\end{aligned}$$

and a similar result can be shown for the  $R^2$  using the expression above. ■

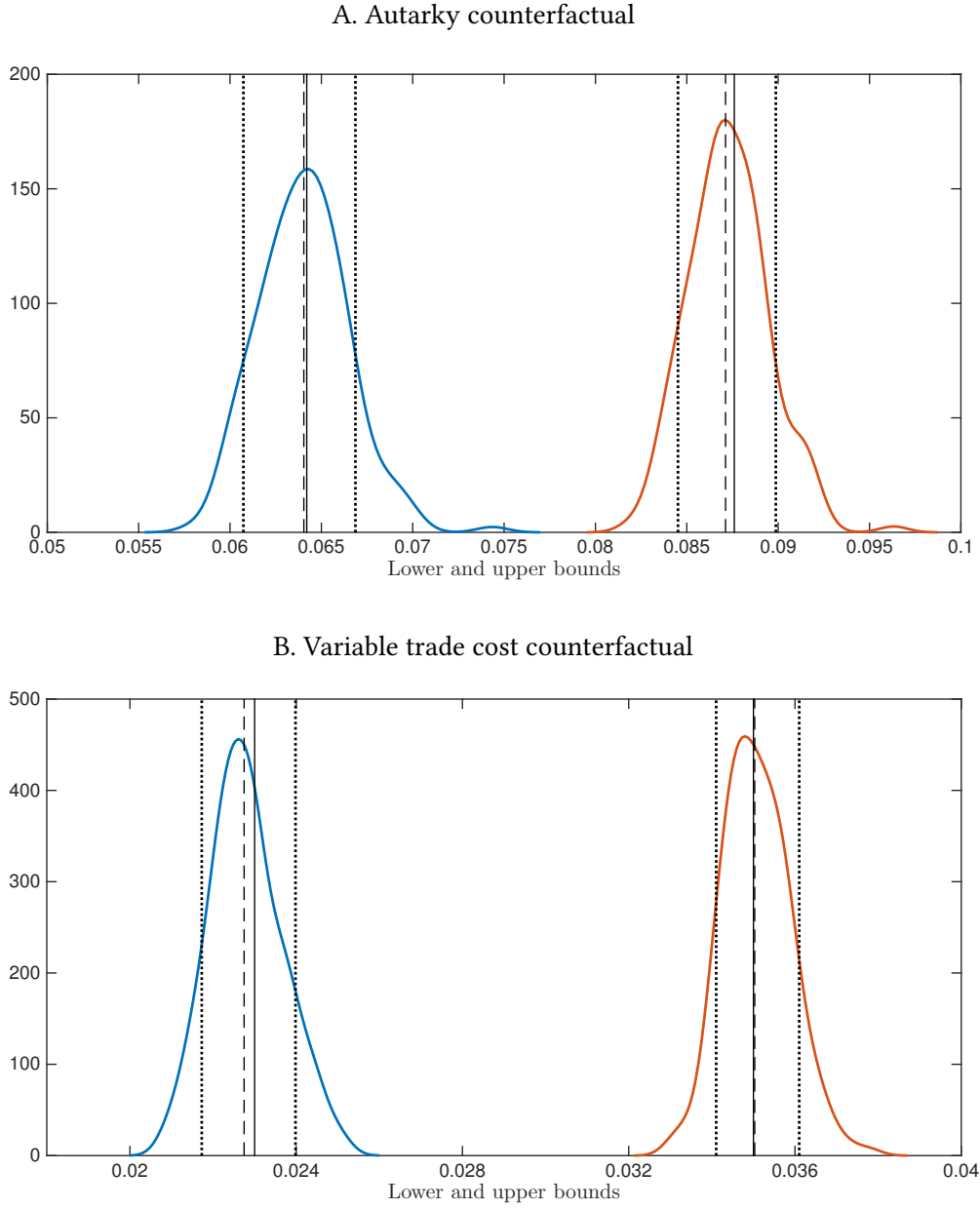
## Bootstrap confidence intervals for identified set and GMM bounds

We now describe our procedure for assessing the statistical precision of the identified set and the implied bounds for the counterfactual effect of trade on wage inequality in Section 5.3 of the paper. We use the data (cross-section for a given year) to draw  $B = 200$  bootstrap samples, randomly with replacement. Each observation in the data corresponds to a firm with a total number of firms given by  $N$  (in 1994, our benchmark year,  $N = 91,410$ ).<sup>5</sup> We assume that idiosyncratic draws are iid across firms so that random bootstrap sampling with replacement captures the statistical sampling error in the data. For each bootstrap sample we follow the same steps as in Section 5.3:

1. we calculate the set of moments and use them to identify the GMM set of model parameters consistent with these moment conditions;
2. for each element of the GMM identified set we conduct two inequality counterfactuals (the autarky counterfactual and a reduction in variable trade costs that raises the exporter employment share by 10 percentage points).

<sup>5</sup>Given this large size of the sample ( $N > 90,000$ ), the variation in the moments across the bootstrap samples is limited, and hence the statistical sampling error is small, resulting in tight confidence intervals below.

Figure D.1: Bootstrap confidence intervals for bounds on counterfactual inequality effects



Note: solid blue and red lines are kernel densities of the distribution of lower and upper bounds across  $B = 200$  bootstrap samples; thin black lines are bounds estimates in the empirical sample; dashed lines are the median values of the bounds across bootstrap samples; dotted lines are the 10th and 90th percentiles respectively of the respective bootstrap distributions for the bounds.

Table D.1: Sampling variation in the trade inequality counterfactual

Moment	Estimated value	Variation across bootstrap samples						
		Mean	St.dev.	5%	10%	50%	90%	95%
A. Autarky counterfactual								
Lower bound	0.064	0.064	0.002	0.060	0.061	0.064	0.067	0.068
Upper bound	0.088	0.087	0.002	0.084	0.085	0.087	0.090	0.091
B. Local variable-trade-cost counterfactual								
Lower bound	0.023	0.023	0.001	0.021	0.022	0.023	0.024	0.024
Upper bound	0.035	0.035	0.001	0.034	0.034	0.035	0.036	0.037
C. Variation in the GMM identified set								
$\mu_h$	0.112	0.112	0.022	0.078	0.086	0.112	0.141	0.148
$\bar{\mu}_h$	2.041	2.041	0.017	2.015	2.019	2.040	2.062	2.068

This procedure results in a bootstrap distribution for each of the moments used in the GMM bounds analysis. The variation in these moments across bootstrap samples results in variation in the GMM identified set of model parameters, characterizing the statistical sampling uncertainty (error). Instead of fully describing the variation in the identified set, we focus on the variation in the trade inequality counterfactuals (presented in Figure 2 of the draft) that are the main goal of our analysis. For each bootstrap sample, we construct the corresponding identified GMM set and carry out the counterfactual analysis as in Section 5.3 for each element in this identified set. This results in a bootstrap sample of counterfactual effects (each is an interval), and we take its minimum and maximum for each bootstrap sample. This gives us a bootstrap distribution for the lower and upper bounds of the counterfactual effects of trade on wage inequality, which we plot in Figure D.1.

We report additional results in Table D.1. The first column of the table restates our estimated bounds on inequality effects from the paper: 6.4% to 8.8% for the autarky counterfactual (in Panel A) and 2.3% to 3.5% for the local variable trade cost counterfactual (in Panel B). The remaining columns report the mean and standard deviation for these bounds across the bootstrap samples, as well as the corresponding percentiles of the bootstrap distributions for these bounds. Comparing the first and second columns, our estimated bounds from the paper are close to the mean of the bootstrap samples. From the third column, we find little variation in these bounds across the bootstrap samples, with standard deviations of 0.2% and 0.1% for the two counterfactuals respectively. Using the percentiles of the bootstrap distributions, we construct 90% confidence intervals for each of the bounds, as reported in the paper. Finally, Panel C of the table provides information on the variation in the boundaries of the identified set: specifically, it reports the minimal and the maximal values of  $\mu_h$  (denoted with  $\underline{\mu}_h$  and  $\bar{\mu}_h$  respectively). Consistent with the previous discussion, we find limited variation across the bootstrap samples in the extreme values of the employment market access premium  $\mu_h$  within the identified set. In particular, based on the bootstrap distribution for  $\underline{\mu}_h$ , we cannot reject the hypothesis that  $\mu_h > 0$ . Similar results obtain for the other parameters of the model (not reported for brevity).

## D.5 Identification

We first report the results of a Monte Carlo exercise, in which we show that our estimation procedure correctly recovers the true parameters when the data are generated according to the model. We next provide an analytical characterization of the relationship between the model's parameters and moments in the data. For our maximum likelihood estimator, we derive closed-form expressions for the score of the likelihood function. For the overidentified GMM estimator considered in Section D.3 above, we derive closed-form expressions for the relationship between the parameters of the model and the first and second moments of the wage and employment distributions conditional on export status.

**Monte Carlo:** We assume the following data generation process:

$$\begin{cases} h &= \alpha_h + \mu_h \iota + u, \\ w &= \alpha_w + \mu_w \iota + \zeta u + v, \\ \iota &= \mathbb{I}\{z \geq f\}. \end{cases} \quad (\text{D.2})$$

$$(u, v, z)' \sim \mathcal{N}(\mathbf{0}, \Sigma_R), \quad \Sigma_R = \begin{pmatrix} \sigma_u^2 & & \\ 0 & \sigma_v^2 & \\ \rho_u \sigma_u & \rho_v \sigma_u & 1 \end{pmatrix}. \quad (\text{D.3})$$

We assume the following parameter values:

$$\begin{aligned} \alpha_h &= 0 & \alpha_w &= 0 \\ \mu_h &= 1 & \mu_w &= 0.5 \\ \sigma_u &= 1.1 & \sigma_v &= 0.8 \\ \rho_u &= 0.4 & \rho_v &= 0.2 \\ \zeta &= 0.5, \end{aligned}$$

which satisfy the inequality constraint (C.5) implied by our structural covariance restriction:

$$\sigma_{\theta\eta} = 0 \quad \Rightarrow \quad \zeta \leq \frac{\mu_w}{\mu_h} < \zeta + \frac{\sigma_v^2}{(1 + \zeta)\sigma_u^2}.$$

We consider 50 replications of the model. For each replication, we draw 100,000 realizations of  $(u, v, z)$  for hypothetical firms from the joint normal distribution (D.3). Given these realizations for  $(u, v, z)$ , we compute employment, wages and export status  $(h, w, \iota)$  using the structure of the reduced-form model (D.2). Given these values for  $(h, w, \iota)$ , we estimate the parameters of the model using maximum likelihood. We repeat this exercise for each of the 50 replications.

In Figure D.2, we display the distribution of the estimated parameters across the 50 replications. Each panel corresponds to a different parameter. Each panel shows the true value of the parameter (as the red vertical line) and the histogram of the parameter estimates across the 50 replications. We find that the estimated parameters are tightly clustered around the true values of the model's parameters. Therefore our estimation procedure indeed correctly recovers the true values of the parameters when



the data are generated according to the model.

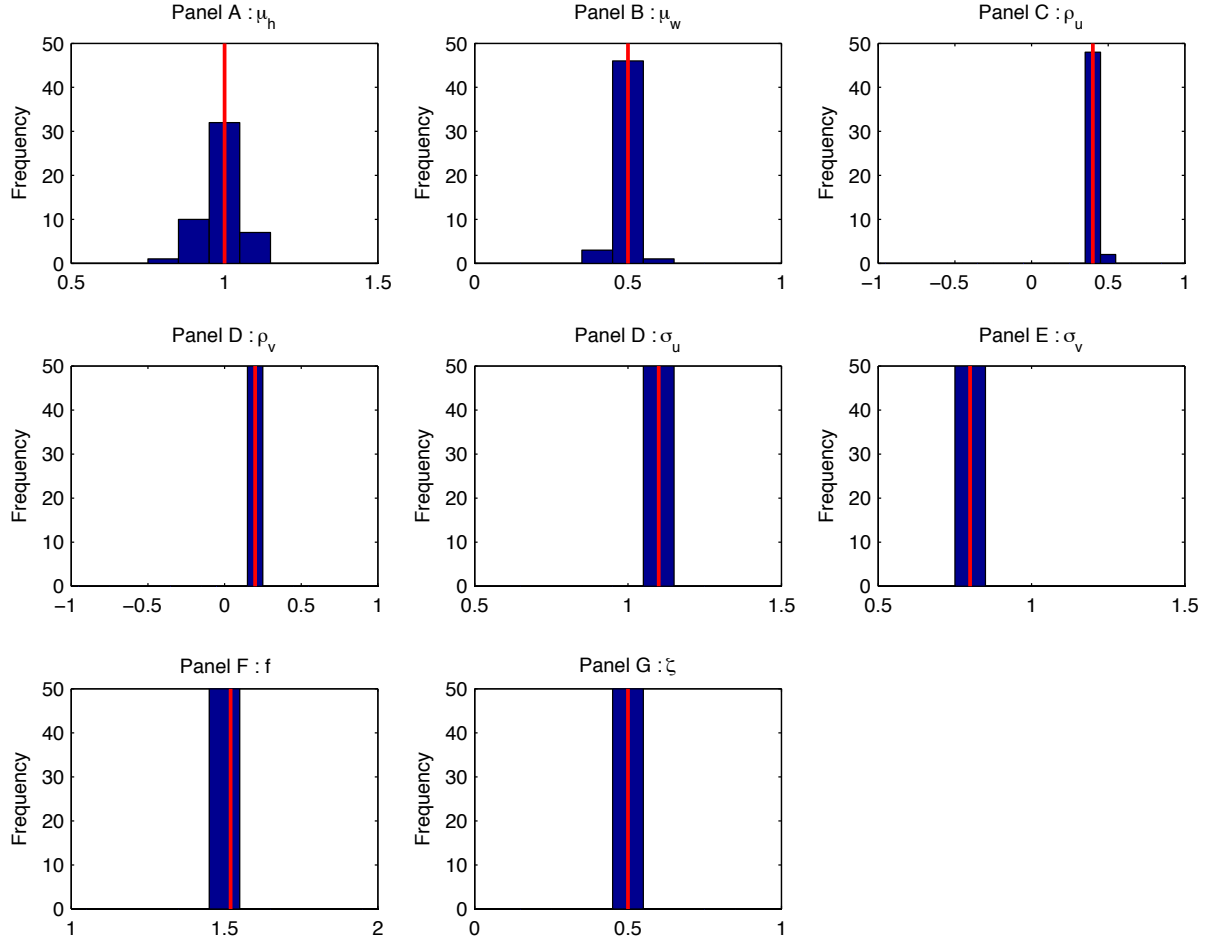


Figure D.2: Monte Carlo Results

**Score of the Likelihood Function:** We now provide closed-form solutions for the first-order conditions of the log likelihood function with respect to the parameters (the score of the likelihood function). These closed-form solutions directly relate the estimated parameters of the model to moments in the data. The log likelihood function is:

$$\ln L = \left\{ \sum_{i=0} -\ln \sigma_u + \ln \phi(\hat{u}) - \ln \sigma_v + \ln \phi(\hat{v}) + \ln \Phi(\hat{f}) \right\} \\ + \left\{ \sum_{i=1} -\ln \sigma_u + \ln \phi(\hat{u}) - \ln \sigma_v + \ln \phi(\hat{v}) + \ln [1 - \Phi(\hat{f})] \right\},$$

where  $(\hat{u}, \hat{v}, \hat{f})$  are related to the observed data  $(h, w, \iota)$  and model parameters  $(\alpha_h, \alpha_w, \mu_h, \mu_w, \sigma_u, \sigma_v, \rho_u, \rho_v, \zeta)$  as follows:

$$\begin{aligned}\hat{u} &= \frac{h - \alpha_h - \mu_h \iota}{\sigma_u}, \\ \hat{v} &= \frac{(w - \alpha_w - \mu_w \iota) - \zeta(h - \alpha_h - \mu_h \iota)}{\sigma_v}, \\ \hat{f} &= \tilde{f} - \tilde{\rho}_u \hat{u} - \tilde{\rho}_v \hat{v}, \\ \tilde{f} &= \frac{f}{\sqrt{1 - \rho_u^2 - \rho_v^2}}, \quad \tilde{\rho}_u = \frac{\rho_u}{\sqrt{1 - \rho_u^2 - \rho_v^2}}, \quad \tilde{\rho}_v = \frac{\rho_v}{\sqrt{1 - \rho_u^2 - \rho_v^2}},\end{aligned}$$

where  $\phi(\cdot)$  is the standard normal density function and  $\Phi(\cdot)$  is the standard normal cumulative distribution function.

#### First-order condition for $\sigma_u$

$$\begin{aligned}\frac{\partial \ln L}{\partial \sigma_u} &= \left\{ \sum_{\iota=0} -\frac{1}{\sigma_u} + \frac{\partial \phi(\hat{u}) / \partial \sigma_u}{\phi(\hat{u})} + \frac{\partial \Phi(\hat{f}) / \partial \sigma_u}{\Phi(\hat{f})} \right\} + \left\{ \sum_{\iota=1} -\frac{1}{\sigma_u} + \frac{\partial \phi(\hat{u}) / \partial \sigma_u}{\phi(\hat{u})} - \frac{\partial \Phi(\hat{f}) / \partial \sigma_u}{1 - \Phi(\hat{f})} \right\}. \\ \frac{\partial \phi(\hat{u})}{\partial \sigma_u} &= \frac{\partial \phi(\hat{u})}{\partial \hat{u}} \frac{\partial \hat{u}}{\partial \sigma_u}, \\ \frac{\partial \phi(\hat{u})}{\partial \hat{u}} &= -\hat{u} \phi(\hat{u}), \\ \frac{\partial \hat{u}}{\partial \sigma_u} &= -\frac{\hat{u}}{\sigma_u}, \\ \frac{\partial \Phi(\hat{f})}{\partial \sigma_u} &= -\frac{\partial \Phi(\hat{f})}{\partial \hat{f}} \tilde{\rho}_u \frac{\partial \hat{u}}{\partial \sigma_u}, \\ \frac{\partial \Phi(\hat{f})}{\partial \hat{f}} &= \phi(\hat{f}),\end{aligned}$$

where we have used  $\partial \phi(\hat{v}) / \partial \sigma_u = 0$ .

#### First-order condition for $\sigma_v$

$$\begin{aligned}\frac{\partial \ln L}{\partial \sigma_v} &= \left\{ \sum_{\iota=0} -\frac{1}{\sigma_v} + \frac{\partial \phi(\hat{v}) / \partial \sigma_v}{\phi(\hat{v})} + \frac{\partial \Phi(\hat{f}) / \partial \sigma_v}{\Phi(\hat{f})} \right\} + \left\{ \sum_{\iota=1} -\frac{1}{\sigma_v} + \frac{\partial \phi(\hat{v}) / \partial \sigma_v}{\phi(\hat{v})} - \frac{\partial \Phi(\hat{f}) / \partial \sigma_v}{1 - \Phi(\hat{f})} \right\}. \\ \frac{\partial \phi(\hat{v})}{\partial \sigma_v} &= \frac{\partial \phi(\hat{v})}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial \sigma_v}, \\ \frac{\partial \phi(\hat{v})}{\partial \hat{v}} &= -\hat{v} \phi(\hat{v}),\end{aligned}$$

$$\begin{aligned}\frac{\partial \hat{v}}{\partial \sigma_v} &= -\frac{\hat{v}}{\sigma_v}, \\ \frac{\partial \Phi(\hat{f})}{\partial \sigma_v} &= -\frac{\partial \Phi(\hat{f})}{\partial \hat{f}} \tilde{\rho}_v \frac{\partial \hat{v}}{\partial \sigma_v}, \\ \frac{\partial \Phi(\hat{f})}{\partial \hat{f}} &= \phi(\hat{f}),\end{aligned}$$

where we have used  $\partial \phi(\hat{u}) / \partial \sigma_v = 0$ .

**First-order condition for  $\zeta$**

$$\begin{aligned}\frac{\partial \ln L}{\partial \zeta} &= \left\{ \sum_{\iota=0} \frac{\partial \phi(\hat{v}) / \partial \zeta}{\phi(\hat{v})} + \frac{\partial \Phi(\hat{f}) / \partial \zeta}{\Phi(\hat{f})} \right\} + \left\{ \sum_{\iota=1} \frac{\partial \phi(\hat{v}) / \partial \zeta}{\phi(\hat{v})} - \frac{\partial \Phi(\hat{f}) / \partial \zeta}{1 - \Phi(\hat{f})} \right\}. \\ \frac{\partial \phi(\hat{v})}{\partial \zeta} &= \frac{\partial \phi(\hat{v})}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial \zeta}, \\ \frac{\partial \phi(\hat{v})}{\partial \hat{v}} &= -\hat{v} \phi(\hat{v}), \\ \frac{\partial \hat{v}}{\partial \zeta} &= -\frac{(h - \alpha_h - \mu_h \iota)}{\sigma_v}, \\ \frac{\partial \Phi(\hat{f})}{\partial \zeta} &= -\frac{\partial \Phi(\hat{f})}{\partial \hat{f}} \tilde{\rho}_v \frac{\partial \hat{v}}{\partial \zeta}, \\ \frac{\partial \Phi(\hat{f})}{\partial \hat{f}} &= \phi(\hat{f}).\end{aligned}$$

**First-order condition for  $\alpha_h$**

$$\begin{aligned}\frac{\partial \ln L}{\partial \alpha_h} &= \left\{ \sum_{\iota=0} \frac{\partial \phi(\hat{u}) / \partial \alpha_h}{\phi(\hat{u})} + \frac{\partial \phi(\hat{v}) / \partial \alpha_h}{\phi(\hat{v})} + \frac{\partial \Phi(\hat{f}) / \partial \alpha_h}{\Phi(\hat{f})} \right\} + \left\{ \sum_{\iota=1} \frac{\partial \phi(\hat{u}) / \partial \alpha_h}{\phi(\hat{u})} + \frac{\partial \phi(\hat{v}) / \partial \alpha_h}{\phi(\hat{v})} - \left[ \frac{\partial \Phi(\hat{f}) / \partial \alpha_h}{1 - \Phi(\hat{f})} \right] \right\}, \\ \frac{\partial \phi(\hat{u})}{\partial \alpha_h} &= \frac{\partial \phi(\hat{u})}{\partial \hat{u}} \frac{\partial \hat{u}}{\partial \alpha_h}, \\ \frac{\partial \phi(\hat{u})}{\partial \hat{u}} &= -\hat{u} \phi(\hat{u}), \\ \frac{\partial \hat{u}}{\partial \alpha_h} &= -\frac{1}{\sigma_u}, \\ \frac{\partial \phi(\hat{v})}{\partial \alpha_h} &= \frac{\partial \phi(\hat{v})}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial \alpha_h}, \\ \frac{\partial \phi(\hat{v})}{\partial \hat{v}} &= -\hat{v} \phi(\hat{v}),\end{aligned}$$

$$\begin{aligned}\frac{\partial \hat{v}}{\partial \alpha_h} &= \frac{\zeta}{\sigma_v}, \\ \frac{\partial \Phi(\hat{f})}{\partial \alpha_h} &= - \left[ \tilde{\rho}_u \frac{\partial \hat{u}}{\partial \alpha_h} + \tilde{\rho}_v \frac{\partial \hat{v}}{\partial \alpha_h} \right] \frac{\partial \Phi(\hat{f})}{\partial \hat{f}}, \\ \frac{\partial \Phi(\hat{f})}{\partial \hat{f}} &= \phi(\hat{f}).\end{aligned}$$

**First-order condition for  $\mu_h$**

$$\begin{aligned}\frac{\partial \ln L}{\partial \mu_h} &= \left\{ \sum_{\iota=0} \frac{\partial \phi(\hat{u})/\partial \mu_h}{\phi(\hat{u})} + \frac{\partial \phi(\hat{v})/\partial \mu_h}{\phi(\hat{v})} + \frac{\partial \Phi(\hat{f})/\partial \mu_h}{\Phi(\hat{f})} \right\} + \left\{ \sum_{\iota=1} \frac{\partial \phi(\hat{u})/\partial \mu_h}{\phi(\hat{u})} + \frac{\partial \phi(\hat{v})/\partial \mu_h}{\phi(\hat{v})} - \left[ \frac{\partial \Phi(\hat{f})/\partial \mu_h}{1 - \Phi(\hat{f})} \right] \right\}, \\ \frac{\partial \phi(\hat{u})}{\partial \mu_h} &= \frac{\partial \phi(\hat{u})}{\partial \hat{u}} \frac{\partial \hat{u}}{\partial \mu_h}, \\ \frac{\partial \phi(\hat{u})}{\partial \hat{u}} &= -\hat{u} \phi(\hat{u}), \\ \frac{\partial \hat{u}}{\partial \mu_h} &= -\frac{\iota}{\sigma_u}, \\ \frac{\partial \phi(\hat{v})}{\partial \mu_h} &= \frac{\partial \phi(\hat{v})}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial \mu_h}, \\ \frac{\partial \phi(\hat{v})}{\partial \hat{v}} &= -\hat{v} \phi(\hat{v}), \\ \frac{\partial \hat{v}}{\partial \mu_h} &= \frac{\zeta \iota}{\sigma_v}, \\ \frac{\partial \Phi(\hat{f})}{\partial \mu_h} &= - \left[ \tilde{\rho}_u \frac{\partial \hat{u}}{\partial \mu_h} + \tilde{\rho}_v \frac{\partial \hat{v}}{\partial \mu_h} \right] \frac{\partial \Phi(\hat{f})}{\partial \hat{f}}, \\ \frac{\partial \Phi(\hat{f})}{\partial \hat{f}} &= \phi(\hat{f}).\end{aligned}$$

**First-order condition for  $\alpha_w$**

$$\begin{aligned}\frac{\partial \ln L}{\partial \alpha_w} &= \left\{ \sum_{\iota=0} \frac{\partial \phi(\hat{v})/\partial \alpha_w}{\phi(\hat{v})} + \frac{\partial \Phi(\hat{f})/\partial \alpha_w}{\Phi(\hat{f})} \right\} + \left\{ \sum_{\iota=1} \frac{\partial \phi(\hat{v})/\partial \alpha_w}{\phi(\hat{v})} - \left[ \frac{\partial \Phi(\hat{f})/\partial \alpha_w}{1 - \Phi(\hat{f})} \right] \right\}, \\ \frac{\partial \phi(\hat{v})}{\partial \alpha_w} &= \frac{\partial \phi(\hat{v})}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial \alpha_w}, \\ \frac{\partial \phi(\hat{v})}{\partial \hat{v}} &= -\hat{v} \phi(\hat{v}), \\ \frac{\partial \hat{v}}{\partial \alpha_w} &= -\frac{1}{\sigma_v},\end{aligned}$$

$$\frac{\partial \Phi(\hat{f})}{\partial \alpha_w} = -\tilde{\rho}_v \frac{\partial \hat{v}}{\partial \alpha_w} \frac{\partial \Phi(\hat{f})}{\partial \hat{f}},$$

$$\frac{\partial \Phi(\hat{f})}{\partial \hat{f}} = \phi(\hat{f}).$$

**First-order condition for  $\mu_w$**

$$\frac{\partial \ln L}{\partial \mu_w} = \left\{ \sum_{\iota=0} \frac{\partial \phi(\hat{v}) / \partial \mu_w}{\phi(\hat{v})} + \frac{\partial \Phi(\hat{f}) / \partial \mu_w}{\Phi(\hat{f})} \right\} + \left\{ \sum_{\iota=1} \frac{\partial \phi(\hat{v}) / \partial \mu_w}{\phi(\hat{v})} - \left[ \frac{\partial \Phi(\hat{f}) / \partial \mu_w}{1 - \Phi(\hat{f})} \right] \right\},$$

$$\frac{\partial \phi(\hat{v})}{\partial \mu_w} = \frac{\partial \phi(\hat{v})}{\partial \hat{v}} \frac{\partial \hat{v}}{\partial \mu_w},$$

$$\frac{\partial \phi(\hat{v})}{\partial \hat{v}} = -\hat{v} \phi(\hat{v}),$$

$$\frac{\partial \hat{v}}{\partial \mu_w} = -\frac{\iota}{\sigma_v},$$

$$\frac{\partial \Phi(\hat{f})}{\partial \mu_w} = -\tilde{\rho}_v \frac{\partial \hat{v}}{\partial \mu_w} \frac{\partial \Phi(\hat{f})}{\partial \hat{f}},$$

$$\frac{\partial \Phi(\hat{f})}{\partial \hat{f}} = \phi(\hat{f}).$$

**First-order condition for  $\tilde{\rho}_u$**

$$\frac{\partial \ln L}{\partial \tilde{\rho}_u} = \left\{ \sum_{\iota=0} \frac{\partial \Phi(\hat{f}) / \partial \tilde{\rho}_u}{\Phi(\hat{f})} \right\} - \left\{ \sum_{\iota=1} \frac{\partial \Phi(\hat{f}) / \partial \tilde{\rho}_u}{1 - \Phi(\hat{f})} \right\},$$

$$\frac{\partial \Phi(\hat{f})}{\partial \tilde{\rho}_u} = \frac{\partial \Phi(\hat{f})}{\partial \hat{f}} \frac{\partial \hat{f}}{\partial \tilde{\rho}_u},$$

$$\frac{\partial \hat{f}}{\partial \tilde{\rho}_u} = -\hat{u},$$

$$\frac{\partial \Phi(\hat{f})}{\partial \hat{f}} = \phi(\hat{f}).$$

**First-order condition for  $\tilde{\rho}_v$**

$$\frac{\partial \ln L}{\partial \tilde{\rho}_v} = \left\{ \sum_{\iota=0} \frac{\partial \Phi(\hat{f}) / \partial \tilde{\rho}_v}{\Phi(\hat{f})} \right\} - \left\{ \sum_{\iota=1} \frac{\partial \Phi(\hat{f}) / \partial \tilde{\rho}_v}{1 - \Phi(\hat{f})} \right\},$$

$$\begin{aligned}\frac{\partial \Phi(\hat{f})}{\partial \tilde{\rho}_v} &= \frac{\partial \Phi(\hat{f})}{\partial \hat{f}} \frac{\partial \hat{f}}{\partial \tilde{\rho}_v}, \\ \frac{\partial \hat{f}}{\partial \tilde{\rho}_v} &= -\hat{v}, \\ \frac{\partial \Phi(\hat{f})}{\partial \hat{f}} &= \phi(\hat{f}).\end{aligned}$$

**First-order condition for  $\tilde{f}$**

$$\begin{aligned}\frac{\partial \ln L}{\partial \tilde{f}} &= \left\{ \sum_{\iota=0} \frac{\partial \Phi(\hat{f}) / \partial \tilde{f}}{\Phi(\hat{f})} \right\} - \left\{ \sum_{\iota=1} \frac{\partial \Phi(\hat{f}) / \partial \tilde{f}}{1 - \Phi(\hat{f})} \right\}, \\ \frac{\partial \Phi(\hat{f})}{\partial \tilde{f}} &= \frac{\partial \Phi(\hat{f})}{\partial \hat{f}} \frac{\partial \hat{f}}{\partial \tilde{f}}, \\ \frac{\partial \hat{f}}{\partial \tilde{f}} &= 1, \\ \frac{\partial \Phi(\hat{f})}{\partial \hat{f}} &= \phi(\hat{f}).\end{aligned}$$

**Exactly-identified GMM:** We now characterize the relationship between the model parameters and the moments in the data used in the overidentified GMM estimator considered in Section D.3 above. We consider an exactly-identified specification of the GMM system (D.1) in terms of first and second moments of wages and employment conditional on export status. In this case, the mapping between the model parameters and moments in the data is particularly transparent, as the exactly-identified GMM system has a recursive structure, in which we can sequentially solve for the model parameters using the moments in the data. In a first equation bloc, the export threshold  $f$  can be determined from the observed fraction of firms that export:

$$\bar{\iota}_1 = \mathbb{E} \iota_j = 1 - \Phi(f),$$

which implies:

$$f = \Phi^{-1}(1 - \bar{\iota}_1), \quad \Phi(f) = 1 - \bar{\iota}_1,$$

where a bar above a variable denotes a value observed in the data.

In a second block of equations, the market access, standard deviation and correlation parameters ( $\mu_h, \sigma_u, \rho_u$ ) can be determined from the conditional and unconditional first and second moments of

employment in (D.1) and the solution for  $f$  above, which imply:

$$\begin{aligned}\rho_u^2 \sigma_u^2 &= \frac{\bar{s}_0^2 - \bar{s}_1^2}{\Lambda(f) - \Lambda(-f)}, \\ \mu_h &= [\bar{m}_{h1} - \bar{m}_{h0}] - \rho_u \sigma_u [\lambda(f) - \lambda(-f)], \\ \sigma_u^2 &= \bar{s}_h^2 - \mu_h^2 \Phi(f) [1 - \Phi(f)] - 2\mu_h \rho_u \sigma_u \phi(f).\end{aligned}$$

In a third block of equations, the market access, standard deviation and correlation parameters ( $\mu_w, \sigma_v, \rho_v$ ) can be determined from the conditional and unconditional first and second moments of wages in (D.1) and the solution for  $f$  above, which imply:

$$\begin{aligned}\rho_\omega^2 \sigma_\omega^2 &= \frac{\bar{s}_{w0}^2 - \bar{s}_{w1}^2}{\Lambda(f) - \Lambda(-f)}, \\ \mu_w &= [\bar{w}_1 - \bar{w}_0] - \rho_\omega \sigma_\omega [\lambda(f) - \lambda(-f)], \\ \sigma_\omega^2 &= \bar{s}_w^2 - \mu_w^2 \Phi(f) [1 - \Phi(f)] - 2\mu_w \rho_\omega \sigma_\omega \phi(f).\end{aligned}$$

In a fourth block of equations, the covariance parameter  $\zeta$  can be determined from the conditional covariance of wages and employment in (D.1) and the solutions for ( $f, \sigma_u, \sigma_\omega, \rho_u, \rho_\omega$ ) above, which imply:

$$\zeta = \bar{c}_{hw} - \{\mu_h \mu_w \Phi(f) [1 - \Phi(f)] + \mu_h \rho_\omega \sigma_\omega \phi(f) + \mu_w \rho_u \sigma_u \phi(f)\}.$$

Having determined  $\{f, \alpha_h, \alpha_w, \mu_h, \mu_w, \sigma_u, \sigma_\omega, \rho_u, \rho_\omega, \zeta\}$  from the above moments in the data, we obtain ( $\sigma_v, \rho_v$ ) from the definitions of ( $\sigma_\omega, \rho_\omega$ ):

$$\begin{aligned}\sigma_v &= [\sigma_\omega^2 - \zeta^2 \sigma_u^2]^{\frac{1}{2}}, \\ \rho_v &= \frac{\rho_\omega \sigma_\omega - \zeta \rho_u \sigma_u}{\sigma_v}.\end{aligned}$$

## E Extensions and Generalizations

### E.1 Isomorphisms

**Class of theoretical models:** We now provide a formal analysis of the class of models that are isomorphic in terms of their predictions for wages, employment and export status to the Helpman, Itskhoki, Muendler and Redding (henceforth HIMR) model developed above. This class of models is defined by the following assumptions: (a) Revenues and employment are power functions of export status and two stochastic shocks, (b) Profits and wage bills are constant shares of revenues, (c) Fixed exporting costs are subject to a third stochastic shock, (d) The three stochastic shocks are joint normally distributed. The class of models defined by these assumptions can be represented as follows:

$$R = \kappa_r [1 + \iota (\Upsilon_x - 1)]^{\psi_r} (e^\theta)^{\xi_r} (e^\eta)^{\phi_r}, \quad (\text{E.4})$$

$$H = \kappa_h [1 + \iota (\Upsilon_x - 1)]^{\psi_h} (e^\theta)^{\xi_h} (e^\eta)^{\phi_h}, \quad (\text{E.5})$$

$$W = \kappa_w [1 + \iota (\Upsilon_x - 1)]^{\psi_w} (e^\theta)^{\xi_w} (e^\eta)^{\phi_w}, \quad (\text{E.6})$$

$$\iota = \mathbb{I} \left\{ \kappa_\pi \left( \Upsilon_x^{\psi_r} - 1 \right) (e^\theta)^{\xi_r} (e^\eta)^{\phi_r} \geq F_x e^\varepsilon \right\}, \quad (\text{E.7})$$

$$\Upsilon_x > 1, \quad F_x > 0,$$

$$\xi_h + \xi_w = \xi_r,$$

$$\phi_h + \phi_w = \phi_r,$$

$$\psi_h + \psi_w = \psi_r.$$

**Reduced-form representation:** Taking logarithms in (E.4)-(E.7), we obtain:

$$h = \alpha_h + \mu_h \iota + \xi_h \theta + \phi_h \eta, \quad (\text{E.8})$$

$$w = \alpha_w + \mu_w \iota + \xi_w \theta + \phi_w \eta, \quad (\text{E.9})$$

$$\iota = \mathbb{I} \left\{ \frac{1}{\sigma} (\xi_r \theta + \phi_r \eta - \varepsilon) \geq f \right\}, \quad (\text{E.10})$$

$$\alpha_s = \log \kappa_s, \quad s \in \{r, h, w, \pi\},$$

$$\mu_s = \psi_s \log \Upsilon_x, \quad s \in \{h, w\},$$

$$f = \frac{1}{\sigma} \left( -\alpha_\pi + \log F_x - \log \left[ \Upsilon_x^{\psi_r} - 1 \right] \right).$$

We now transform this empirical system by orthogonalizing the errors in the employment and wage equations. We define:

$$u = \xi_h \theta + \phi_h \eta,$$

$$v = \left( \phi_w - \frac{\xi_w}{\xi_h} \phi_h \right) \eta - \pi u,$$

$$\zeta = \frac{\xi_w}{\xi_h} + \pi,$$

$$z = \frac{1}{\sigma} [(1 + \zeta) u + v - \varepsilon],$$

where  $\pi$  is the projection coefficient of  $\eta$  on  $u$ . Using these definitions, we can re-write the empirical system (E.8)-(E.10) as:

$$h = \alpha_h + \mu_h \iota + u, \quad (\text{E.11})$$

$$w = \alpha_w + \mu_w \iota + \zeta u + v, \quad (\text{E.12})$$

$$\iota = \mathbb{I} \{ z \geq f \}. \quad (\text{E.13})$$



Under the assumptions above, we have the following theoretical restriction:

$$\mu_h + \mu_w = \log \Upsilon_x > 0.$$

Under the additional assumptions that  $\psi_h, \psi_w > 0$ , we also obtain the following additional theoretical restriction:

$$\mu_h, \mu_w > 0.$$

Under the assumption that  $\{\theta, \eta, \varepsilon\}$  are joint normally distributed,  $\{u, v, z\}$  are also joint normally distributed.

The reduced-form equations (E.11)-(E.13) are identical to those in the HIMR model. Therefore any structural model that can be mapped to the mathematical structure in (E.4)-(E.7) has the same reduced-form econometric specification as the HIMR structural model.

**Likelihood function:** Since the reduced-form equations (E.11)-(E.13) take exactly the same form as in the HIMR model, the likelihood function also takes exactly the same form as in the HIMR model.

$$\mathcal{L}(\Theta|\mathbf{X}) = \prod_j \mathbb{P}\{h_j, w_j, \iota_j = 0\} \prod_j \mathbb{P}\{h_j, w_j, \iota_j = 1\},$$

$$\begin{aligned} \mathbb{P}\{h, w, \iota = 0\} &= \mathbb{P}\{u = h - \alpha_h, v = (w - \alpha_w) - \zeta(h - \alpha_h), z < f\} \\ &= \frac{1}{\sigma_u} \phi(\hat{u}(x)) \frac{1}{\sigma_v} \phi(\hat{v}(x)) \Phi\left(\frac{f - \rho_u \hat{u}(x) - \rho_v \hat{v}(x)}{\sqrt{1 - \rho_u^2 - \rho_v^2}}\right), \\ \mathbb{P}\{h, w, \iota = 1\} &= \mathbb{P}\{u = h - \alpha_h - \mu_h, v = (w - \alpha_w - \mu_w) - \zeta(h - \alpha_h - \mu_h), z \geq f\} \\ &= \frac{1}{\sigma_u} \phi(\hat{u}(x)) \frac{1}{\sigma_v} \phi(\hat{v}(x)) \left[1 - \Phi\left(\frac{f - \rho_u \hat{u}(x) - \rho_v \hat{v}(x)}{\sqrt{1 - \rho_u^2 - \rho_v^2}}\right)\right], \end{aligned}$$

where

$$\begin{aligned} \hat{u}(x) &= \frac{h - \alpha_h - \mu_h \iota}{\sigma_u}, \\ \hat{v}(x) &= \frac{(w - \alpha_w - \mu_w \iota) - \zeta(h - \alpha_h - \mu_h \iota)}{\sigma_v}. \end{aligned}$$

**Counterfactuals:** For all models within this class, the reduced-form coefficients  $\Theta \equiv \{\alpha_h, \alpha_w, \zeta, \sigma_u, \sigma_v, \rho_u, \rho_v, \mu_h, \mu_w, f\}$  are sufficient statistics for wages, employment and export status (and hence wage inequality). Therefore, for all models within this class, counterfactuals can be undertaken following exactly the same procedure as for the HIMR model above.

**Fair wages example:** Finally, we provide an example of another model within the class defined by assumptions (a)-(d) above that is isomorphic to the HIMR model. This example is based on an extension of the fair wages model of [Egger and Kreickemeier \(2012\)](#). As in the HIMR model, each firm faces a

fixed exporting cost ( $e^\varepsilon F_x$ ) and an iceberg variable trade cost ( $\tau$ ). A firm of productivity  $\theta$  that employs a measure  $h$  of workers produces the following measure of output:

$$y = e^\theta h.$$

The firm revenue function is:

$$r = [1 + \iota(\Upsilon_x - 1)]^{1-\beta} A_d \left( e^\theta h \right)^\beta$$

$$\Upsilon_x = 1 + \tau^{-\frac{\beta}{1-\beta}} \left( \frac{A_x}{A_d} \right)^{\frac{1}{1-\beta}}.$$

The firm faces a fair wage constraint such that workers exert no effort unless the firm pays a wage of at least  $\hat{w}$ . As is standard in fair wage models, the firm pays a wage of  $w = \hat{w}$  and workers exert effort in equilibrium. Extending the specification in [Egger and Kreickemeier \(2012\)](#), the fair wage constraint is assumed to take the following form:

$$\hat{w} = e^\eta r^\gamma b^{1-\gamma}, \quad (\text{E.14})$$

where  $\eta$  is a stochastic shock to the firm fair wages constraint and  $b$  is common to all firms within the sector (and typically depends on the sectoral unemployment rate that each firm takes as given when making its choices). The firm's problem is:

$$\max_h \left\{ [1 + \iota(\Upsilon_x - 1)]^{1-\beta} A_d \left( e^\theta h \right)^\beta - e^\eta [1 + \iota(\Upsilon_x - 1)]^{(1-\beta)\gamma} A_d^\gamma \left( e^\theta \right)^{\beta\gamma} b^{1-\gamma} h^{\beta\gamma+1} \right\}.$$

The first-order condition implies:

$$h = \frac{\beta}{1 + \beta\gamma} \frac{r^{1-\gamma}}{e^\eta b^{1-\gamma}}. \quad (\text{E.15})$$

Using the first-order condition in the revenue function, we can solve for the equilibrium revenue function for the firm:

$$r = \left( \frac{\beta}{1 + \beta\gamma} \right)^{\frac{\beta}{1-\beta(1-\gamma)}} \left[ \frac{[1 + \iota(\Upsilon_x - 1)]^{1-\beta} A_d}{(e^\eta b^{1-\gamma})^\beta} \right]^{\frac{1}{1-\beta(1-\gamma)}} \left( e^\theta \right)^{\frac{\beta}{1-\beta(1-\gamma)}}. \quad (\text{E.16})$$

Combining this expression for equilibrium revenue with the first-order condition ([E.15](#)), we can also solve for equilibrium employment:

$$h = \left( \frac{\beta}{1 + \beta\gamma} \right)^{\frac{1}{1-\beta(1-\gamma)}} \left( \frac{1}{e^\eta b^{1-\gamma}} \right)^{\frac{1}{1-\beta(1-\gamma)}} \left( [1 + \iota(\Upsilon_x - 1)]^{1-\beta} A_d \right)^{\frac{1-\gamma}{1-\beta(1-\gamma)}} \left( e^\theta \right)^{\frac{\beta(1-\gamma)}{1-\beta(1-\gamma)}}. \quad (\text{E.17})$$

Using the fair wage constraint (E.14) and equilibrium revenue (E.16), we can also solve for the equilibrium wage:

$$w = \left( \frac{\beta}{1 + \beta\gamma} \right)^{\frac{\beta\gamma}{1 - \beta(1 - \gamma)}} (e^\eta b^{1 - \gamma})^{\frac{1 - \beta}{1 - \beta(1 - \gamma)}} \left( [1 + \iota(\Upsilon_x - 1)]^{1 - \beta} A_d \right)^{\frac{\gamma}{1 - \beta(1 - \gamma)}} (e^\theta)^{\frac{\beta\gamma}{1 - \beta(1 - \gamma)}}. \quad (\text{E.18})$$

Using the fair wage constraint (E.15) and the first-order condition (E.15), we also have:

$$wh = \frac{\beta}{1 + \beta\gamma} r,$$

and hence:

$$\pi = \frac{1 - \beta(1 - \gamma)}{1 + \beta\gamma} r.$$

A firm exports if:

$$\pi_{d+x} - \pi_d \geq e^\varepsilon F_x, \quad \left\{ \begin{array}{l} \left[ [1 + \iota(\Upsilon_x - 1)]^{\frac{1 - \beta}{1 - \beta(1 - \gamma)}} - 1 \right] \left( \frac{1 - \beta(1 - \gamma)}{1 + \beta\gamma} \right) \left( \frac{\beta}{1 + \beta\gamma} \right)^{\frac{\beta}{1 - \beta(1 - \gamma)}} \\ \left[ \frac{A_d}{(e^\eta b^{1 - \gamma})^\beta} \right]^{\frac{1}{1 - \beta(1 - \gamma)}} (e^\theta)^{\frac{\beta}{1 - \beta(1 - \gamma)}} \end{array} \right\} \geq e^\varepsilon F_x. \quad (\text{E.19})$$

Note that (E.16)-(E.19) take exactly the same form as the system of equations (E.4)-(E.7), where:

$$\begin{aligned} \psi_r &= \frac{1 - \beta}{1 - \beta(1 - \gamma)} = \psi_h + \psi_w, \\ \psi_h &= \frac{(1 - \beta)(1 - \gamma)}{1 - \beta(1 - \gamma)}, \quad \psi_w = \frac{(1 - \beta)\gamma}{1 - \beta(1 - \gamma)}, \\ \xi_r &= \frac{\beta}{1 - \beta(1 - \gamma)} = \xi_h + \xi_w, \\ \xi_h &= \frac{\beta(1 - \gamma)}{1 - \beta(1 - \gamma)}, \quad \xi_w = \frac{\beta\gamma}{1 - \beta(1 - \gamma)}, \\ \phi_r &= -\frac{\beta}{1 - \beta(1 - \gamma)} = \phi_h + \phi_w, \\ \phi_h &= -\frac{1}{1 - \beta(1 - \gamma)}, \quad \phi_w = \frac{1 - \beta}{1 - \beta(1 - \gamma)}. \end{aligned}$$

While this extension of Egger and Kreickemeier (2012) falls within the class that are isomorphic to HIMR, it implies a different pattern of correlations between wages, employment and export status. In this fair wage model, a high wage conditional on productivity implies low profitability, because the firm must pay workers a high wage to induce them to exert effort. Therefore, controlling for employment, this fair wage model predicts that exporters should be relatively low-wage firms (rather than relatively high-wage firms), reflecting their low fair wage requirements and high profitability.

## E.2 Two sources of heterogeneity

In this subsection, we consider a special case of the Helpman, Itskhoki, Muendler and Redding (henceforth HIMR) model with only two sources of heterogeneity: (a) a stochastic shock to productivity and (b) a stochastic shock to fixed exporting costs.

**Theoretical model:** The model with only productivity and fixed exporting cost shocks takes the following form:

$$R = \kappa_r [1 + \iota (\Upsilon_x - 1)]^{\frac{1-\beta}{\Gamma}} \left( e^\theta \right)^{\frac{\beta}{\Gamma}}, \quad (\text{E.20})$$

$$H = \kappa_h [1 + \iota (\Upsilon_x - 1)]^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} \left( e^\theta \right)^{\frac{\beta(1-k/\delta)}{\Gamma}}, \quad (\text{E.21})$$

$$W = \kappa_w [1 + \iota (\Upsilon_x - 1)]^{\frac{k(1-\beta)}{\delta\Gamma}} \left( e^\theta \right)^{\frac{\beta k}{\delta\Gamma}}, \quad (\text{E.22})$$

$$\iota = \mathbb{I} \left\{ \kappa_\pi \left( \Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1 \right) \left( e^\theta \right)^{\frac{\beta}{\Gamma}} \geq F_x e^\varepsilon \right\}, \quad (\text{E.23})$$

$$\Upsilon_x > 1, \quad F_x > 0.$$

**Reduced form representation:** Taking logarithms in the theoretical system (E.20)-(E.23), we obtain the following empirical system:

$$h = \alpha_h + \mu_h \iota + \theta, \quad (\text{E.24})$$

$$w = \alpha_w + \mu_w \iota + \zeta \theta, \quad (\text{E.25})$$

$$\iota = \mathbb{I} \{ z \geq f \}, \quad (\text{E.26})$$

$$z = \frac{1}{\sigma} \left( \frac{\beta}{\Gamma} \theta - \varepsilon \right) = \frac{1}{\sigma} ((1 + \zeta) u - \varepsilon),$$

$$\alpha_s = \log \kappa_s, \quad s \in \{r, h, w, \pi\},$$

$$\mu_h = \left( 1 - \frac{k}{\delta} \right) \frac{1-\beta}{\Gamma} \log \Upsilon_x,$$

$$\mu_w = \frac{k}{\delta} \frac{1-\beta}{\Gamma} \log \Upsilon_x,$$

$$\mu_h + \mu_w = \frac{1-\beta}{\Gamma} \log \Upsilon_x,$$

$$\zeta = \frac{k}{\delta - k},$$

$$f = \frac{1}{\sigma} \left( -\alpha_\pi + \log F_x - \log \left[ \Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1 \right] \right).$$

$$(\theta, \varepsilon) \sim N(0, \Sigma), \quad \Sigma = \begin{pmatrix} \sigma_\theta^2 & \\ \sigma_{\theta\varepsilon} & \sigma_\varepsilon^2 \end{pmatrix},$$

$$(u, z) \sim N(0, \Sigma_R), \quad \Sigma_R = \begin{pmatrix} \sigma_u^2 & \\ \rho_u \sigma_u & 1 \end{pmatrix}.$$

**Empirical predictions:** We now show that this special case of the model with only stochastic shocks to productivity and fixed exporting costs has two sets of empirical predictions that are strongly rejected by the data. Therefore the introduction of a third source of heterogeneity in the form of stochastic shocks to wages conditional on productivity (stochastic shocks to human resources management or screening costs in the HIMR model) is quantitatively important in accounting for the data.

First, the special case of the model with only productivity and fixed exporting cost shocks implies an exporter wage premium conditional on productivity, but implies no exporter wage premium conditional on employment. Using the employment equation (E.24), the wage equation (E.25) can be written as:

$$w = (\alpha_w - \zeta \alpha_h) + (\mu_w - \zeta \mu_h) \iota + \zeta h.$$

Using the definition of  $\zeta$ , we obtain:

$$w = (\alpha_w - \zeta \alpha_h) + \zeta h, \tag{E.27}$$

which implies no exporter wage premium conditional on employment.

In contrast to these predictions, we find strong evidence of an exporter wage premium conditional on employment in the data. The HIMR model with three sources of heterogeneity generates this exporter wage premium, because of selection on the stochastic shocks to wages conditional on employment. The presence of a fixed exporting cost implies that exporters are on average high profitability firms conditional on employment, which implies that exporters on average have low realizations of the stochastic shock to screening costs conditional on employment.

Second, this special case of the model with only productivity and fixed exporting cost shocks implies that wages can be perfectly predicted by employment, as is immediately apparent from (E.27). This prediction is also inconsistent with the data and implies a log likelihood of minus infinity.

To further explore the predictions of this special case of the model for employment and wages, we compute the following nine first and second moments of employment and wages conditional on export status in terms of the eight parameters  $\{f, \alpha_h, \alpha_w, \mu_h, \mu_w, \sigma_u, \rho_u, \zeta\}$ :

$$\begin{aligned} \iota_1 &= \mathbb{E} \iota_j = 1 - \Phi(f), \\ h_0 &= \mathbb{E}\{h_j | \iota_j = 0\} = \alpha_h - \rho_u \sigma_u \lambda(-f), \\ h_1 &= \mathbb{E}\{h_j | \iota_j = 1\} = \alpha_h + \mu_h + \rho_u \sigma_u \lambda(f), \\ w_0 &= \mathbb{E}\{w_j | \iota_j = 0\} = \alpha_w - \zeta \rho_u \sigma_u \lambda(-f), \\ w_1 &= \mathbb{E}\{w_j | \iota_j = 1\} = \alpha_w + \mu_w + \zeta \rho_u \sigma_u \lambda(f), \end{aligned}$$

$$\begin{aligned}
s_{h0}^2 &= \mathbb{V}\{h_j | \iota_j = 0\} = \sigma_u^2 - \rho_u^2 \sigma_u^2 \lambda(-f)[\lambda(-f) + f], \\
s_{h1}^2 &= \mathbb{V}\{h_j | \iota_j = 1\} = \sigma_u^2 - \rho_u^2 \sigma_u^2 \lambda(f)[\lambda(f) - f], \\
s_{w0}^2 &= \mathbb{V}\{w_j | \iota_j = 0\} = \zeta^2 \sigma_u^2 - \zeta^2 \rho_u^2 \sigma_u^2 \lambda(-f)[\lambda(-f) + f], \\
s_{w1}^2 &= \mathbb{V}\{w_j | \iota_j = 1\} = \zeta^2 \sigma_u^2 - \zeta^2 \rho_u^2 \sigma_u^2 \lambda(f)[\lambda(f) - f],
\end{aligned}$$

where

$$\begin{aligned}
\lambda(f) &= \phi(f)/[1 - \Phi(f)], \\
\lambda(-f) &= \phi(-f)/[1 - \Phi(-f)] = \phi(f)/\Phi(f), \\
\Lambda(-f) &= \lambda(-f)[\lambda(-f) + f], \\
\Lambda(f) &= \lambda(f)[\lambda(f) - f],
\end{aligned}$$

and where  $\phi(\cdot)$  is the standard normal probability density function and  $\Phi(\cdot)$  is the standard normal cumulative distribution function.

Since this special case of the model implies that wages can be perfectly predicted by employment, it implies that the variances of wages conditional on export status are the same constant multiple of the variances of employment conditional on export status for both exporters and non-exporters,  $s_{w0}^2 = \zeta^2 s_{h0}^2$  and  $s_{w1}^2 = \zeta^2 s_{h1}^2$ , which is rejected by the data.

### E.3 Measurement error

Consider the following cross-sectional Mincer regression with firm fixed effects:

$$w_i = x_i' \vartheta + \psi_j + \nu_i,$$

where  $i$  indexes the worker and  $j$  indexes the firm that employs worker  $i$  (formally  $j(i)$ ). As discussed in the paper, we make the theoretical assumption that the firm observes the wage component  $\psi_j$  and that the model is about this wage component, which can be therefore taken as data in its estimation. We now discuss the implications of relaxing this theoretical assumption for our wage inequality decompositions, in which case the firm-specific wage effects  $\psi_j$  may be subject to measurement error in the presence of wage residuals  $\{\nu_i\}_{i \in j}$  that are non-zero on average for firms of finite size. To illustrate the implications of this kind of measurement error, we treat  $x_i' \vartheta$  as known (our sample size for workers is huge so that  $\vartheta$  is estimated relatively precisely compared to  $\psi_j$ ). Then we can write

$$\hat{\psi}_j = \frac{1}{H_j} \sum_{i \in j} (w_i - x_i' \vartheta) = \psi_j + \bar{\nu}_j,$$

where  $\bar{\nu}_j = (1/H_j) \sum_{i \in j} \nu_i$  is the firm average residual and  $H_j$  is employment of firm  $j$ . Therefore, our variance decomposition is:

$$\text{var}(w_i) = \text{var}(x_i' \vartheta) + \text{var}(\hat{\psi}_j) + 2\text{cov}(x_i' \vartheta, \hat{\psi}_j) + \text{var}(\hat{\nu}_i),$$

where  $\hat{\nu}_i = \nu_i - \bar{\nu}_j$ .

We now adopt the assumption

$$\nu_i | (j, x_i) \sim \mathcal{N}(0, \sigma_\nu^2),$$

that is the distribution of the residual neither depends on any firm characteristic, including firm size, nor on worker observables.<sup>6</sup> Given this assumption, we have:

$$\bar{\nu}_j | (\{x'_i \vartheta\}_{i \in j}, H_j, \psi_j) \sim \mathcal{N}\left(0, \frac{\sigma_\nu^2}{H_j}\right).$$

More generally, the variance term can be allowed to depend on  $H_j$  in more complicated ways that can be estimated in our data, but we adopt this assumption as a natural benchmark to illustrate the adjustments to our estimates.

With these assumptions, measurement error  $\bar{\nu}_j$  does not affect  $\text{var}(x'_i \vartheta)$  or  $\text{cov}(x'_i \vartheta, \psi_j) = \text{cov}(x'_i \vartheta, \hat{\psi}_j)$ , because  $\bar{\nu}_j$  is orthogonal to  $x'_i \vartheta$ . The two remaining terms are affected by measurement error in the following way:<sup>7</sup>

$$\begin{aligned} \text{var}(\hat{\psi}_j) &= \text{var}(\psi_j) + \text{var}(\bar{\nu}_j) = \text{var}(\psi_j) + \sigma_\nu^2 / \bar{H}, \\ \text{var}(\hat{\nu}_i) &= \text{var}(\nu_i - \bar{\nu}_j) = \sigma_\nu^2 (1 - 1/\bar{H}), \end{aligned}$$

where  $\bar{H} = \mathbb{E}H_j$  is mean employment. Note that the sum of the two terms does not depend on measurement error, but we overstate the firm-effect variance and understate the residual variance. However, based on these formulas, the correction is straightforward to calculate according to

$$\hat{\sigma}_\nu^2 = \frac{\widehat{\text{var}}(\hat{\nu}_i)}{1 - 1/\bar{H}},$$

where  $\widehat{\text{var}}$  is the sample residual variance across workers and  $\hat{\sigma}_\nu^2$  is the corrected measure of residual inequality. Next, we compute

$$\text{var}(\psi_j) = \widehat{\text{var}}(\hat{\psi}_j) - \frac{\hat{\sigma}_\nu^2}{\bar{H}} = \widehat{\text{var}}(\hat{\psi}_j) - \frac{\widehat{\text{var}}(\hat{\nu}_i)}{\bar{H} - 1}.$$

Therefore, the only additional quantity we need for correction is  $\bar{H}$ , and the correction is smaller the larger is the average size of a firm  $\bar{H}$ . In our data,  $\bar{H} \approx 70$ , and hence the corrections of the wage inequality decompositions are quantitatively trivial.

**Maximum Likelihood estimation** As discussed above, we make the theoretical assumption that the firm observes the wage component  $\psi_j$  and that the model is about this wage component, which

<sup>6</sup>This is restrictive but in the data  $\text{var}(\nu_i | H_j)$  is likely to increase in  $H_j$  (and also possibly in worker skill), which can be estimated and would only reduce the bias from measurement error in our results. The normality assumption is without substantial loss of generality because we can also invoke a central limit theorem, by which the firm-average wage should be approximately normally distributed for employment of five and more workers under many underlying distributions.

<sup>7</sup>Note that  $\text{var}(\bar{\nu}_j) = (1/\sum_j H_j) \sum_j H_j (\sigma_\nu^2 / H_j) = \sigma_\nu^2 / \bar{H}$  since the variance is across workers, and all workers within a firm have the same  $\bar{\nu}_j$ . By the same argument,  $\text{cov}(\nu_i, \bar{\nu}_j) = \sigma_\nu^2 / \bar{H}$ .

can be therefore taken as data in its estimation. We now discuss the implications of relaxing this assumption for our Maximum Likelihood estimation. In this case, the model applies to  $\psi_j$  but we observe  $\hat{\psi}_j = \psi_j + \bar{v}_j$  instead. We can then write the reduced-form of our structural model as:

$$\begin{aligned} h_j &= \alpha_h + \mu_h \iota_j + u_j, \\ \hat{w}_j &= \alpha_w + \mu_w \iota_j + \zeta u_j + \hat{v}_j, \\ \iota_j &\equiv \mathbb{I} \left\{ z_j \equiv \frac{1}{\sigma} [(1 + \zeta)u_j + v_j - \varepsilon_j] \geq f \right\}, \end{aligned}$$

where

$$\hat{v}_j = v_j + \frac{\sigma_v}{\exp\{h_j/2\}} \xi_j$$

and  $\xi_j = \bar{v}_j / \sqrt{\sigma_v^2 / H_j}$  denotes the normalized measurement error. The distributional assumption on  $(u_j, v_j, z_j)$  is as before given by (C.3). Now  $\xi_j \sim \mathcal{N}(0, 1)$  is standard normal, which in turn is jointly orthogonal to  $(u_j, v_j, z_j)$ . This fully specifies the model, and the likelihood function can be derived in this case following similar steps as before.<sup>8</sup> If one ignores the presence of  $\xi_j$ , the estimate of  $\sigma_{\hat{v}}$  overstates the variance  $\sigma_v$  and the estimate of  $\rho_{\hat{v}}$  accordingly understates the correlation coefficient  $\rho_v$ , without affecting the estimates of  $(\sigma_u, \rho_u; \rho_v \sigma_v)$ .

#### E.4 Estimation with sector-region heterogeneity

One potential concern is that employment and wages could vary systematically across industries and regions in ways that are correlated with export status. To address this concern, we extend our econometric model to allow the constants of the employment and wage equations  $(\alpha_h, \alpha_w)$  and the export threshold  $f$  to vary by sector-region:

$$\begin{cases} h_i &= \alpha_{h,m(i)} + \mu_h \iota_i + u_i, \\ w_i &= \alpha_{w,m(i)} + \mu_w \iota_i + \zeta u_i + v_i, \\ \iota_i &= \mathbb{I}\{z_i \geq \alpha_{f,m(i)}\} \end{cases}$$

where  $i = 1, \dots, N$  is a firm index,  $m = 1, \dots, M$  is the index of the sector-region bin,  $m(i)$  is a mapping of a given firm to the sector-region bin it belongs to,  $\alpha_{\cdot,m}$  is the sector-region specific constant in the respective equation. The remaining assumptions are the same, including the distributional assumption for  $(u, v, z)$  and the inequality constraint (C.5) implied by our structural covariance restriction. We are interested in estimating the  $7 \times 1$  coefficient vector  $\tilde{\Theta} = (\zeta, \sigma_u, \sigma_v, \rho_u, \rho_v, \mu_w, \mu_h)'$ , while  $\{\alpha_{h,m}, \alpha_{w,m}, \alpha_{f,m}\}_m$  are nuisance parameters.

The likelihood function in this case is an immediate extension of (17) in the paper (as derived in

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<sup>8</sup>We report the expression for the likelihood function under measurement error (omitting the derivation):

$$\mathbb{P}\{h, w, \iota\} = \frac{1}{\sigma_u} \phi \left( \frac{h - \alpha_h}{\sigma_u} \right) \int_{-\infty}^{\infty} \left[ \Phi(\hat{z}(\tilde{\xi})) \right]^{1-\iota} \left[ 1 - \Phi(\hat{z}(\tilde{\xi})) \right]^{\iota} \frac{1}{\sigma_u} \phi \left( \frac{\omega - \sigma_v \tilde{\xi} / \exp\{h/2\}}{\sigma_v} \right) \frac{\phi(\tilde{\xi})}{\sigma_v / \exp\{h/2\}} d\tilde{\xi}$$

where  $\hat{z}(\tilde{\xi}) \equiv [1 - \rho_u^2 - \rho_v^2]^{-1/2} [f - \rho_u \frac{h - \alpha_h}{\sigma_u} - \rho_v \frac{\omega - \sigma_v \tilde{\xi} / \exp\{h/2\}}{\sigma_v}]$  and  $\omega = (w - \alpha_w - \mu_w \iota) - \zeta(h - \alpha_h - \mu_h \iota)$ .



subsection D.1 above), but instead of maximizing the log likelihood with respect to  $7 + 3M$  coefficients, we adopt an alternative two-step procedure. Denote by  $(\tilde{\Theta}, A)$  the coefficient vector, where  $A' = \{\alpha_{h,m}, \alpha_{w,m}, \alpha_{f,m}\}_m$ . Given an initial guess for  $\hat{\Theta}^{(0)}$ , we solve for  $\hat{A}^{(1)}$  from the following moment conditions for  $m = 1, \dots, M$ :

$$\begin{cases} \hat{\alpha}_{h,m}^{(1)} &= \frac{1}{n_m} \sum_{i: m(i)=m} (h_i - \mu_h \iota_i), \\ \hat{\alpha}_{w,m}^{(1)} &= \frac{1}{n_m} \sum_{i: m(i)=m} (w_i - \mu_w \iota_i), \\ \hat{\tilde{\alpha}}_{f,m}^{(1)} &= \Phi^{-1} \left( \frac{1}{n_m} \sum_{i: m(i)=m} (1 - \iota_i) \right), \end{cases} \quad (\text{E.28})$$

where  $n_m$  is the number of observations in region-sector bin  $m$ . Note the tilde over  $\tilde{\alpha}_{f,m}$  which indicates that it is rather a transformed parameter:

$$\tilde{\alpha}_{f,m} = \frac{\alpha_{f,m}}{\sqrt{1 - \rho_u^2 - \rho_v^2}}.$$

Also note that these moment conditions ensure that we match exactly the means of  $h_i$ ,  $w_i$  and  $\iota_i$  within every bin  $m = 1, \dots, M$ , given the vector of coefficients  $\hat{\Theta}$ .<sup>9</sup> At the next step, given  $\hat{A}^{(1)}$ , we do MLE with respect to  $\tilde{\Theta}$  to obtain  $\hat{\Theta}^{(1)}$ . Note that this step is equivalent to a GMM where the moment function corresponds to the respective score vector. Given  $\hat{\Theta}^{(1)}$  we repeat to obtain  $\hat{A}^{(2)}$ , and we proceed until convergence of  $(\tilde{\Theta}, A)$ .<sup>10</sup> We denote:  $\hat{\Theta} = \hat{\Theta}^{(B)}$ , where  $B$  is the step of the procedure on which numerical convergence is achieved.

This iterative procedure is equivalent to GMM with  $3M$  first-moment conditions for  $h_i$ ,  $w_i$  and  $\iota_i$  and 7 moment conditions corresponding to the derivative of the log-likelihood with respect to  $\tilde{\Theta}$ .<sup>11</sup> Therefore, we can use the properties of the GMM estimator. Note that our  $3M$  dummies are estimated from substantially smaller samples than the other 7 coefficients which pool all  $N = \sum_{m=1}^M n_m$  observations. To guarantee consistency of  $\hat{\Theta}$ , we assume  $n_m \rightarrow \infty$  for all  $m = 1, \dots, M$ , which is a reasonable assumption given that the average  $\bar{n}_m = N/M \approx 100,000/(136 \cdot 12) > 60$ . Therefore, under this asymptotic assumption, we have consistency:  $\hat{\Theta} \xrightarrow{p} \Theta$ .

In estimating this generalized model, we find similar market access and selection effects  $(\mu_h, \mu_w, \rho_u, \rho_v)$  and similar predicted impacts of trade on wage inequality as in our baseline estimation. Specifically, the predicted wage inequality in 1994 in the model with sector-region heterogeneity is 8.8% above the counterfactual autarky level, slightly above the value for our baseline estimates of 7.6%. This suggests that our findings are robust to unobserved heterogeneity across sectors and regions.

<sup>9</sup>This is why we do not have  $\hat{u}_i$  and  $\hat{v}_i$  in the third set of moment conditions (for  $\alpha_{f,m}$ ) since their means within each bin are exactly zero due to the first two sets of moment conditions.

<sup>10</sup>The specific convergence criterion used is  $\sqrt{(\max |\hat{A}^{(i)} - \hat{A}^{(i-1)}|)^2 + (\max |\hat{\Theta}^{(i)} - \hat{\Theta}^{(i-1)}|)^2} < \text{Tol. Cutoff}$ .

<sup>11</sup>Note that we separate the linear and non-linear conditions, have a closed-form expression (E.28) for 'linear' parameters, and maximize over 'non-linear' parameters. This is the reason why the computational burden is significantly smaller than under a full maximization over  $3M + 7$  parameters when  $M$  is large.

## E.5 Multiple destinations

In the data, we split all firms into four market access bins,  $j = 0, 1, 2, 3$ . Bin  $j = 0$  corresponds to non-exporters, and the other three bins correspond to three categories of exporters based on the number of destinations they serve. In the model we capture this in a very stylized way as follows. Consider a model with a domestic market and three ranked export markets,  $j = 1, 2, 3$ . Each export market is characterized by a demand shifter  $A_{x,j}$  and a common component of the fixed cost of exporting  $F_{x,j}$ . For simplicity we assume that the variable trade costs  $\tau$  are the same to all destinations, but this is without loss of generality since the export-destination-specific component of variable trade costs is absorbed into the export market demand shifter  $A_{x,j}$ . We then have the following generalization of the market access variable:

$$\tilde{\Upsilon}_i = 1 + \sum_{j=1,2,3} \iota_{j,i} \cdot \tau^{-\frac{\beta}{1-\beta}} \left( \frac{A_{x,j}}{A_d} \right)^{\frac{1}{1-\beta}},$$

where  $i$  indexes the firm and  $\iota_{j,i}$  is dummy variable equal to 1 if firm  $i$  serves market  $j$ . The rest of the model immediately generalizes, with firm revenue given by  $R_i = \tilde{\Upsilon}_i^{1-\beta} A_d Y_i^\beta$ , and the equilibrium equations for revenues, employment and wages given by equations (9)–(11) in the paper, but with  $[1 + \iota(\Upsilon_x - 1)]$  replaced with  $[1 + \iota(\tilde{\Upsilon}_i - 1)]$ . Finally, the exporting decision in equation (12) in the paper is instead characterized by:

$$\iota_{j,i} = \mathbb{I} \left\{ \kappa_\pi \left( \Upsilon_j^{\frac{1-\beta}{\Gamma}} - \Upsilon_{j-1}^{\frac{1-\beta}{\Gamma}} \right) (e^{\theta_i})^{\frac{\beta}{\Gamma}} (e^{\eta_i})^{\frac{\beta(1-\gamma k)}{\delta \Gamma}} \geq e^{\varepsilon_i} F_{x,j} \right\}, \quad j = 1, 2, 3,$$

where  $\Upsilon_j \equiv 1 + \sum_{\ell=1}^j \tau^{-\frac{\beta}{1-\beta}} \left( \frac{A_{x,\ell}}{A_d} \right)^{\frac{1}{1-\beta}}$  and hence  $\Upsilon_0 = 1$ . Note that we assumed here that the idiosyncratic export cost shock  $\varepsilon_i$  affects proportionally fixed costs in all destinations for a given firm. We rank destinations by  $\left( \Upsilon_j^{\frac{1-\beta}{\Gamma}} - \Upsilon_{j-1}^{\frac{1-\beta}{\Gamma}} \right) / F_{x,j}$  in decreasing order, so that no firm serves destination  $j + 1$  before it served destination  $j$ . This fully describes the extension to the theoretical model.

The reduced-form model for this extension can be written following the same steps as in Subsection C.2 as follows:

$$\begin{cases} h_i &= \alpha_h + \sum_{j=1,2,3} \mu_{h,j} \iota_{j,i} + u_i, \\ w_i &= \alpha_w + \sum_{j=1,2,3} \mu_{w,j} \iota_{j,i} + \zeta u_i + v_i, \\ \iota_{j,i} &= \mathbb{I} \{ f_{j-1} \leq z_i \leq f_j \}, \quad j = 1, 2, 3, \end{cases}$$

where

$$\mu_{h,j} \equiv \frac{\chi}{1 + \chi} \left[ \log \Upsilon_j^{\frac{1-\beta}{\Gamma}} - \log \Upsilon_{j-1}^{\frac{1-\beta}{\Gamma}} \right], \quad \mu_{w,j} = \chi \mu_{h,j}, \quad \chi = \frac{k/\delta}{1 - k/\delta},$$

and

$$f_j \equiv \frac{1}{\sigma} \left( \alpha_\pi + F_{x,j} - \left[ \log \Upsilon_j^{\frac{1-\beta}{\Gamma}} - \log \Upsilon_{j-1}^{\frac{1-\beta}{\Gamma}} \right] \right), \quad j = 1, 2, 3,$$

and  $f_0 \equiv -\infty$ . We maintain the same distributional assumption (C.3) for  $(u_i, v_i, z_i)$ . The theoretical restrictions imposed by the model are a generalization of those for the model with a single export destination. We have the following theoretical restrictions:  $0 \leq \mu_{h,1} \leq \mu_{h,2} \leq \mu_{h,3}$  and  $\mu_{w,j} = \chi \mu_{h,j}$  for  $j = 1, 2, 3$ . We impose our structural covariance restriction that  $\sigma_{\theta\eta} = 0$ , which implies the

following reduced-form inequality:

$$\zeta \leq \chi < \zeta + \frac{\sigma_v^2}{(1+\zeta)\sigma_u^2}. \quad (\text{E.29})$$

The coefficient vector is now 16-dimensional:

$$\Theta = (\alpha_h, \alpha_w, \zeta, \sigma_u, \sigma_v, \rho_u, \rho_v, \mu_{h,1}, \mu_{h,2}, \mu_{h,3}, \mu_{w,1}, \mu_{w,2}, \mu_{w,3}, f_1, f_2, f_3)'$$

In the data we now observe a  $5 \times 1$  vector for each firm,  $x_i = (h_i, w_i, \iota_{1,i}, \iota_{2,i}, \iota_{3,i})$  with  $\iota_{j,i} \in \{0, 1\}$ ,  $\iota_{j+1,i} \geq \iota_{j,i}$  and firm  $i$  being a non-exporter when  $\iota_{j,i} = 0$  for  $j = 1, 2, 3$ . Since it is not the case that in the data the export destinations are perfectly ranked, we split all exporting firms into bins of exporters by the number of destinations that they serve, so that these bins are necessarily ranked.

The likelihood function of observing  $x_i$  given the coefficient vector  $\Theta$  is a direct generalization of (17) in the text, and the proof follows the same lines as in Subsection D.1. We simply state the expression for the likelihood function here:

$$\begin{aligned} \mathbb{P}_{\Theta}\{x_i\} &= \frac{1}{\sigma_u} \phi(\hat{u}_i) \cdot \frac{1}{\sigma_v} \phi(\hat{v}_i) \cdot \left[ \Phi \left( \frac{f_1 - \rho_u \hat{u}_i - \rho_v \hat{v}_i}{\sqrt{1 - \rho_u^2 - \rho_v^2}} \right) \right]^{\mathbb{I}\{\iota_{j,1}=\iota_{j,2}=\iota_{j,3}=0\}} \\ &\times \left[ \Phi \left( \frac{f_2 - \rho_u \hat{u}_i - \rho_v \hat{v}_i}{\sqrt{1 - \rho_u^2 - \rho_v^2}} \right) - \Phi \left( \frac{f_1 - \rho_u \hat{u}_i - \rho_v \hat{v}_i}{\sqrt{1 - \rho_u^2 - \rho_v^2}} \right) \right]^{\mathbb{I}\{\iota_{j,1}=1, \iota_{j,2}=\iota_{j,3}=0\}} \\ &\times \left[ \Phi \left( \frac{f_3 - \rho_u \hat{u}_i - \rho_v \hat{v}_i}{\sqrt{1 - \rho_u^2 - \rho_v^2}} \right) - \Phi \left( \frac{f_2 - \rho_u \hat{u}_i - \rho_v \hat{v}_i}{\sqrt{1 - \rho_u^2 - \rho_v^2}} \right) \right]^{\mathbb{I}\{\iota_{j,1}=\iota_{j,2}=1, \iota_{j,3}=0\}} \\ &\times \left[ 1 - \Phi \left( \frac{f_3 - \rho_u \hat{u}_i - \rho_v \hat{v}_i}{\sqrt{1 - \rho_u^2 - \rho_v^2}} \right) \right]^{\mathbb{I}\{\iota_{j,1}=\iota_{j,2}=\iota_{j,3}=1\}}, \end{aligned}$$

where

$$\begin{aligned} \hat{u}_i &= \frac{1}{\sigma_u} \left( h_i - \alpha_h - \sum_{j=1,2,3} \mu_{h,j} \iota_{j,i} \right), \\ \hat{v}_i &= \frac{1}{\sigma_v} \left[ \left( w_i - \alpha_w - \sum_{j=1,2,3} \mu_{w,j} \iota_{j,i} \right) - \zeta \sigma_u \hat{u}_i \right]. \end{aligned}$$

We maximize this likelihood function with respect to the parameter vector  $\Theta$  subject to the inequality (E.29) implied by our structural covariance restriction.

## F Dynamic model

In this section of the online supplement we lay out and characterize the solution of a dynamic extension of our model, following the analysis of a simpler model without worker heterogeneity in [Itskhoki and Helpman \(2014\)](#). We show that the steady-state of this dynamic extension exhibits similar properties as the equilibrium of our static model. In order to keep the dynamic analysis tractable, we assume

that worker ability  $a$  is match specific. Therefore there is no learning about a systematic component of worker ability upon separation from a firm. In the static model, this assumption was not required, and worker ability could contain both a match-specific component and a persistent worker component. While we view this as a more realistic description of the world, incorporating these two components of worker ability would substantially complicate the dynamic analysis, and hence we leave this further extension for future work. Under the assumption of purely match-specific ability, upon separation all workers become homogenous again in the pool of unemployed, and therefore share a common value of unemployment,  $J^U$ . The equilibrium determination of  $J^U$  is characterized in [Itskhoki and Helpman \(2014\)](#), and here we provide the analysis for any arbitrary (yet common across workers) value of  $J^U$ .

The remaining assumptions are the direct extensions of those used in the static model to the dynamic environment. In particular, match-specific ability is drawn from a Pareto distribution with shape parameter  $k$ , and we normalize the lower bound  $a_{\min} = 1$  without loss of generality. With this assumption, the fraction of workers with match productivity draws above  $a_c$  is  $(a_c)^{-k}$  and their mean productivity is  $ka_c/(k-1)$ . We next describe bargaining and wage setting, set up the dynamic firm's problem, and then characterize the cross-sectional properties of the steady state equilibrium.

**Wage bargaining** Wages are set as an outcome of [Stole and Zwiebel \(1996b\)](#) bargaining between the firm and its workers, as we discuss in detail in [Itskhoki and Helpman \(2014\)](#). The bargaining happens after all the hiring, production and exporting decisions have been made by the firm and are sunk. Therefore the bargaining is over the revenues of the firm. As in the static model, the revenues of the firm are given by:

$$R = Ay^\beta, \quad y = \theta h^\gamma \bar{a}, \quad (\text{F.30})$$

where  $h$  is the number (mass) of workers and  $\bar{a}$  is their mean ability. The firm demand shifter  $A$  includes export market access as considered above in the static model:

$$A = [1 + \iota(\Upsilon_x - 1)]^{1-\beta} A_d, \quad \Upsilon_x = 1 + \tau^{-\frac{\beta}{1-\beta}} \left( \frac{A_x}{A_d} \right)^{\frac{1}{1-\beta}}. \quad (\text{F.31})$$

We prove the following result:

**Lemma S.3** *The Stole-Zwiebel wage bargaining outcome for a worker  $i$  with (expected) ability  $a_i$  in a firm with revenues given by (F.30) is:*

$$w_i = \frac{\beta\gamma}{1 + \beta\gamma} \frac{R}{h} \left( 1 + \frac{1}{\gamma} \frac{a_i - \bar{a}}{\bar{a}} \right) + \frac{1}{2} r J^U, \quad (\text{F.32})$$

where  $r J^U$  is the flow value of unemployment.

**Proof:** We prove this result for the case of two worker groups, and the logic extends to an arbitrary number of groups. Consider the case in which there are  $h_i$  workers with expected ability  $a_i$  for  $i = 1, 2$ .

The Stole-Zwiebel bargaining condition for each group is:

$$w_i - rJ^U = \frac{\partial}{\partial h_i} \left[ R - \sum_i w_i h_i \right],$$

where the left-hand side is the flow surplus to the worker from employment and the right-hand side is the flow surplus of the firm from employing the worker.<sup>12</sup> We can substitute the wage schedules from (F.32) inside the square bracket on the right hand side of the bargaining condition:

$$R - \sum_i w_i h_i = \frac{1}{1+\beta\gamma} R - \frac{1}{2} r J^U h,$$

where  $h = h_1 + h_2$  and we used the fact that  $h_1 a_1 + h_2 a_2 = h \bar{a}$ , and that the outside option  $rJ^U$  is the same for both groups of workers. This implies:

$$w_i = \frac{1}{1+\beta\gamma} \frac{\partial R}{\partial h_i} + \frac{1}{2} r J^U \quad \text{and} \quad \frac{\partial R}{\partial h_i} = \beta\gamma \frac{R}{h} \left( 1 + \frac{1}{\gamma} \frac{a_i - \bar{a}}{\bar{a}} \right),$$

where we used the fact that  $R = A\theta^\beta h^{\beta\gamma} \bar{a}^\gamma$  and the expressions for  $h$  and  $\bar{a}$  above. Note that increasing  $h_i$  increases both  $h$  and affects  $\bar{a}$  if  $a_i \neq \bar{a}$ . This establishes that the wage schedule in (F.32) indeed solves the bargaining game. ■

An important implication of (F.32) is that the surplus of the firm (operating profit) is equal to

$$\varphi(h, \bar{a}) = R - \sum_i w_i h_i - F = \frac{1}{1+\beta\gamma} R - \frac{1}{2} r J^U h - F, \quad (\text{F.33})$$

where  $F$  is the operating fixed cost (including the exporting fixed cost if the firm exports). Since  $R$  only depends on  $h$  and  $\bar{a}$ , this implies that the operating surplus of the firm does not depend on the specific ability mix in the firm as long as  $(h, \bar{a})$  are held fixed, and so this pair is the state vector in the firm's problem, which we formulate next.

**Firm's problem** In any period, the firm can match with additional workers  $n$  at the cost  $bn$ . Then the firm can screen these workers at the cost  $\frac{c}{\delta} (a_c)^\delta n^\psi$ , where  $\delta > k$  and  $\psi \in [0, 1]$ , and  $c = c_0/\eta$ , where  $\eta$  is firm-specific screening cost draw. Note that  $(\theta, \eta)$  in the current specification (and throughout Section F of this online supplement) correspond to the exponentiated firm productivity draws in the main text of the paper. Note that we generalized the screening cost to allow for  $\psi > 0$ . In the dynamic model, assuming  $\psi = 0$  results in a non-convex optimization, as the firm will hire and screen workers infrequently to take advantage of the extreme increasing returns in screening. In contrast, with  $\psi = 1$ , the returns in screening are constant, and there is no incentive to bunch hiring over time. When  $\psi \approx 1$  (and above a cutoff that we specify below), the mild increasing returns in screening keep the problem of the firm convex, and allow for a simple characterization of the steady state.

Given this structure of hiring (matching and screening) costs, the evolution of the state variables

<sup>12</sup>In Itskhoki and Helpman (2014) we show that the continuation values for both the firm and the worker are equalized and thus cancel from this condition, justifying the focus on the equalization of the flow values only.

of the firm can be written as follows:

$$h' = (1 - \sigma)h + (a_c)^{-k}n, \quad (\text{F.34})$$

$$\bar{a}' = \frac{\bar{a}(1 - \sigma)h + \frac{k}{k-1}(a_c)^{1-k}n}{(1 - \sigma)h + (a_c)^{-k}n}, \quad (\text{F.35})$$

where  $\sigma$  is the exogenous attrition rate of the labor force of the firm. Given the firm's state vector  $(h, \bar{a})$ , the firm's control variables are number of new matches  $n$  and the screening cutoff  $a_c$ , and its new state vector is  $(h', \bar{a}')$ .

The problem of the firm is characterized by the following Bellman equation:

$$J^F(h, \bar{a}) = \max_{n \geq 0, a_c} \left\{ \varphi(h, \bar{a}) - bn - \frac{c}{\delta}(a_c)^\delta n^\psi + \frac{1-d}{1+r} J^F(h', \bar{a}') \right\}$$

subject to (F.34) and (F.35), where  $d > 0$  is the death rate of the firm and  $r > 0$  is the discount rate. We denote with  $s$  the overall separation rate of the workers from the firm, due to either labor attrition or the death of the firm, which is defined by  $(1 - s) = (1 - d)(1 - \sigma)$ .

Given this problem, we can characterize the optimal choices of the firm in an aggregate steady state, determining the steady-state cross-sectional distributions of employment and wages across firms. Denoting with  $\lambda$  and  $\mu$  the Lagrange multipliers on (F.34) and (F.35), the first order conditions and the Envelope theorem for the firm problem can be written as:

$$\begin{aligned} -b - \frac{\psi c}{\delta}(a_c)^\delta n^{\psi-1} + \lambda(a_c)^{-k} + \mu \frac{\partial \bar{a}'}{\partial n} &= 0, \\ -c(a_c)^{\delta-1} n^\psi - \lambda k(a_c)^{-k-1} n + \mu \frac{\partial \bar{a}'}{\partial a_c} &= 0, \\ -\lambda + \frac{1-d}{1+r} \frac{\partial J^F(h', \bar{a}')}{\partial h'} &= 0, \\ -\mu + \frac{1-d}{1+r} \frac{\partial J^F(h', \bar{a}')}{\partial \bar{a}'} &= 0, \\ \frac{\partial J^F(h, \bar{a})}{\partial h} &= \frac{\partial \varphi(h, \bar{a})}{\partial h} + \lambda(1 - \sigma) + \mu \frac{\partial \bar{a}'}{\partial h}, \\ \frac{\partial J^F(h, \bar{a})}{\partial \bar{a}} &= \frac{\partial \varphi(h, \bar{a})}{\partial \bar{a}} + \mu \frac{\partial \bar{a}'}{\partial \bar{a}}. \end{aligned}$$

In steady state,  $h' = h$  and  $\bar{a}' = \bar{a}$ , which implies  $(a_c)^{-k}n = \sigma h$  and  $ka_c/(k-1) = \bar{a}$ . It also implies  $\partial J^F(h, \bar{a})/\partial h = (1+r)\lambda/(1-d)$  and  $\partial J^F(h, \bar{a})/\partial \bar{a} = (1+r)\mu/(1-d)$ . Lastly, we can evaluate the derivatives of  $\bar{a}'$  in steady state as:

$$\frac{\partial \bar{a}'}{\partial n} = \frac{\partial \bar{a}'}{\partial h} = 0, \quad \frac{\partial \bar{a}'}{\partial a_c} = \frac{k\sigma}{k-1}, \quad \frac{\partial \bar{a}'}{\partial \bar{a}} = 1 - \sigma.$$

Combining all this, we can simplify the conditions for the optimal choice of the firm in steady state as:

$$\begin{aligned}\frac{\partial \varphi(h, \bar{a})}{\partial h} &= \frac{r+s}{1-d} \lambda, \\ \frac{\partial \varphi(h, \bar{a})}{\partial \bar{a}} &= \frac{r+s}{1-d} \mu, \\ \lambda(a_c)^{-k} &= b + \frac{\psi c}{\delta} (a_c)^\delta n^{\psi-1}, \\ \mu \frac{\sigma k}{k-1} &= c(a_c)^{\delta-1} n^\psi + \lambda k(a_c)^{-k-1} n.\end{aligned}$$

The first two conditions state that the steady-state marginal operating profit flow from an increase in  $h$  and an increase in  $\bar{a}$  respectively is equal to the flow costs, where costs are given by  $\lambda$  and  $\mu$  and the term  $\frac{r+s}{1-d}$  converts the costs into the flow equivalents. The second two conditions define  $\lambda$  and  $\mu$  recursively. Intuitively, increasing  $h$  holding  $\bar{a}$  constant requires increasing  $n$  by  $1/(a_c)^{-k}$ , and the marginal cost of increasing  $n$  is given by the right-hand side of the expression for  $\lambda$ . The marginal cost of increasing  $\bar{a}$  is more involved: a marginal increase in  $a_c$  at marginal cost  $c(a_c)^{\delta-1} n^\psi$  increases steady state  $\bar{a}$  by  $k\sigma/(k-1)$  and each unit of increase has a value of  $\mu$ , but also reduces steady state  $h$  by  $k(a_c)^{-k-1} n$  with each unit of reduction costing  $\lambda$ .

Next we substitute out for  $\lambda$  and  $\mu$ , and use (F.33) to evaluate the derivatives of  $\varphi(h, \bar{a})$ :

$$\frac{\beta\gamma}{1+\beta\gamma} \frac{R}{h} = \frac{r+s}{1-d} b(a_c)^k \left[ 1 + \frac{\psi c}{b\delta} (a_c)^\delta n^{\psi-1} \right] + \frac{1}{2} r J^U, \quad (\text{F.36})$$

$$\frac{\beta}{1+\beta\gamma} \frac{R}{\bar{a}} = \frac{r+s}{1-d} \frac{k h}{\sigma \bar{a}} b(a_c)^k \left[ 1 + \left( 1 + \frac{\delta}{k\psi} \right) \frac{\psi c}{b\delta} (a_c)^\delta n^{\psi-1} \right], \quad (\text{F.37})$$

$$w = \frac{\beta\gamma}{1+\beta\gamma} \frac{R}{h} + \frac{1}{2} r J^U, \quad (\text{F.38})$$

where the last equation is the equilibrium expression for wages, which follows from (F.32). Using the expression for  $R$  and the relationships between  $(a_c, n)$  and  $(h, \bar{a})$ , the three conditions above allow to solve for equilibrium  $(\bar{a}, h, w)$  as functions of firm parameters  $(A, \theta, \eta)$ . They generalize the first-order conditions of the firm's static profit maximization problem in the paper (discussed in Section C.1 of this online supplement).<sup>13</sup> These generalized conditions, however, do not admit a closed-form solution to parallel (9)–(11) in the text of the paper. Nonetheless, we can take a log-linear expansion to these conditions to approximate the solution for variation in equilibrium employment  $h$  and wages  $w$  across firms with different  $(A, \theta, \eta)$ . This leads to the main result of this section:

**Proposition S.4** *Assume the parameters of the model satisfy*

$$0 \leq 1 - \psi < \frac{1 - \beta\gamma}{\Omega}, \quad (\text{F.39})$$

where  $\Omega \in (0, 1)$  is a derived parameter defined below. Then the second-order conditions for firm maxi-

<sup>13</sup>Indeed, setting  $J^U = 0$  and  $\psi = 0$ , the first condition becomes equivalent to the static condition given by  $\frac{\beta\gamma}{1+\beta\gamma} \frac{R}{h} = b(a_c)^k$ , adjusting for discounting. Somewhat more involved manipulations of the second condition allow to recover the second static optimality condition  $\frac{\beta(1-\gamma k)}{1+\beta\gamma} \frac{R}{a_c} = c(a_c)^{\delta-1}$ .

mization are satisfied, and (F.36)–(F.38) determine the firm's steady-state employment  $h$ , workforce composition  $\bar{a}$ , and wage  $w$ . A first-order log approximation to these conditions (around a typical firm) yields the following pattern of variation in employment and wages across firms:

$$\hat{h} = \phi_{hA}\hat{A} + \phi_{h\theta}\hat{\theta} + \phi_{h\eta}\hat{\eta}, \quad (\text{F.40})$$

$$\hat{w} = \phi_{wA}\hat{A} + \phi_{w\theta}\hat{\theta} + \phi_{w\eta}\hat{\eta}, \quad (\text{F.41})$$

where a hat denotes a log-deviation. The coefficients  $\{\phi_{hA}, \phi_{h\theta}, \phi_{h\eta}, \phi_{wA}, \phi_{w\theta}, \phi_{w\eta}\}$  are functions of the revenue, employment and screening cutoff of the typical firm around which the log approximation is undertaken and satisfy the following inequalities:  $\phi_{hA} > 0$ ,  $\phi_{h\theta} > 0$  and  $\phi_{w\eta} > 0$ , and  $\phi_{wA} \geq 0$  and  $\phi_{w\theta} \geq 0$ . Furthermore, when  $\psi < 1$ , the inequalities are strict (and they are equalities when  $\psi = 1$ ).

We provide a formal proof of this result below, and start here with a brief discussion. First, note that (F.40)–(F.41) offer a generalization to the reduced-form model (13) of the main paper.<sup>14</sup> As discussed above, foreign market access increases the firm's demand shifter  $A$ , and therefore the coefficients  $\phi_{hA}$  and  $\phi_{wA}$  correspond to the reduced-form market access premia  $\mu_h$  and  $\mu_w$  in the main text of the paper. Next, in order for the wage to increase with market access and firm productivity, it is important to have some increasing returns in screening (i.e.,  $\psi < 1$ ), while with constant returns in screening ( $\psi = 1$ ), the wage only increases with  $\eta$  and does not depend on  $A$  or  $\theta$ . Our static model assumed an extreme form of increasing returns in screening ( $\psi = 0$ ), and our dynamic model generalizes it to  $\psi \in [0, 1]$ . Finally, condition (F.39) imposes a lower bound on  $\psi$ , which ensures that the problem of the firm is convex and has an internal solution, as discussed in the beginning of this section (formally, it is a second-order condition for the firm). Note that there always exists a  $\psi < 1$ , but sufficiently close to 1, which satisfies condition (F.39), since its right-hand side is strictly positive. Lastly, the proposition characterizes the steady state for the firm, which also corresponds to the cross-sectional steady state since new entrants (in case  $d > 0$  and there is entry in steady state) will immediately jump to their steady state employment and workforce composition since they face no decreasing returns in either matching or screening ( $\psi \leq 1$ ), a point discussed in more detail in the simpler model without worker heterogeneity in [Itskhoki and Helpman \(2014\)](#).

**Proof:** We log-linearize (F.36)–(F.38) around the values for some typical (average) firm. We make use of the following useful relationships:

$$\begin{aligned} \hat{a} &= \hat{\bar{a}} = \hat{a}_c, \\ \hat{h} &= \hat{n} - k\hat{a}, \\ \hat{R} &= \hat{A} + \beta\hat{\theta} + \gamma\hat{a} - (1 - \beta\gamma)\hat{h}, \end{aligned}$$

where a hat denotes a log change;  $\hat{a}$  is a short-hand for the derivative of both the screening cutoff and average workforce productivity (since they are linearly related,  $\bar{a} = ka_c/(k - 1)$ ); the second line is

<sup>14</sup>The selection equation can be also derived in a similar way, as we formally do in the simpler model without worker heterogeneity in [Itskhoki and Helpman \(2014\)](#).



the expansion of the steady state relationship  $\sigma h = (a_c)^{-k} n$ ; and the last line is the expansion of the revenue function in (F.30).

Using these relationships, we log-linearize (F.36)–(F.38) to yield:

$$\begin{aligned}\hat{A} + \beta\hat{\theta} + \beta\hat{a} - (1 - \beta\gamma)\hat{h} &= k\hat{a} + \Phi_2(\delta\hat{a} - (1 - \psi)\hat{n} - \hat{\eta}), \\ \Phi_0[k\hat{a} + \Phi_1(\delta\hat{a} - (1 - \psi)\hat{n} - \hat{\eta})] &= k\hat{a} + \Phi_2(\delta\hat{a} - (1 - \psi)\hat{n} - \hat{\eta}), \\ \hat{w} &= \frac{1}{2 - \Phi_0}[\hat{A} + \beta\hat{\theta} + \beta\hat{a} - (1 - \beta\gamma)\hat{h}],\end{aligned}$$

where the constants  $\Phi_0, \Phi_1, \Phi_2$  are defined by:

$$\begin{aligned}\Phi_0 &\equiv 1 - \frac{(1 + \beta\gamma)rJ^U h^*}{2\beta\gamma R^*} \in (0, 1], \\ \Phi_1 &\equiv \frac{\frac{\psi c}{b\delta}(a_c^*)^\delta (n^*)^{\psi-1}}{1 + \frac{\psi c}{b\delta}(a_c^*)^\delta (n^*)^{\psi-1}} \in (0, 1), \\ \Phi_2 &\equiv \frac{\left(1 + \frac{\delta}{k\psi}\right) \frac{\psi c}{b\delta}(a_c^*)^\delta (n^*)^{\psi-1}}{1 + \left(1 + \frac{\delta}{k\psi}\right) \frac{\psi c}{b\delta}(a_c^*)^\delta (n^*)^{\psi-1}} \in (\Phi_1, 1),\end{aligned}$$

where an asterisk denotes the value of a variable for the typical firm around which the log linearization is undertaken, such that  $\{R^*, h^*, a_c^*, n^*\}$  are the revenue, employment, screening cutoff, and measure of sampled workers for the typical firm respectively.

Manipulating the log-linearized equations, we obtain:

$$\begin{aligned}[\Phi_2\delta - \beta(1 - k\gamma)]\hat{a} &= \hat{A} + \beta\hat{\theta} + \Phi_2\hat{\eta} - [1 - \beta\gamma - (1 - \psi)\Phi_2]\hat{n}, \\ \hat{a} &= \frac{\Phi_2 - \Phi_0\Phi_1}{k(1 - \Phi_0) + \delta(\Phi_2 - \Phi_0\Phi_1)}[(1 - \psi)\hat{n} + \hat{\eta}], \\ \hat{h} &= \frac{k(1 - \Phi_0) + [\delta - (1 - \psi)k](\Phi_2 - \Phi_0\Phi_1)}{k(1 - \Phi_0) + \delta(\Phi_2 - \Phi_0\Phi_1)}\hat{n} - \frac{(\Phi_2 - \Phi_0\Phi_1)k\hat{\eta}}{k(1 - \Phi_0) + \delta(\Phi_2 - \Phi_0\Phi_1)}, \\ \hat{w} &= \frac{1}{2 - \Phi_0}[\hat{A} + \beta\hat{\theta} + [\beta + k(1 - \beta\gamma)]\hat{a} - (1 - \beta\gamma)\hat{n}].\end{aligned}$$

Note that  $\Phi_2 > \Phi_0\Phi_1$ ,  $\Phi_0 \leq 1$  and  $\delta > (1 - \psi)k$ . Combining the first two equations, we can solve for:

$$\hat{n} = \frac{\hat{A} + \beta\hat{\theta} + \Omega\hat{\eta}}{1 - \beta\gamma - (1 - \psi)\Omega},$$

where

$$\Omega \equiv \frac{k\Phi_2(1 - \Phi_0) + \beta(1 - k\gamma)(\Phi_2 - \Phi_0\Phi_1)}{k(1 - \Phi_0) + \delta(\Phi_2 - \Phi_0\Phi_1)} \in (0, 1),$$

and the stability (second-order) condition requires that the denominator in the expression for  $\hat{n}$  is positive. This requirement is equivalent to (F.39) in the proposition. Substituting the solution for  $\hat{n}$  into the expression for  $\hat{h}$  above, we can immediately establish (F.40), and in particular that  $\phi_{hA}, \phi_{h\theta} > 0$ , while the sign of  $\phi_{h\eta}$  is in general ambiguous.

Lastly, we manipulate the equation for wages to obtain:

$$\hat{w} = \frac{1}{2 - \Phi_0} \left[ \hat{A} + \beta \hat{\theta} + \frac{[\beta + k(1 - \beta\gamma)](\Phi_2 - \Phi_0\Phi_1)}{k(1 - \Phi_0) + \delta(\Phi_2 - \Phi_0\Phi_1)} \hat{\eta} - \left[ 1 - (1 - \psi) \frac{[k + \frac{\beta}{1 - \beta\gamma}](\Phi_2 - \Phi_0\Phi_1)}{k(1 - \Phi_0) + \delta(\Phi_2 - \Phi_0\Phi_1)} \right] (1 - \beta\gamma) \hat{n} \right],$$

and then substitute the solution for  $\hat{n}$ :

$$\hat{w} = \frac{1}{2 - \Phi_0} \frac{k\Phi_0(\Phi_2 - \Phi_1)}{k(1 - \Phi_0) + \delta(\Phi_2 - \Phi_0\Phi_1)} \frac{(1 - \psi)(\hat{A} + \beta\hat{\theta}) + (1 - \beta\gamma)\hat{\eta}}{1 - \beta\gamma - (1 - \psi)\Omega}.$$

This establishes (F.41), and the fact that  $\phi_{wA}, \phi_{w\theta} \geq 0$  and  $\phi_{w\eta} > 0$  (since  $\Phi_2 > \Phi_1$ ), and the inequality for  $\phi_{wA}, \phi_{w\theta}$  is strict whenever  $\psi < 1$ , completing the proof of the proposition. ■

Proposition S.4 establishes that the steady-state of the dynamic model yields similar reduced-form relationships between employment, wages and export status to those in the static model: firms with greater physical productivity  $\theta$  employ more workers and pay them higher wages, and exporting firms characterized by discontinuously higher revenue shifter  $A$  (see (F.31)) have greater employments and wages compared to non-exporting firms (the market access effects). One difference is that the coefficients of these relationships in the dynamic model depend on the revenue, employment and screening threshold of the typical firm around which the log linearization is undertaken. Note however that the approximation is carried out around the same firm (characterized by a triplet of shifters  $(A, \theta, \eta)$  exogenous to the firm) both before and after the trade liberalization, so that the coefficients of approximation in (F.40)–(F.41) are constant and do not depend on trade costs.

## G Data Appendix

Our main data source is the linked employer-employee database RAIS (*Relação Anual de Informações Sociais*) for the period 1986-1998, a nationwide administrative register of workers formally employed in any sector of Brazil's economy (including the public sector). Brazilian law requires every firm to submit annual reports with detailed employment and demographic information on every formally employed worker to the ministry of labor (*Ministério de Trabalho*, MTE). The original intention of the RAIS records is to provide information for a federal wage supplement program (*Abono Salarial*), by which every worker with formal employment during the calendar year receives the equivalent of a monthly minimum wage. A strong incentive for compliance is that workers' benefits depend on RAIS so that workers follow up on their records (payment of the worker's annual public wage supplement (*Abono Salarial*) is exclusively based on RAIS records). The ministry of labor estimates that coverage of workers exceeded 90 percent throughout the 1990s.

In our baseline specification, we restrict our sample to the manufacturing sector and to firms with at least five workers. We also undertake robustness tests including all manufacturing workers in RAIS, including workers in the agricultural, manufacturing and mining sectors in RAIS, and using household survey data from *Pesquisa Nacional por Amostra de Domicílios* (PNAD) (see Section H.18 below). Throughout our analysis of the RAIS data, we aggregate the monthly worker-plant information to firms

and to calendar years. For this purpose, we first restrict the worker sample to all proper worker identifiers (*PIS/PASEP*) with the required 11 digits. For every worker, we then retain her or his last employment per year with a strictly positive wage (the final employment spell may or may not occur in December).<sup>15</sup> If a worker holds more than one simultaneous job at that time, we keep the job with the highest pay (randomly dropping ties). Our according definition of a firm's employment is the count of workers whose employment spell at the firm is their final and highest-paid job of the year (so that the counted workers may not hold simultaneous jobs at the firm). In the so restricted manufacturing sample of firms with at least five workers from 1986-1998, we have 83.0 million observations of 20.4 million workers at 352 thousand plants and 270 thousand firms in 348 (CBO) occupations and 12 (subsector IBGE) sectors (or 309 CNAE industries in the 1994-1998 subsample).

For wage information, we use the RAIS reported average monthly wage that the worker earns over the course of the job spell during the calendar year. As a rule, worker payments are part of the RAIS wage if they are taxable income or are subject to Brazilian social security contributions.<sup>16</sup> RAIS reports the average wage in multiples of the current minimum wage, which we transform into the annual equivalent wage in current US-Dollars, multiplying the monthly average wage by twelve and using the end-of-year nominal exchange rate along with the prevailing minimum wage for the currency conversion. As a check on the quality of the Brazilian matched employer-employee data, [Menezes-Filho, Muendler, and Ramey \(2008\)](#) show that these data exhibit many of the same properties as the matched employee-employer data for France and the United States. We provide further checks on the quality of the Brazilian wage and employment data in the paper, where we show that they exhibit similar patterns for wage inequality as for other countries including the United States. As an additional check, this online supplement reports the results of re-estimating our econometric model using Colombian firm-level data and demonstrates a similar pattern of results.

We categorize worker demographics into age, education, gender, and experience (tenure) groups. The eight age categories are Child (10-14), Youth (15-17), Adolescent (18-24), Nascent Career (25-29), Early Career (30-39), Peak Career (40-49), Late Career (50-64) and Post Retirement (65+). Our choice of educational categories is guided by the existing labor economics literature, including Autor, Katz, and Krueger (1998) and Katz and Autor (1999). In our baseline specification, we distinguish the following four categories: Primary School or less education (up to 8 grades of education including illiteracy), Some High School education (up to 12 grades of education), Some College education (college enrollment without college degree), and College Graduate. We also report the results of a robustness test using nine more disaggregated educational categories: Illiterate, some primary, complete primary, some middle, complete middle, some high, complete high, some college, and complete college. There are two gender

<sup>15</sup>In contrast, [Menezes-Filho, Muendler, and Ramey \(2008\)](#) only retain observations of jobs on December 31st of the calendar year for prime-age workers.

<sup>16</sup>According to the RAIS manuals, the reported wage has to include: salaries; overtime compensation for contracted extra hours; extraordinary additions, supplements and bonuses (but not participation in the employer's profits outside the employment contract such as through equity holdings in the employing firm); tips and gratuities; commissions and fees; contracted premia; hazard compensation; executive compensation; cost reimbursement components if they exceed 50 percent of the base salary and are for travel or transfers necessary for the execution of the job; payments for periods of vacation, holidays and parental leave (but not severance payments for layoffs and not indemnity payments for permanent maternal leave); vacation gratuities if they exceed 20 days of salary; piece wages; and in-kind remunerations such as room and board.

categories (male and female). We measure experience by the number of months for which a worker has been employed by the firm. We form quintiles of the experience distribution by year and classify every worker by his or her quintile.

**Occupations** For most of our analysis, we classify occupations into five categories, which correspond closely to those used in existing research for other countries. Occupation information in RAIS 1986-1998 is reported under the *CBO* system of 1994 (*Classificação Brasileira de Ocupações*), which we convert to the internationally comparable *ISCO-88* categories following [Muendler, Poole, Ramey, and Wajnberg \(2004\)](#). For the conversion, we reset unknown *CBO* codes in RAIS at the four-digit level to the nearest applicable miscellaneous occupation category at the four-digit level. We then lump the *ISCO-88* categories into five broad occupation groups: Professional and Managerial occupations (including professionals, senior officials, and managers), Technical and Supervisory occupations (including technicians and associate professionals), Other White Collar occupations (including clerks, service workers, shop and market sales workers), Skill Intensive Blue Collar occupations (including plant and machine operators and assemblers, craft and related workers, skilled agricultural and fishery workers), and Other Blue Collar occupations (elementary occupations in *ISCO-88*). We report the concordance between the 348 disaggregated *CBO* occupations and the 5 aggregated occupations in Table [I.1](#) at the end of this online supplement. Using these five occupation groups, we obtain 1.2 million firm-occupation cells at 462.9 thousand firms after our sample restriction to workers' highest-paid final jobs in a calendar year and manufacturing firms with at least five such workers during the sample period 1986-1998.

**Industries and locations** We infer a firm's sector and municipality in RAIS as its worker mode sector and mode municipality across the firm's plants. For most of our analysis, we use a firm's sector according to the classification by *Instituto Brasileiro de Geografia e Estatística (IBGE)*, which disaggregates manufacturing into twelve sectors roughly corresponding to two-digit International Standard Industrial Classification (ISIC) sectors. These sectors again correspond closely to those used in existing research for other countries. In robustness tests, we use the 310 disaggregated CNAE industries for which information is available from 1994 onwards. We report the concordance between the 310 disaggregated CNAE industries and the twelve IBGE sectors in Table [I.2](#) at the end of this online supplement.

**Exporter data** We obtain information on the employer's export status from national customs records. The exports records are available to us from SECEX (*Secretaria de Comércio Exterior*) through FUNCEX Rio de Janeiro for 1986-1998. We set the indicator variable for a firm's export status to one if SECEX records show exports by the firm of any product to any destination in a given year.<sup>17</sup> We link the export-status indicator to RAIS at the firm level.

When we combine the SECEX exporter data with the linked employer-employee information from RAIS for the period 1986-1998 (no firm size restriction), we find that 21,686 manufacturing firms (IBGE subsectors 2 through 13) are exporters in at least one sample year, and there are 92,624 exporter-year

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<sup>17</sup>We do not use a minimum exports per domestic sales ratio to define the export indicator because domestic sales information is only available for a small subsample of firms from a manufacturing survey.

observations. This implies that around 4.7 percent of formal manufacturing firms are exporters during the period 1986-1998, similar to the around 5 percent exporter share in the U.S. universe of manufacturing firms (Bernard, Jensen, and Schott 2009). Single-employee and other small firms enter the RAIS records, explaining the apparently low share of exporter firms in the total, compared to data for most other developing countries that censor their samples at a minimum employment level. In terms of workforce size, manufacturing exporters employ 47.4 percent of the formal RAIS workers (in their last held top-paid job per year) during the sample period 1986-1998.

## H Additional Empirical Results and Robustness

### H.1 Export Participation

As discussed in the paper, our sample period includes trade liberalization and fluctuations in the real exchange rate. Tariffs are lowered in 1988 and further reduced between 1990 and 1993, whereas non-tariff barriers are dropped by presidential decree in March 1990. Following this trade liberalization, the share of exporting firms nearly doubles between 1990 and 1993, and their employment share increases by around 10 percentage points, as shown in Figure H.1. In contrast, following Brazil's real exchange rate appreciation of the mid-1990s, both the share of firms that export and the employment share of exporters decline by around the same magnitude, as also shown in Figure H.1. Aggregate exports for the manufacturing sector display a similar pattern over time as the share of firms that export and the employment share of exporters, as shown in Figure H.2.

### H.2 Wage inequality within versus between sectors and occupations

In Figure H.3, we provide further evidence on the decomposition of overall wage inequality into its within and between-group components using sectors (Panel A), occupations (Panel B) and sector-occupations (Panel C). For each variable, we subtract the 1986 value of the variable to generate an index that takes the value zero in 1986, which allows us to quantify the contribution of the within and between components to the *change* in overall wage inequality after 1986. Whether we use sectors, occupations or sector-occupations, we find that the within component of wage inequality closely mirrors the time-series evolution of overall wage inequality and accounts for most of its growth over our sample period. For each within component, we observe the same inverted U-shaped pattern as for overall wage inequality. This finding is also robust to the use of alternative base years to 1986.

### H.3 Non-manufacturing industries

In this subsection, we demonstrate the robustness of our results for wage inequality within and between sectors and occupations in Table 1 in the paper to the inclusion of non-manufacturing industries. We expect the mechanism in our model to apply in any sector characterized by heterogeneous firm profitability, firm-level wage determination and selection into export markets. In Column (1) of Table H.1, we replicate our results from Table 1 in the paper for our baseline sample including thirteen IBGE man-

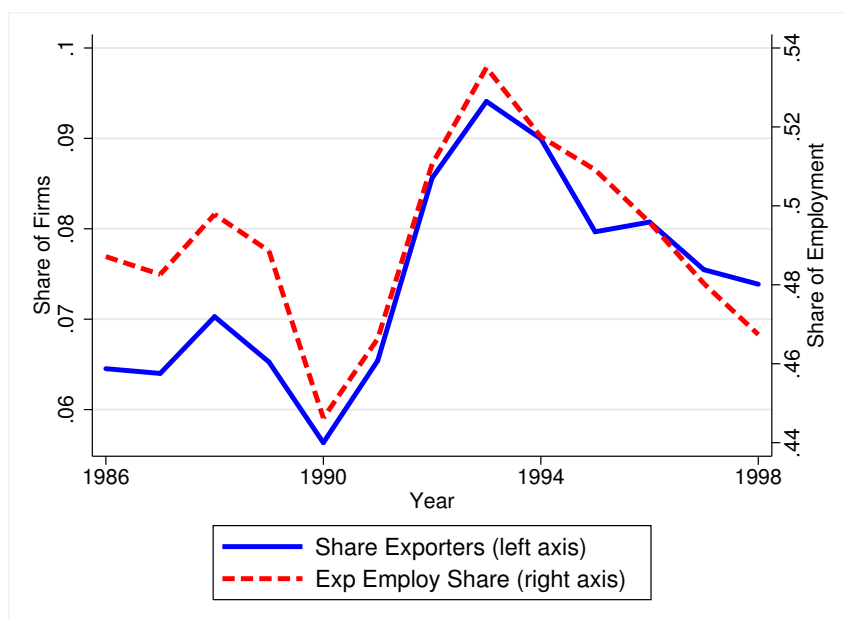


Figure H.1: Export Participation Over Time

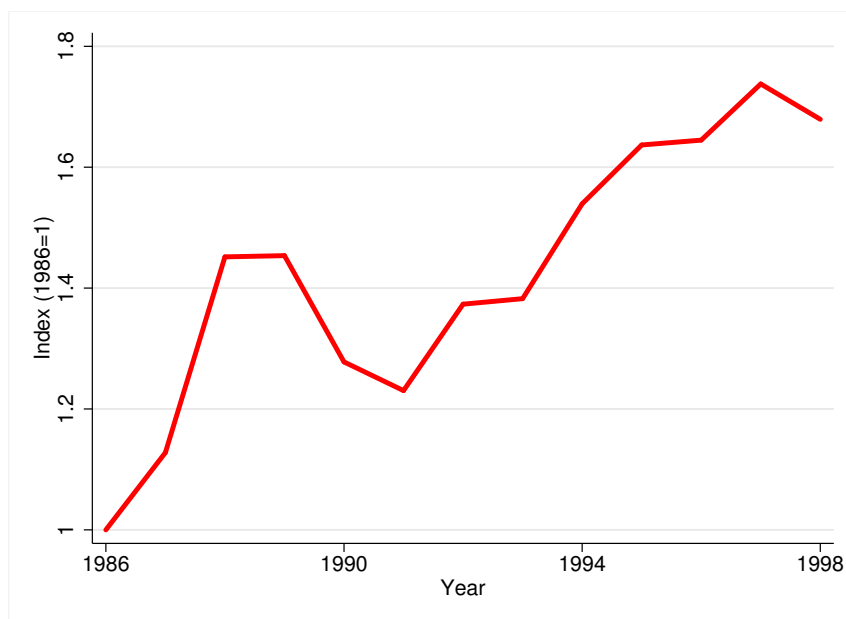


Figure H.2: Real Aggregate Manufacturing Exports

Note: Aggregate Brazilian manufacturing exports from the NBER World Trade Database in U.S. dollars, delated by the U.S. GDP deflator (1990=1).

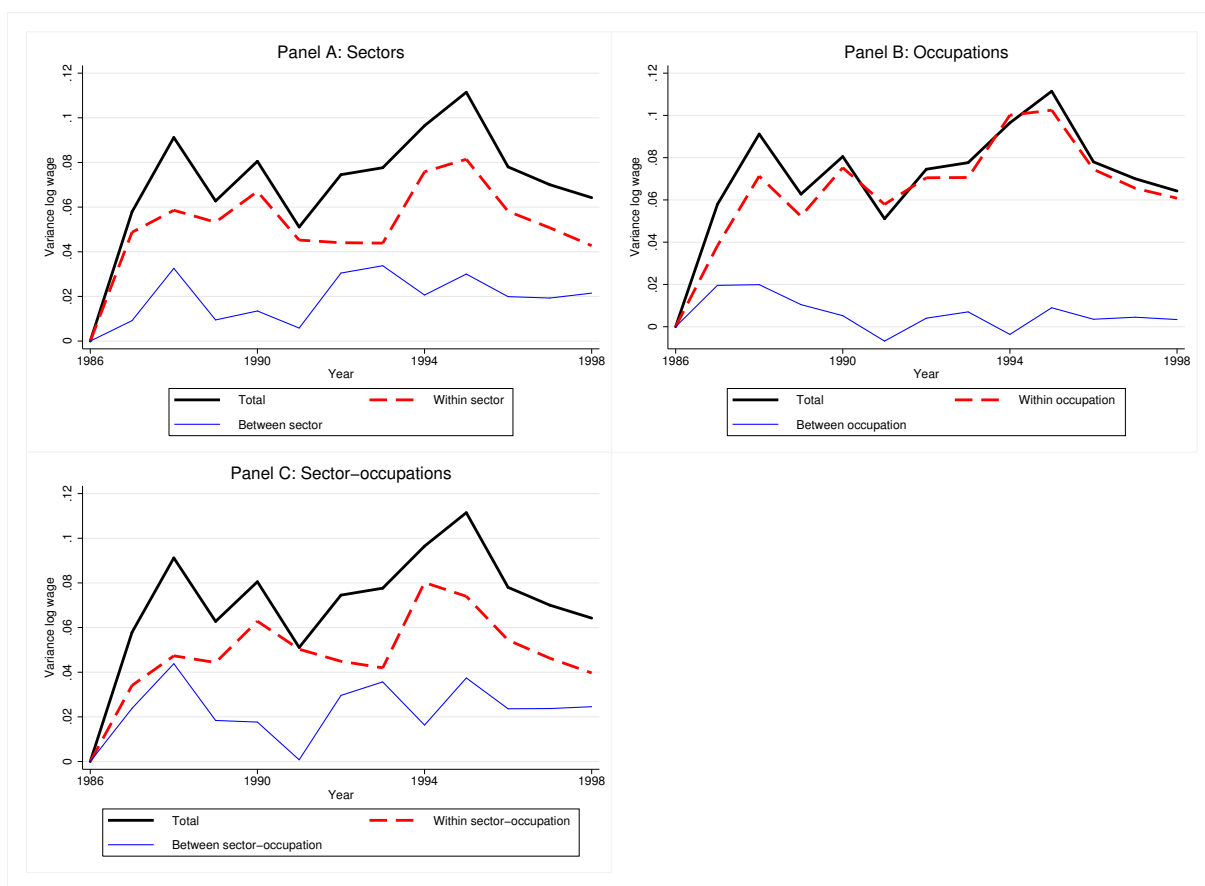


Figure H.3: Changes in Log Wage Inequality and its Components

*Note:* Decomposition of overall log wage inequality into its within and between components. 1986 is used as the base year, that is each series expressed as difference from its 1986 value.

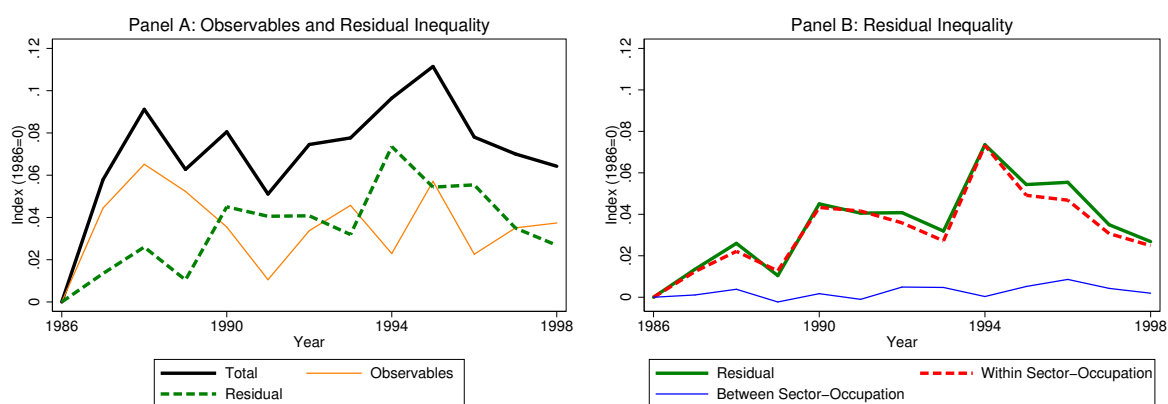


Figure H.4: Changes in Observable and Residual Log Wage Inequality

*Note:* Left panel: Decomposition of overall log wage inequality into the contributions of the worker observables and residual components. Right panel: Decomposition of residual log wage inequality into within and between sector-occupation components. 1986 is used as the base year.

ufacturing sectors. In Column (2) of Table H.1, we expand the sample to include the mining sector, while in Column (3) of Table H.1 we expand the sample to include both the agricultural and mining sectors. In both specifications, we find that the within component makes a substantial contribution towards overall wage inequality, of around the same magnitude as for our baseline sample including only manufacturing industries.

Table H.1: Contribution of the Within Component to Log Wage Inequality

Within component	Level (percent) 1994	Level (percent) 1994 including Mining	Level (percent) 1994 including Mining & Agric
Within occupation	82	83	82
Within sector	83	82	78
Within sector-occupation	68	67	65
Within detailed-occupation	61	61	59
Within sector-detailed-occupation	56	55	53

*Note:* Each cell in the table reports the contribution of the within component to total log wage inequality. The unreported between component is 100 percent minus the reported within component. The within component exceeds 100 percent when the between component moves in the opposite direction partially offsetting its effect.

#### H.4 Worker observables and residual wage inequality

In Figure H.4, we provide further evidence on the contributions of worker observables and residual wage inequality to the evolution of overall wage inequality over time. We plot the changes in the components of wage inequality over time taking 1986 as the base year. The left panel plots the change in overall wage inequality, as well as its worker-observable and residual components. While both components of overall wage inequality initially increase from 1986 onwards, overall wage inequality inherits its inverted U-shaped pattern from residual wage inequality, which rises until 1994 and declines thereafter. The right panel decomposes changes in residual wage inequality into its within and between sector-occupation components, again relative to the base year of 1986. The time-series evolution of residual wage inequality is entirely dominated by the evolution of the within sector-occupation component, while the between component remains relatively stable over time.<sup>18</sup> As is clear from the two panels of the figure, our conclusions are not sensitive to the choice of the base year, and if anything the role of the residual inequality becomes more pronounced as we move forward the base year.

#### H.5 Regional Robustness

In Table H.2, we demonstrate the robustness of our results for overall and residual wage inequality to controlling for region. In the first row, we restate our baseline results. In the second row, we report results for the state of São Paulo, which accounts for around 45 percent of formal manufacturing employment in our sample. In the third and fourth rows, we report results using sector-occupation-region

<sup>18</sup>Since the within component dominates using sector-occupation cells, it follows that it also dominates using sector and occupation cells separately, and hence for brevity we do not report these results.



cells instead of sector-occupation cells, where we define regions in terms of either 27 states or 136 meso regions. These specifications abstract from any variation in wages across workers within sector-occupations that occurs between regions. Nonetheless, in each specification, we continue to find that a sizeable fraction of wage inequality is a within phenomenon. This is particularly notable for residual wage inequality, where the within component still accounts for over two thirds of the level and around half of the growth of residual inequality even for the detailed meso-regions.

Table H.2: Regional Robustness

	OVERALL INEQUALITY		RESIDUAL INEQUALITY	
	Level 1994	Change 1986–95	Level 1994	Change 1986–95
Within sector-occupation	68	66	89	90
Within sector-occupation, São Paulo	64	49	89	71
Within sector-occupation-state	58	38	76	56
Within sector-occupation-meso	54	30	72	49

*Note:* All entries are in percent. The first line duplicates the baseline results from Table 1 (overall inequality in Panel A and residual inequality in Panel B). The second line reports the same decomposition for the state of São Paulo. The last two lines report the within component using sector-occupation-region cells, where regions are first 27 states and second 136 meso regions.

## H.6 Returns to education and tenure

In this section, we report additional results from the Mincer wage regressions (equation (1)) in Section 3.2 of the paper. Table H.3 reports the estimated coefficients on the indicator variables for observable worker characteristic cells for 1994. Figure H.5 displays the estimated coefficients on the indicator variables for education cells over time, where primary education is the excluded category. Figure H.6 displays the estimated coefficients on the indicator variables for experience (tenure) cells over time, where the first quintile of experience is the excluded category. We find an increase in the estimated returns to both education and experience during our sample period, which is consistent with the results in Attanasio, Goldberg, and Pavcnik (2004) and Menezes-Filho, Muendler, and Ramey (2008).

## H.7 Robustness test using more disaggregated education measures

In this section, we report the results of a robustness test using the nine more disaggregated education categories instead of the four education categories used in our baseline specification. These more disaggregated education categories are as follows: Illiterate, some primary, complete primary, some middle, complete middle, some high, complete high, some college, and complete college. We re-estimate the Mincer regression of log worker wages on observed worker characteristics (equation (1) in the paper) using the more disaggregated education categories. Table H.4 reports the estimated coefficients and standard errors for 1994. We find a similar pattern of results using these more disaggregated education categories. The residual component accounts for 57 percent of the level (1994) of wage inequality

Table H.3: Mincer Regression Results, 1994

Variable	Coefficient	Standard Error
Age 10-14	0.070***	0.008
Age 15-17	0.090***	0.006
Age 18-24	0.311***	0.006
Age 25-29	0.460***	0.006
Age 30-39	0.576***	0.006
Age 40-49	0.658***	0.006
Age 50-64	0.488***	0.006
Age 65+	0.323***	0.008
Female	-0.397***	0.001
High School	0.432***	0.001
Some College	0.980***	0.002
College Graduate	1.441***	0.002
Second Experience Quintile	0.012***	0.001
Third Experience Quintile	0.127***	0.001
Fourth Experience Quintile	0.366***	0.001
Fifth Experience Quintile	0.699***	0.001
R-squared	0.41	
Observations	6,043,768	

*Note:* Table reports estimated coefficients on indicator variables for worker observables from the Mincer regression in Section 3.2 of the paper. The Mincer regression is estimated for each year separately. Coefficients reported are for 1994. Standard errors in third column are heteroscedasticity robust.

and 46 percent of the growth (1986–1995) of wage inequality. Around 90 percent of both the level and growth of this residual wage inequality is again explained by the within sector-occupation component.

## H.8 Between-sector wage differentials

In this section, we provide evidence on the evolution of between-sector wage differentials controlling for worker observables and region (see for example [Goldberg and Pavcnik 2005](#)). To do so, we augment the Mincer regression of log wages on worker observables (equation (1) in the paper) with sector fixed effects and region fixed effects:

$$w_{it} = z'_{it}\vartheta_t + \mu_{\ell t} + \kappa_{rt} + \nu_{it}, \quad (\text{H.1})$$

where  $i$  denotes workers,  $t$  is time;  $\ell$  is sectors;  $r$  is regions;  $z_{it}$  is a vector of observable worker characteristics,  $\vartheta_t$  is a vector of returns to worker observables, and  $\nu_{it}$  is a residual. We estimate this regression separately for each year, allowing the sector fixed effects ( $\mu_{\ell t}$ ), the region fixed effects ( $\kappa_{rt}$ ), and the coefficients on worker observables ( $\vartheta_t$ ) to change over time. In Figure [H.7](#), we display the evolution of the sector fixed effects ( $\mu_{\ell t}$ ) over time. As shown in the figure, the estimated sector fixed effects are relatively stable over time. Therefore the observed pattern of rising and declining wage inequality in the data is not explained by changes in the sector fixed effects over time.

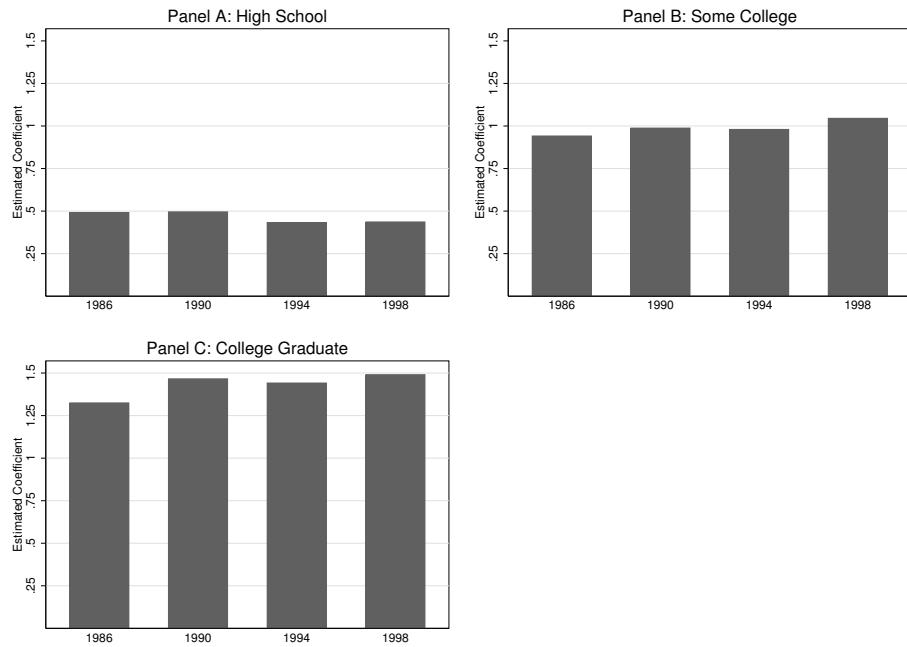


Figure H.5: Returns to Education Over Time

Note: Primary School or less education is the excluded category.

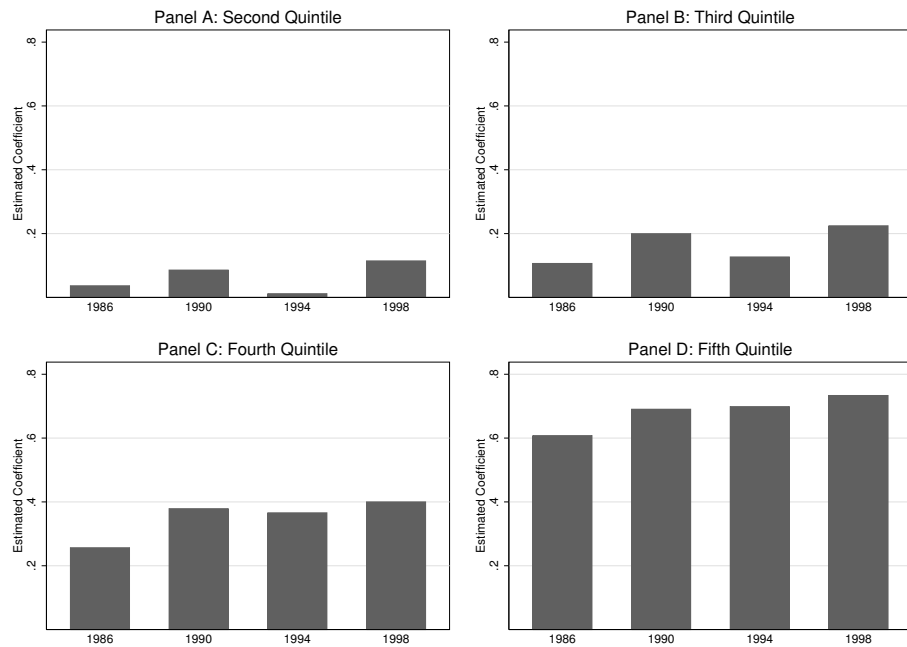


Figure H.6: Returns to Experience Over Time

Note: First quintile is the excluded category.

Table H.4: Mincer Regression Results, 1994 (More Disaggregated Education Categories)

Variable	Coefficient	Standard Error
Age 10-14	0.033***	0.009
Age 15-17	0.065***	0.006
Age 18-24	0.282***	0.006
Age 25-29	0.427***	0.006
Age 30-39	0.551***	0.006
Age 40-49	0.652***	0.006
Age 50-64	0.509***	0.006
Age 65+	0.357***	0.008
Female	-0.409***	0.001
Some Primary	0.111***	0.002
Complete Primary	0.238***	0.002
Some Middle	0.312***	0.002
Complete Middle	0.411***	0.002
Some High	0.554***	0.002
Complete High	0.825***	0.002
Some College	1.255***	0.003
Complete College	1.710***	0.002
Second Experience Quintile	0.006***	0.001
Third Experience Quintile	0.116***	0.001
Fourth Experience Quintile	0.357***	0.001
Fifth Experience Quintile	0.691***	0.001
R-squared	0.425	
Observations	6,043,768	

*Note:* Table reports estimated coefficients from a Mincer regression of log worker wages on indicator variables for observed worker characteristics using the nine more disaggregated education categories. The Mincer regression is estimated for each year separately. Coefficients reported are for 1994. Standard errors in the third column are heteroscedasticity robust.

## H.9 Between versus within-firm wage inequality

In Figure H.8, we provide further evidence on the decomposition of wage inequality within sector-occupations into the following components: (a) worker observables; (b) between-firm component (conditional firm wage component); (c) covariance between worker observables and the conditional firm wage component; (d) the within-firm component (residual). We display the change in wage inequality within sector-occupations and its components relative to the base year of 1986. We confirm that between-firm wage dispersion dominates the evolution of wage inequality within sector-occupations and drives the inverted U-shaped pattern in wage inequality within sector-occupations (which in turn drives the inverted U-shaped pattern in overall wage inequality).

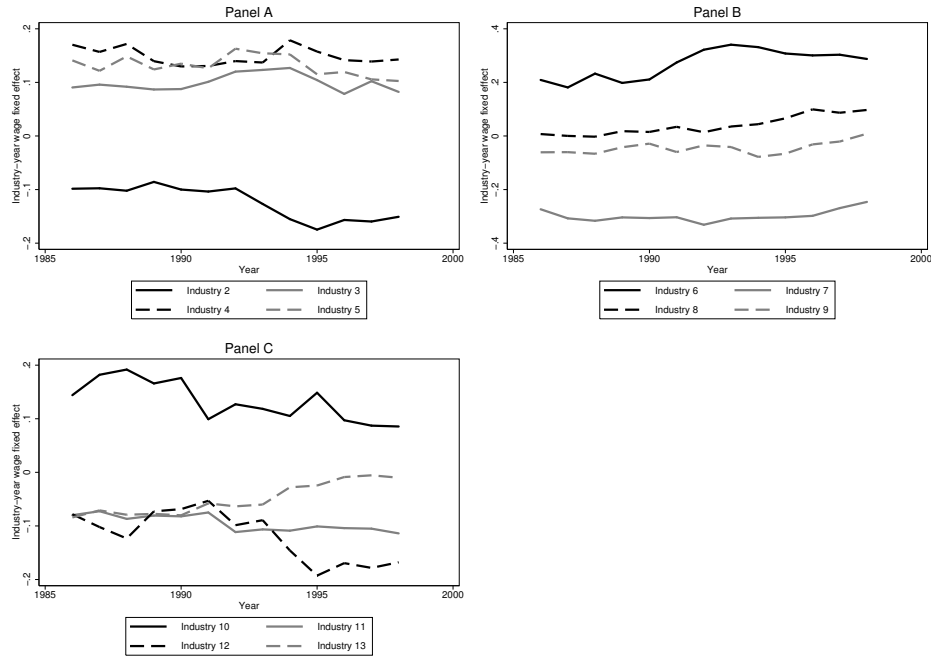


Figure H.7: Sector-Year Wage Fixed Effects

*Note:* Sector fixed effects from estimating a Mincer regression for each year of log wages on observed worker characteristics, sector fixed effects and region fixed effects. Sector fixed effects normalized to sum to zero for each year.

## H.10 Between versus within-firm wage inequality by occupation

In Table H.5, we report the results of our decomposition of wage inequality within sector-occupations for each occupation separately. In the first column, we decompose the inequality in raw wages within sector-occupations into the contributions of the between-firm and within-firm components. In the second column, we report the results of our Mincer equation estimation that controls for observable worker characteristics (equation (2) in the paper). We decompose within-sector-occupation wage inequality into the following components: (a) worker observables; (b) between-firm component (conditional firm wage component); (c) the covariance between worker observables and the conditional firm wage component; (d) the within-firm component (residual). For each of the occupations, we find a substantial between-firm component, confirming that our results are robust across occupations. The main difference across occupations is that worker observables and the covariance between the conditional firm wage component and worker observables are more important for (i) professional and managerial workers and (ii) skilled white collar workers.

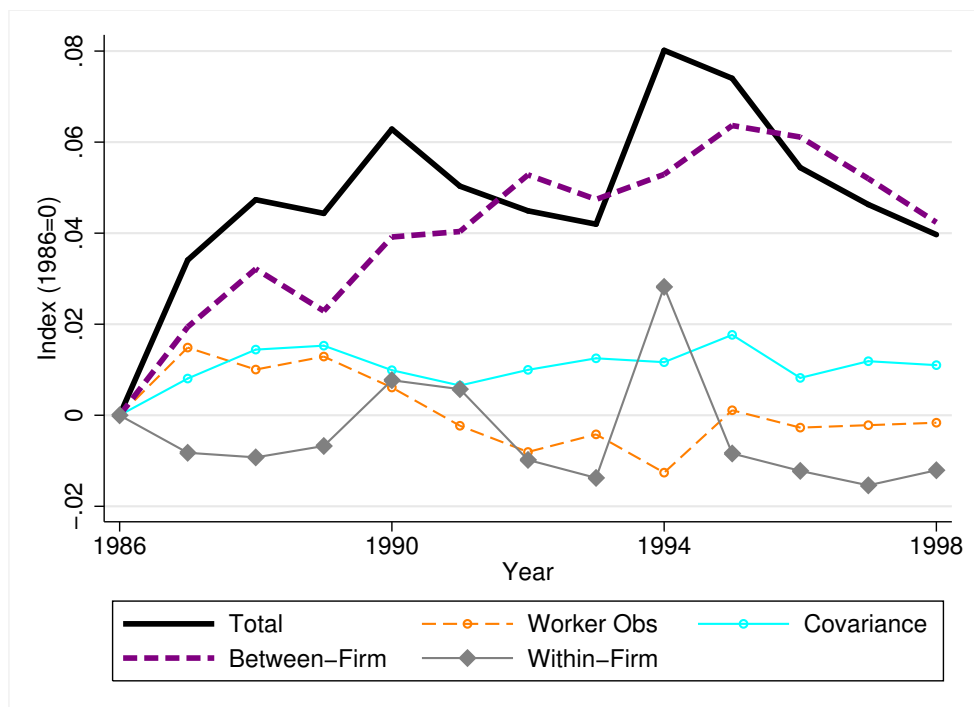


Figure H.8: Changes in Log Wage Inequality *within* Sector-Occupations and its Components

Note: Decomposition of log wage inequality within sector-occupations (employment-weighted) into the following components: (a) worker observables; (b) between-firm component (conditional firm wage component); (c) the covariance between worker observables and the conditional firm wage component; (d) the within-firm component (residual); changes relative to the base year of 1986.

Table H.5: Decomposition of Log Wage Inequality *within* Sector-Occupations (Results by Occupation)

	UNCONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^U$ Level 1994	CONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^C$ Level 1994
<b>1. Professional and Managerial</b>		
Between-firm wage inequality	0.557	0.327
Within-firm wage inequality	0.443	0.288
Worker observables		0.233
Covar observables–firm effects		0.151
<b>2. Skilled White Collar</b>		
Between-firm wage inequality	0.613	0.386
Within-firm wage inequality	0.387	0.289
Worker observables		0.158
Covar observables–firm effects		0.166
<b>3. Unskilled White Collar</b>		
Between-firm wage inequality	0.565	0.381
Within-firm wage inequality	0.435	0.350
Worker observables		0.133
Covar observables–firm effects		0.135
<b>4. Skilled Blue Collar</b>		
Between-firm wage inequality	0.528	0.399
Within-firm wage inequality	0.472	0.400
Worker observables		0.104
Covar observables–firm effects		0.097
<b>5. Unskilled Blue Collar</b>		
Between-firm wage inequality	0.534	0.479
Within-firm wage inequality	0.466	0.441
Worker observables		0.037
Covar observables–firm effects		0.042

*Note:* All entries are in percent. Decomposition of the level and growth of wage inequality within sector-occupations by occupation (employment-weighted average of the results for each sector within an occupation). The decomposition in the first column corresponds to the unconditional firm wage component that does not control for worker observables. The decomposition in the last column corresponds to the conditional firm wage component that controls for worker observables. Figures may not sum exactly to 100 percent due to rounding.

### **H.11 Between versus within-firm wage inequality by sector**

In Tables [H.6](#) and [H.7](#), we report the results of our decomposition of wage inequality within sector-occupations for each sector separately. In the first column, we decompose the inequality in raw wages within sector-occupations into the contributions of the between-firm and within-firm components. In the second column, we report the results of our Mincer equation estimation that controls for observable worker characteristics (equation (2) in the paper). We decompose within-sector-occupation wage inequality into the following components: (a) worker observables; (b) between-firm component (conditional firm wage component); (c) the covariance between worker observables and the conditional firm wage component; (d) the within-firm component (residual). In both columns and for each sector, we find a substantial between-firm component, which is larger than the within-firm component for the majority of sectors. Therefore our results are not driven by a small number of sectors but are rather a robust feature of the data across sectors.

### **H.12 Between versus Within-Firm Wage Inequality for Exporters and Non-Exporters**

In Table [H.8](#), we report the results of our decomposition of wage inequality within sector-occupations for exporters and non-exporters separately. In the first column, we decompose the inequality in raw wages within sector-occupations into the contributions of the between-firm and within-firm components. In the second column, we report the results of our Mincer equation estimation that controls for observable worker characteristics (equation (2) in the paper). We use this estimation to decompose within-sector-occupation wage inequality into the following components: (a) worker observables; (b) between-firm component (conditional firm wage component); (c) the covariance between worker observables and the conditional firm wage component; (d) the within-firm component (residual). Panel A reproduces the results for all firms from the paper, while Panels B and C report results for exporters and non-exporters respectively. We find that the between-firm and within-firm components make roughly equal contributions to wage inequality within sectors, whether we use raw wages or the conditional firm wage component, and whether we consider exporters, non-exporters or all firms together. Therefore our findings of an important between-firm component that is of around the same magnitude as the within-firm component are robust to considering exporters and non-exporters separately as different types of firms.



Table H.6: Decomposition of Log Wage Inequality *within* Sector-Occupations (Results by Sector)

	UNCONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^U$ Level 1994	CONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^C$ Level 1994
<b>1. Non-metallic Minerals</b>		
Between-firm wage inequality	0.668	0.546
Within-firm wage inequality	0.332	0.285
Worker observables		0.074
Covar observables–firm effects		0.096
<b>2. Metallic Products</b>		
Between-firm wage inequality	0.585	0.381
Within-firm wage inequality	0.415	0.315
Worker observables		0.154
Covar observables–firm effects		0.15
<b>3. Mach., Equip. and Instruments</b>		
Between-firm wage inequality	0.527	0.35
Within-firm wage inequality	0.473	0.325
Worker observables		0.198
Covar observables–firm effects		0.127
<b>4. Electrical &amp; Telecomm. Equip.</b>		
Between-firm wage inequality	0.533	0.325
Within-firm wage inequality	0.467	0.333
Worker observables		0.197
Covar observables–firm effects		0.145
<b>5. Transport Equip.</b>		
Between-firm wage inequality	0.632	0.4
Within-firm wage inequality	0.368	0.253
Worker observables		0.163
Covar observables–firm effects		0.185
<b>6. Wood &amp; Furniture</b>		
Between-firm wage inequality	0.53	0.459
Within-firm wage inequality	0.47	0.428
Worker observables		0.063
Covar observables–firm effects		0.051

*Note:* All entries are in percent. Decomposition of the level and growth of wage inequality within sector-occupations by sector (employment-weighted average of the results for each occupation within a sector). The decomposition in the first column corresponds to the unconditional firm wage component that does not control for worker observables. The decomposition in the last column corresponds to the conditional firm wage component that controls for worker observables. Figures may not sum exactly to 100 percent due to rounding.

Table H.7: Decomposition of Log Wage Inequality *within* Sector-Occupations (Results by Sector)

	UNCONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^U$ Level 1994	CONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^C$ Level 1994
<b>7. Paper &amp; Printing</b>		
Between-firm wage inequality	0.55	0.387
Within-firm wage inequality	0.45	0.352
Worker observables		0.151
Covar observables–firm effects		0.11
<b>8. Rubber, Tobacco, Leather, etc.</b>		
Between-firm wage inequality	0.594	0.401
Within-firm wage inequality	0.406	0.315
Worker observables		0.145
Covar observables–firm effects		0.139
<b>9. Chemical &amp; Pharm. Products</b>		
Between-firm wage inequality	0.628	0.398
Within-firm wage inequality	0.372	0.282
Worker observables		0.15
Covar observables–firm effects		0.17
<b>10. Apparel &amp; Textiles</b>		
Between-firm wage inequality	0.578	0.441
Within-firm wage inequality	0.422	0.363
Worker observables		0.097
Covar observables–firm effects		0.099
<b>11. Footwear</b>		
Between-firm wage inequality	0.301	0.251
Within-firm wage inequality	0.699	0.607
Worker observables		0.11
Covar observables–firm effects		0.031
<b>12. Food, Beverages &amp; Alcohol</b>		
Between-firm wage inequality	0.468	0.361
Within-firm wage inequality	0.532	0.47
Worker observables		0.092
Covar observables–firm effects		0.077

*Note:* All entries are in percent. Decomposition of the level and growth of wage inequality within sector-occupations by sector (employment-weighted average of the results for each occupation within a sector). The decomposition in the first column corresponds to the unconditional firm wage component that does not control for worker observables. The decomposition in the last column corresponds to the conditional firm wage component that controls for worker observables. Figures may not sum exactly to 100 percent due to rounding.

Table H.8: Decomposition of Log Wage Inequality *within* Sector-Occupations

PANEL A: ALL FIRMS		
	UNCONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^U$	CONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^C$
	Level 1994	Level 1994
Between-firm wage inequality	55	39
Within-firm wage inequality	45	37
Worker observables		13
Covar observables–firm effects		11
PANEL B: EXPORTERS		
	UNCONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^U$	CONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^C$
	Level 1994	Level 1994
Between-firm wage inequality	44	45
Within-firm wage inequality	56	31
Worker observables		14
Covar observables–firm effects		10
PANEL C: NON-EXPORTERS		
	UNCONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^U$	CONDITIONAL FIRM WAGE COMPONENT, $\hat{\psi}_{j\ell t}^C$
	Level 1994	Level 1994
Between-firm wage inequality	55	43
Within-firm wage inequality	45	38
Worker observables		13
Covar observables–firm effects		7

*Note:* All entries are in percent. Decomposition of the level and growth of wage inequality within sector-occupations (employment-weighted average of the results for each sector-occupation). The decomposition in the first two columns corresponds to the unconditional firm wage component that does not control for worker observables. The decomposition in the last two columns corresponds to the conditional firm wage component that controls for worker observables. Figures may not sum exactly to 100 percent due to rounding.

### H.13 Firm-year rather than firm-occupation-year fixed effects

In this section, we discuss a robustness test in which we estimate firm-year rather than firm-occupation-year wage components. For each sector-year cell, we decompose wage inequality across workers in that cell into within and between-firm components. To do so, we regress log worker wages on firm fixed effects for each sector-year separately:

$$w_{it} = z'_{it}\vartheta_{\ell t} + \psi_{jt} + \nu_{it}, \quad (\text{H.2})$$

where  $i$  again indexes workers,  $j$  indexes firms, and  $\ell$  indexes sector cells; we normalize the firm-year fixed effects  $\psi_{jt}$  to sum to zero for each sector-year, which implies that the regression constant is separately identified; we absorb this regression constant into the worker observables component ( $z'_{it}\vartheta_{\ell t}$ ); we allow the coefficients on observed worker characteristics ( $\vartheta_{\ell t}$ ) to differ across sectors  $\ell$  and time  $t$  to capture variation in their rate of return; and  $\nu_{it}$  is a stochastic error.

As discussed in the paper, the firm-occupation-year wage components in our baseline specification do not capture average differences in wages between occupations within firms, because these firm-occupation-year wage components have a mean of zero for each sector-occupation-year. As a result, we find similar results in this robustness test using firm-year wage components as in our baseline specification using firm-occupation-year wage components. For example, using the estimates from the Mincer regression (H.2) to decompose wage inequality within sectors into its components, we find the following contributions to the level of within-sector wage inequality in 1994: worker observables (21 percent); between-firm component (29 percent); covariance (13 percent); within-firm component (37 percent). Over time, the between-firm component accounts for 76 percent of the growth in wage inequality within sectors from 1986-1995.

### H.14 Constant composition residual wage inequality

As discussed in the paper, one potential concern is that our findings for wage inequality could be influenced by changes in workforce composition. Residual wage inequality is typically higher for older workers, more experienced workers and workers with greater education. Therefore changes in the composition of the workforce according to age, experience and education can influence the magnitude of residual wage inequality and its contribution to overall wage inequality. To address this concern, we follow Lemieux (2006) in using the fact that our controls for worker observables take the form of indicator variables for cells (e.g. age 25-29, college degree etc). As a result, the variance of the residuals in the Mincer regression (equation (1) in the paper) can be expressed as:

$$\text{var}(\nu_{it}) = \sum_{\ell \in \mathcal{L}} s_{it} \text{var}(\nu_{it} | z_{it} \in \ell), \quad (\text{H.3})$$

where  $\ell$  now indexes the cells for observable worker characteristics (education  $\times$  age  $\times$  experience  $\times$  gender) and  $\mathcal{L}$  denotes the set of these cells;  $s_{it}$  is the share of workers in cell  $\ell$  at time  $t$ .

To examine the role of changes in workforce composition, we use (H.3) to construct a counter-

factual measure of residual wage inequality ( $\text{var}(\hat{\nu}_{it})$ ), in which workforce composition across cells  $\ell$  is held constant at its beginning of the sample period values. As shown in Figure H.9, we find that counterfactual residual wage inequality displays the same qualitative pattern as actual residual wage inequality, rising in the late 1980s and early 1990s and declining following the real appreciation of the mid-1990s. As shown in Table H.9, we find that residual wage inequality is quantitatively even more important for overall wage inequality if we hold workforce composition constant at its beginning of the sample period values (compare with Table 1 in the paper). Therefore our findings for residual wage inequality are not being driven by changes in observable workforce composition.

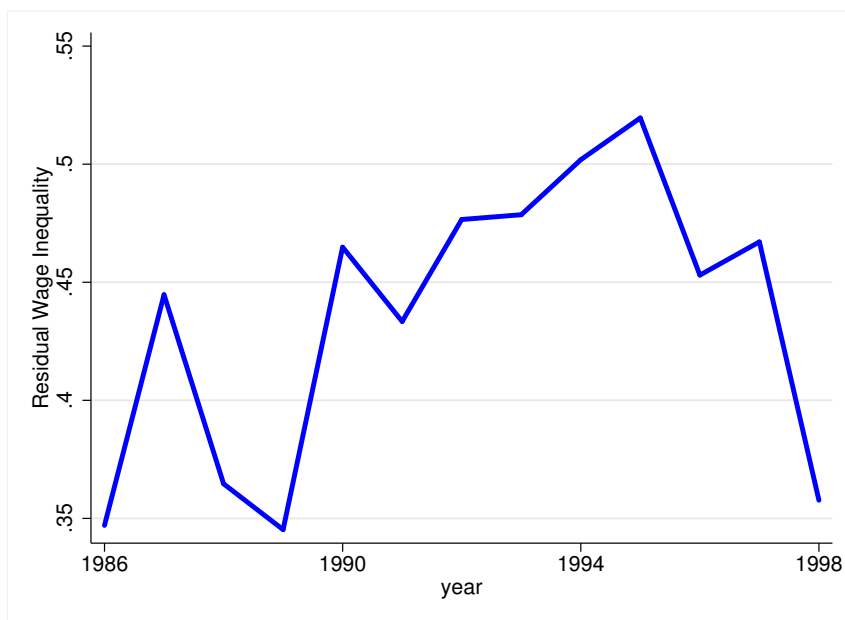


Figure H.9: Constant Composition Residual Wage Inequality

Table H.9: Residual Log Wage Inequality (Constant Workforce Composition)

	Level (percent) 1994	Change (percent) 1986–95
Residual wage inequality	68	155

*Note:* The table decomposes the level and growth of overall log wage inequality into the contributions of worker observables and residual (within-group) wage inequality using equation (1) in the paper, where workforce composition is held constant at its beginning sample period values using equation H.3 above. The unreported contribution of worker observables equals 100 percent minus the reported contribution of residual wage inequality.

## H.15 Firm and Worker Fixed Effects Estimates

In this subsection, we discuss a robustness test, in which we consider an alternative specification of the Mincer wage regression that includes firm and worker fixed effects. Following [Abowd, Creecy, and Kramarz \(2002\)](#), we estimate this specification under the identifying assumptions of no complementarities

between worker abilities and conditional random switching of workers between firms. We emphasize that our theoretical model features complementarities in worker abilities and imperfect assortative matching of workers across firms. Therefore our model implies that these assumptions are invalid and that the firm and worker fixed effects in this specification are not separately identified. Nonetheless, we estimate this specification as a robustness test to show that we continue to find that the between-firm component accounts for a substantial proportion of overall wage inequality even after controlling for time-invariant worker fixed effects, which is consistent with the results in both [Card, Heining, and Kline \(2013\)](#) and [Lopes de Melo \(2013\)](#).

Our data set contains  $N^*$  person-year observations on  $N$  workers and  $J$  establishments. The function  $J(i, t)$  gives the identity of the unique establishment that employs worker  $i$  in year  $t$ . The log wage ( $w_{it}$ ) of worker  $i$  in year  $t$  is assumed to depend on the sum of a time-invariant worker component ( $\alpha_i$ ), a time-invariant establishment component ( $\psi_{J(i,t)}$ ), time-varying worker observables ( $z_{it}$ ), and a stochastic error ( $\nu_{it}$ ):

$$w_{it} = \alpha_i + \psi_{J(i,t)} + z'_{it}\vartheta + \nu_{it}, \quad (\text{H.4})$$

where the stochastic error ( $\nu_{it}$ ) is assumed to be orthogonal to the other wage components, which requires conditional random switching of workers between firms.

As discussed in the data section of this online supplement, our baseline sample includes 83 million observations of 20.4 million workers at 270 thousand firms over the period 1986-1998. To ensure that the estimation is computationally feasible, we use a one percent random sample of all workers that appear in our data over the period 1986-1998. To allow for some variability in the firm component of wages over time, we follow [Card, Heining, and Kline \(2013\)](#) and divide our sample period into sub-periods (1986-89, 1990-93 and 1994-98). For each of these three sub-periods, we estimate the Mincer regression (H.4) using the estimation algorithm of [Abowd, Creedy, and Kramarz \(2002\)](#), as implemented in Stata in the `felsdvreg` command. Under the assumption of conditional random switching of workers between firms, the worker and firm fixed effects are separately identified for a “connected set” of firms that are linked by worker mobility. We use the largest set of connected firms. We normalize the sums of the firm and worker fixed effects to each equal zero. Using the estimates from (H.4), we decompose wage inequality within each sub-period into the following seven terms:

$$\begin{aligned} \text{var}(w_{it}) = & \text{var}\left(z'_{it}\hat{\vartheta}\right) + \text{var}\left(\hat{\psi}_{J(i,t)}\right) + \text{var}\left(\hat{\alpha}_i\right) + 2\text{cov}\left(z'_{it}\hat{\vartheta}, \hat{\psi}_{J(i,t)}\right) \\ & + 2\text{cov}\left(z'_{it}\hat{\vartheta}, \hat{\alpha}_i\right) + 2\text{cov}\left(\hat{\psi}_{J(i,t)}, \hat{\alpha}_i\right) + \text{var}\left(\hat{\nu}_{it}\right). \end{aligned} \quad (\text{H.5})$$

These seven terms are: (1) worker observables, (2) the firm component, (3) the worker component, (4) the covariance between worker observables and the firm component, (5) the covariance between worker observables and the worker component, (6) the covariance between the firm and worker component, (7) the residual component, which by construction is orthogonal to the other terms.

In Table H.10, we summarize the results from these decompositions for each sub-period (Columns (1)-(3)) and the employment-weighted average of the results across all sub-periods (Column (4)). Consistent with the results in both [Card, Heining, and Kline \(2013\)](#) and [Lopes de Melo \(2013\)](#), we find a

substantial contribution of the firm component to wage inequality even after controlling for worker fixed effects, with a minimum contribution of more than 25 percent and an average contribution across all sub-periods of around one third. Of the seven terms in the variance decomposition, the firm and worker components typically make the largest contributions. As in several other empirical studies using this methodology, we find a negative correlation between the firm and worker components. As argued in [Lopes de Melo \(2013\)](#), this negative correlation can arise even if the true data generating process features positive assortative matching. In our theoretical model, the assumptions of no complementarities in worker abilities and conditional random switching of workers between firms are violated. Therefore the firm and worker fixed effects are not separately identified and do not have a structural interpretation. Hence the relative importance of these terms can change over time and the correlation between them does not reveal the true pattern of worker sorting across firms.

Table H.10: Decomposition of Log Wage Inequality within Sub-Periods

	1986-89	1990-93	1994-98	All
Firm component	26	65	29	39
Worker component	70	96	73	79
Worker observables	19	2	7	10
Residual	6	6	6	6
Covariance observables–firm effects	5	2	3	3
Covariance observables–worker effects	-1	1	7	2
Covariance firm–worker effects	-24	-72	-25	-39

*Note:* All entries are in percent. Decomposition of the level of wage inequality within sub-periods (employment-weighted average of the results for each sub-period). Sub-periods are 1986-89, 1990-93 and 1994-98. Figures may not sum exactly to 100 percent due to rounding.

## H.16 Counterfactuals Robustness Test

In this subsection, we report the results of a robustness test, in which we replicate the counterfactuals for fixed exporting costs and variable trade costs using the parameter estimates for each year of our sample. In Figure 1 in the paper, we report these counterfactuals using the parameter estimates for our baseline year of 1994. We vary the fixed exporting cost and variable trade costs from sufficiently small values that all firms export to sufficiently high values that almost no firm exports. In Figure [H.10](#), we report these counterfactuals using the parameter estimates for each year of our sample, where each curve in the figure corresponds to the parameter estimates for a different year.

As discussed in the paper and shown in Figure A1, we expect the market access premia to fluctuate from year to year with changes in trade costs and relative market demand in the export and domestic markets. Nevertheless, the market access premia remain of around the same magnitude throughout our sample period ( $\mu_h$  varies from 1.86 to 2.38, while  $\mu_w$  varies from 0.13 to 0.27). Therefore we find similar results in these counterfactuals using the parameters estimates for different years. As shown in Figure [H.10](#), peak wage inequality varies from around 5-9 percent for fixed exporting costs and from around 7-16 percent for variable trade costs using the parameter estimates for different years.

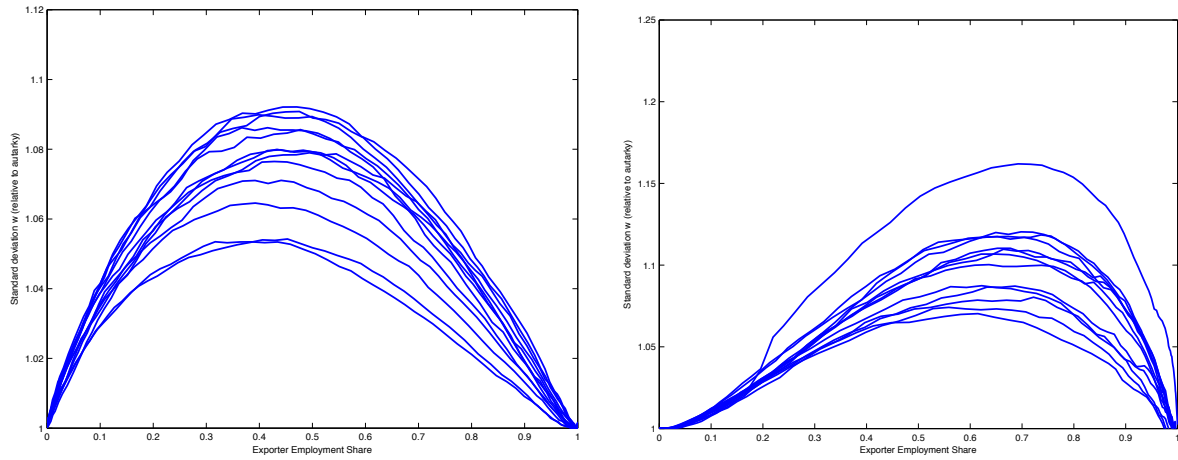


Figure H.10: Counterfactuals Robustness: (a) Fixed export costs and (b) Variable trade cost

Note: The figure plots counterfactual standard deviation of log worker wages (scaled by its counterfactual autarky level) against exporter employment share in the simulated model. The parameters of the model are held constant with the exception of fixed exporting cost  $F_x$  in the left panel and variable trade cost  $\tau$  in the right panel (which are varied from high to low values to cover the full range of the exporter employment share). Each curve shows the counterfactuals using the parameter estimates for a different year of the sample from 1986–98.

## H.17 Robustness Test using Colombian Data

We now report the results of a robustness test in which we re-estimate our model using Colombian data instead of Brazilian data. We show that we find a similar pattern of market access ( $\mu_w, \mu_h$ ) and selection ( $\rho_u, \rho_v$ ) effects as for Brazil. We find that our model is not only successful in explaining the Brazilian distribution of wages across firms and workers but also provides a close approximation to the Colombian data. We obtain counterfactual effects of trade liberalization on wage inequality for Colombia of around the same magnitude as for Brazil.

The data for Colombia are those used by [Clerides, Lach, and Tybout \(1998\)](#). The data were obtained from the Departamento Administrativo Nacional de Estadística (DANE) and cover manufacturing industries (Standard Industrial Classification (SIC) 31–39) for the period 1981–1991. The data report annual information on the inputs, outputs, exports, and characteristics of all plants with at least ten workers. Although the Colombian data are for plants rather than firms, [Clerides, Lach, and Tybout \(1998\)](#) report that, in semi-industrialized countries where the calculation is possible, 95 percent of plants are owned by single-plant firms. Therefore, from now onwards, we follow [Clerides, Lach, and Tybout \(1998\)](#) in referring to the plants as firms.

Our use of linked employer–employee data for Brazil enables us to estimate a worker-level Mincer regression, in which we recover a conditional firm wage component after controlling for worker observables ( $\psi_{jt}^C$ ). Therefore we are able to report results for both this conditional firm wage component ( $\psi_{jt}^C$ ) and raw firm wages ( $w_{jt}$ ). In contrast, for Colombia, we only have firm-level data. Therefore we can only report results for raw firm wages ( $w_{jt}$ ). We compare the results using raw wages for Brazil and Colombia.

In the first two columns of Table [H.11](#), we summarize our estimates for Brazil using raw wages for



our baseline year of 1994. In the last two columns of Table H.11, we report our estimates for Colombia using raw wages for a baseline year of 1986 (the middle of the sample). In our theoretical model, there is no necessary reason for the export premia ( $\mu_h$ ,  $\mu_w$ ) to be exactly the same for Brazil and Colombia. These export premia depend on relative demands in the export and domestic markets, which plausibly differ between Brazil and Colombia. Nonetheless, we find a similar pattern of results for the two countries, with positive estimated market access effects for both employment ( $\mu_h$ ) and wages ( $\mu_w$ ).<sup>19</sup>

Table H.11: Comparison Between Brazil and Colombia Parameter Estimates

	Brazil Raw Wages 1994		Colombia Raw Wages 1986	
	COEFFICIENT	STD ERROR	COEFFICIENT	STD ERROR
$\mu_h$	1.941	0.018	1.231	0.054
$\mu_w$	0.262	0.006	0.232	0.021
$\rho_u$	0.046	0.004	0.020	0.018
$\rho_v$	0.300	0.005	0.249	0.025
$f$	1.343	0.004	1.189	0.013

*Note:* Number of observations for Brazil: 91,410 (firms). Number of observations for Colombia: 6,534 (firms). Maximum likelihood estimates and robust (sandwich-form) asymptotic standard errors.

In Figures H.11 and H.12, we examine the ability of the model to fit the entire distributions of observed employment and wages. Figure H.11 examines the distribution of employment and wages across firms. In the first two panels, we display kernel densities for employment and wages for all firms, both in the data (solid blue line) and the model (dashed red line). In the second two panels, we show kernel densities for firm employment and wages for exporters and non-exporters separately (data is the solid blue line and model is the red dashed line, with exporters shown by crosses and non-exporters shown by hollow circles). As for Brazil, we find that the model is overall successful in fitting these distributions for Colombia. In particular, the model captures both the wide overlap in the employment and wage distributions across exporters and non-exporters, as well as the noticeable rightward shift in the employment and wage distribution of exporters relative to non-exporters.

Figure H.12 examines the distribution of wages across workers. In these distributions, the wage paid by each firm is weighted by the number of workers employed by the firm. In the first two panels, we display kernel densities for wages for all workers, both in the data (solid blue line) and the model (dashed red line). In the second two panels, we show kernel densities for wages for workers employed by exporters and non-exporters separately (data is the solid blue line and model is the red dashed line, with exporters shown by crosses and non-exporters shown by hollow circles). Again the results for Colombia confirm our findings for Brazil. These worker wage distributions combine information from the firm employment and wage distributions. Therefore, since the model is successful in approximating the firm employment and wage distributions, we also find that it is successful in approximating the

<sup>19</sup>For brevity, we report the results for Colombia for the baseline year of 1986, although we have also estimated the model for each year of the sample from 1981-1991. Again, there is no necessary reason for the market access premia ( $\mu_h$ ,  $\mu_w$ ) to be exactly the same in each year, because the relative demands in the domestic and export markets can change over time with trade liberalization and exchange rate fluctuations. Nonetheless, as for Brazil, we find that the market access premia are of a relatively similar magnitude in each year. Therefore we find similar counterfactual effects of trade on wage inequality in Colombia using the parameter estimates for each year in our sample (see Section H.16 of this online supplement for Brazil).

worker wage distributions.

In Figure H.13, we use our estimates for Colombia to compute counterfactual effects of trade liberalization on wage inequality. We undertake these counterfactuals in the same way as discussed in the paper for Brazil. In the first panel of the figure, we examine changes in fixed exporting costs. In the second panel, we consider changes in variable trade costs. In each case, we change trade costs from a sufficiently small value for which all firms export (and the exporter employment share is one) to a sufficiently high value for which no firm exports (and the exporter employment share is zero). We hold all other parameters constant at the values in our baseline estimates for Colombia for 1986 (as reported in Table H.11). In both panels, we display wage inequality (y-axis) against the exporter employment share (x-axis) as we vary trade costs. For ease of interpretation, we express wage inequality on the y-axis as relative to wage inequality under autarky.

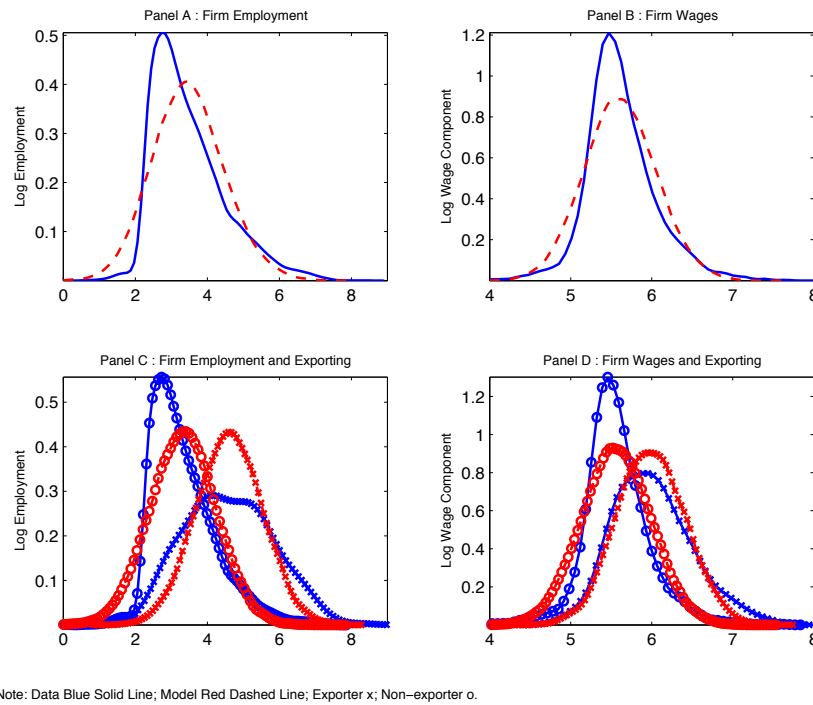
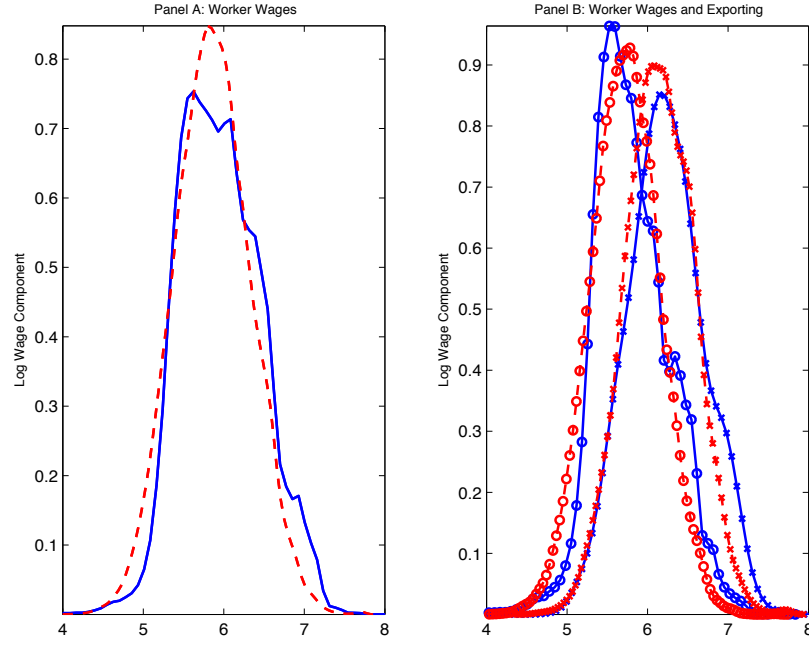


Figure H.11: Colombia Firm Distributions

Confirming our results for Brazil, we find a non-monotonic relationship between wage inequality and trade openness. Starting from a relatively closed economy, reductions in trade costs increase wage inequality. Once the economy is relatively open to trade, further reductions in trade costs decrease wage inequality. In both counterfactuals, we find effects of trade liberalization on wage inequality for Colombia that are around the same magnitude as for Brazil. The peak increase in wage inequality relative to autarky is around 10 percent for fixed exporting costs and around 20 percent for variable trade costs. Both estimates are comparable with the counterfactual predictions for Brazil using raw firm wages (which are slightly higher than the counterfactual predictions for Brazil using the conditional firm wage component  $\psi_{jt}^C$  that are reported in the paper).



Note: Data Blue Solid Line; Model Red Dashed Line; Exporter x; Non-exporter o.

Figure H.12: Colombia Worker Distributions

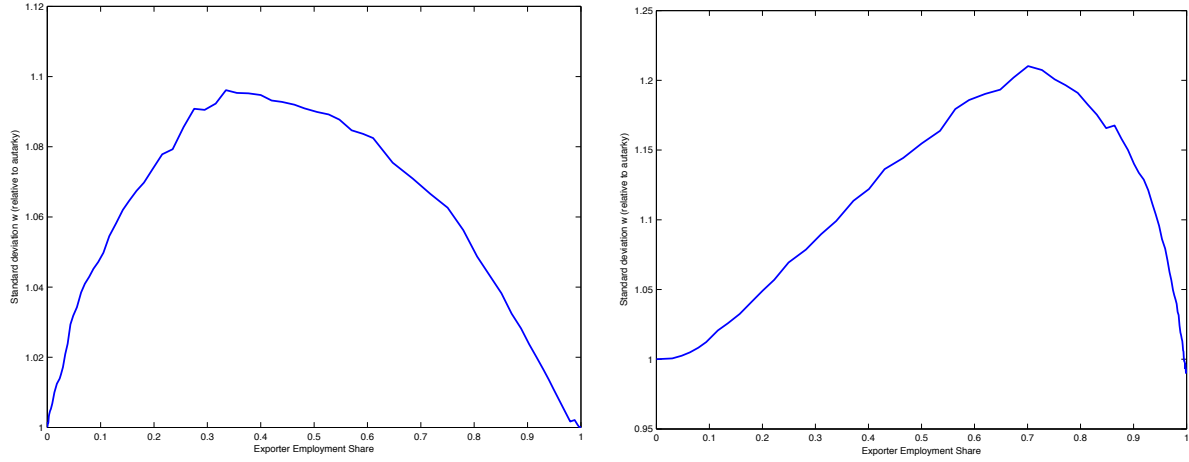


Figure H.13: Colombia Counterfactuals: (a) Fixed export costs and (b) Variable trade cost

Note: The figure plots counterfactual standard deviation of log worker wages (scaled by its counterfactual autarky level) against the exporter employment share in the simulated model. The parameters of the model are held constant at their baseline 1986 estimated values, with the exception of fixed exporting cost  $F_x$  in the left panel and variable trade cost  $\tau$  in the right panel (which are varied from high to low values to cover the full range of exporter employment share).

## H.18 Robustness Test Using Brazilian Household Survey Data

We use Brazilian household survey data to demonstrate the robustness of our results on log wage variation between and within sector-occupations. The Brazilian national household survey *Pesquisa Nacional por Amostra de Domicílios* (PNAD) is designed to be conducted annually during the first week of September and reports both formal and informal employment by household member. All house-

hold members are assigned sampling weights so that we can scale the PNAD records to be nationally representative and comparable to our economy-wide RAIS data on formal employment.<sup>20</sup>

**PNAD preparation:** We prepare the PNAD household data in a way that they reflect wage and categorical information identical to that in RAIS whenever possible.<sup>21</sup> Age, gender and federal state categories are the same in both PNAD and RAIS. We can map uniquely all categorical information on education, occupations and sectors to the related information in RAIS on education, occupations and sectors (at the same IBGE sector level as reported in Table A2 of the paper) using according official concordances (from *Instituto Brasileiro de Geografia e Estatística IBGE*).<sup>22</sup>

Our prepared PNAD data differ to a certain extent from RAIS with regards to five sample features. First, PNAD contains no relevant employer information, so we cannot identify what fraction of the within sector-occupation variation in log wages is attributable to between-firm variation, and we cannot replicate our firm-level structural estimation. While we restrict the RAIS data to firms with five or more employees, we cannot place any such restriction on the PNAD sample. The PNAD sample therefore potentially includes household members employed at firms with four or less employees. Second, PNAD does not include information on tenure at the employer so that Mincer regressions on the PNAD sample exclude tenure predictors, whereas tenure indicators are included in the RAIS sample.

Third, detailed categorical information on industries is limited in PNAD. The “ramo” classification in PNAD cannot be uniquely mapped to the detailed *National Classification of Economic Activities (CNAE)* in RAIS, which breaks down manufacturing into over 250 industries. For the decompositions of log wage variability according to detailed industry categories, in PNAD we can only rely on 28 economic activities in the manufacturing sector.

Fourth, the definition of formal employment in PNAD can potentially differ from the inclusion criterion for formal workers in RAIS. Formal employment in PNAD is based on the household-reported information whether a member holds a signed social-security card (“carteira assinada”) on the principal job, whereas inclusion in RAIS is based on the employer’s choice to report a worker’s formal employment. While it is plausible that a typical employer formally reports a worker in RAIS if and only if the employer also formally signs the worker’s social-security card (“carteira assinada”), some employers may choose to pursue one formalization but not the other.

Fifth, cross sectional sample statistics in our paper are mostly based on the year 1994, but PNAD

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<sup>20</sup>To make PNAD information for the period 1986-2001 consistent over time, we follow the *datazoom* project at PUC Rio de Janeiro and standardize information to the 1981 conventions in PNAD.

<sup>21</sup>For wage information in PNAD we use the monthly wage earned from a household member’s principal occupation in Brazilian Real, deflated by the Consumer Price Index to September 2012. While we convert wages in RAIS to U.S. dollars, the difference has no consequence for statistics because we present summary information in log differences from annual economy-wide means.

<sup>22</sup>We generate in PNAD the same education categories as in RAIS using PNAD’s reported years of study (not counting a person’s repeat years), running from 0 (illiterate) to 17 (graduate studies). We base the occupation concordance from PNAD’s CBO-Domicílio to CBO in RAIS on the official concordance by IBGE, which permits full compatibility with RAIS because PNAD’s CBO-Domicílio is more precise than the 3-digit CBO group level that we use in RAIS. We map the 169 economic activities from PNAD’s “ramo” classification to the subsector IBGE level as in RAIS using the official IBGE concordance from “ramo” to CNAE-Domicílio and general CNAE. “Ramo” is more precise in manufacturing than our 12 IBGE sectors, so we generate a unique many-to-one mapping into our 12 subsectors IBGE in RAIS.

was not conducted in 1994 so that our PNAD statistics are for 1995. We restrict the RAIS sample to the principal (highest-paying) job on December 31st in the paper. The PNAD survey is based on employment in the first week of September. The reported information therefore differs by a little more than eight months between the two data sources.

**Comparisons between RAIS and PNAD:** Table H.12 reports employment and log wages by occupation for RAIS as well as for formal employees in PNAD and all employees in PNAD. Both employment shares and log wage differences from the occupation mean are highly similar for formal workers in RAIS and formal employees in PNAD, with the main difference that employers report relatively more skill white-collar occupations in RAIS and employees in PNAD relatively more unskilled white-collar occupations—consistent with the interpretation that white-collar workers tend to report a lower skill intensity of their jobs than their employers do. The overall PNAD sample for formal and informal workers shows similar employment shares to just the formal sample, suggesting that the incidence of informality is not strongly different across occupations. In all three samples, log wage deviations of unskilled white collar workers from economy-wide average are close to zero, and signs of log wage differences for all other occupation groups are identical.

Table H.12: Occupation Employment Shares and Relative Mean Log Wages

CBO Occupation	RAIS 1994		PNAD formal 1995		PNAD total 1995	
	Empl. sh. (%)	Rel. mean log wage	Empl. sh. (%)	Rel. mean log wage	Empl. sh. (%)	Rel. mean log wage
1 Professional & Managerial	7.2	1.12	7.3	1.12	5.8	1.36
2 Skilled White Collar	10.8	0.38	8.1	0.72	6.7	0.83
3 Unskilled White Collar	8.8	0.07	13.5	−0.03	11.5	0.08
4 Skilled Blue Collar	63.1	−0.14	61.7	−0.15	66.7	−0.14
5 Unskilled Blue Collar	10.0	−0.39	9.5	−0.51	9.4	−0.49

*Note:* Share in manufacturing-sector employment; log wage minus average log wage in manufacturing sector. RAIS information from Table A1 in the paper. PNAD standardized to 1981 conventions using *datazoom* by PUC Rio de Janeiro (employment formal in PNAD if worker holds signed social-security card *carteira assinada* on job). PNAD total includes both formal and informal employees.

A similar pattern emerges for sectors in Table H.13, which reports employment and log wages by sector for RAIS as well as for formal employees in PNAD and all employees in PNAD. Except for a detectably higher share of total employment in wood and furniture for the total PNAD sample, implying a relatively higher incidence of informality in the wood and furniture sector than others, employment shares are remarkably similar across all three samples. Similarly, log wage deviations from the economy-wide average have consistently identical signs and largely similar magnitudes across all three samples. We conclude that RAIS and PNAD data are in close agreement for employment and mean log wage information, and including informal workers in the PNAD all-worker sample typically does not strongly affect statistics compared to formal-worker only samples.

Table H.14 reports the relative contributions of the within component in various decompositions of the variance of the log wage. The within component explains the bulk of log wage variation within oc-

Table H.13: Sectoral Employment Shares and Relative Mean Log Wages

	RAIS 1994		PNAD formal 1995		PNAD total 1995	
	Empl. sh. (%)	Rel. mean ln wage	Empl. sh. (%)	Rel. mean ln wage	Empl. sh. (%)	Rel. mean ln wage
IBGE Sector						
2 Non-metallic Minerals	4.6	−0.21	5.7	−0.17	7.2	−0.30
3 Metallic Products	10.3	0.31	11.6	0.23	10.8	0.33
4 Mach., Equip. and Instruments	5.9	0.48	6.2	0.23	5.0	0.37
5 Electrical & Telecomm. Equip.	4.3	0.41	4.8	0.30	3.7	0.48
6 Transport Equip.	6.0	0.73	7.9	0.49	6.0	0.65
7 Wood & Furniture	6.9	−0.51	6.7	−0.35	12.1	−0.31
8 Paper & Printing	5.5	0.20	6.4	0.17	5.8	0.28
9 Rubber, Tobacco, Leather, etc.	5.1	−0.05	5.6	−0.03	5.3	−0.05
10 Chemical & Pharm. Products	9.4	0.31	9.9	0.24	8.3	0.30
11 Apparel & Textiles	15.1	−0.34	11.9	−0.28	11.8	−0.27
12 Footwear	5.4	−0.44	4.5	−0.52	4.1	−0.42
13 Food, Beverages & Alcohol	21.3	−0.18	18.8	−0.20	19.8	−0.21
All Manufacturing Sectors	100	0.00	100	0.00	100	0.00

Note: Share in manufacturing-sector employment; log wage minus average log wage in manufacturing sector. RAIS information from Table A2 in the paper. PNAD standardized to 1981 conventions using *datazoom* by PUC Rio de Janeiro (employment formal in PNAD if worker holds signed social-security card *carteira assinada* on job). PNAD total includes both formal and informal employees.

Table H.14: Contribution of the Within Component to Formal Manufacturing Log Wage Inequality

	RAIS (%) 1994	PNAD formal (%) 1995	PNAD total (%) 1995
Within occupation	82	72	77
Within sector	83	88	87
Within sector-occupation	68	62	67
Within detailed-occupation	61	58	61
Within sector-detailed-occupation	56	49	53
Within detailed-sector-detailed-occup.	47	46	50

Note: Each cell in the table reports the contribution of the within component to total log wage inequality in the manufacturing-sector. The unreported between component is 100 percent minus the reported within component. RAIS information from Table 1 in the paper. For the detailed-sector classification in PNAD, there are 28 “ramo” activities within manufacturing; for the detailed-sector classification in RAIS, there are 296 CNAE activities within manufacturing.

cupations and sectors in all three samples, including the PNAD sample for formal and informal workers together. For finer and finer sector and occupation definitions, and combined sector-occupation cells, the contribution of the within component naturally drops, and drops in similar ways across all three samples.

Table H.15 reports variance decompositions of the log wage when the log wage is split into its worker observable and residual components through Mincer regressions. As was the case in the preceding table, too, the contributions of the within component are remarkably similar across all three

Table H.15: Worker Observables and Residual Log Wage Inequality

	RAIS (%) 1994	PNAD formal (%) 1995	PNAD total (%) 1995
Residual wage inequality	59	55	58
— within sector-occupation	89	86	88

*Note:* The first row decomposes the level and growth of overall log wage inequality into the contributions of worker observables and residual (within-group) wage inequality. The unreported contribution of worker observables equals 100 percent minus the reported contribution of residual wage inequality. The second row reports the within sector-occupation component of residual wage inequality. RAIS information from Table 1 in the paper.

samples, including the PNAD total sample that combines formal and informal workers.

## I Data Supplement with Industry and Occupation Concordances

In this data supplement, we report details of the disaggregated CNAE industries, for which information is available from 1994 onwards, and details of the disaggregated CBO occupations that we use for robustness tests in the paper. For the year 1994, we show the concordance of 282 CNAE industries to subsectors IBGE and of 348 occupation definitions to our five occupation aggregates, along with the employment distribution. Note that some non-manufacturing CNAE industries occur at the worker level because manufacturing firms by subsector IBGE may report employment at some non-manufacturing plants at the CNAE level.

Table I.1: Occupations and Their Employment Shares, 1994

CBO Occupation	Employment share (percent)	Aggreg. categ.
11 Chemists	.09	1
12 Physicists	.003	1
19 Physicists and related professionals n.e.c.	.01	1
20 Agricultural, forestry and fishing engineers	.03	1
21 Civil engineers, architects and urban planners	.03	1
22 Operations engineers and designers	.04	1
23 Electrical and electronic engineers	.1	1
24 Mechanical engineers	.1	1
25 Chemical engineers	.05	1
26 Metallurgical engineers	.03	1
27 Mining engineers and geologists	.004	1
28 Organizational and methods engineers	.06	1
29 Engineers and related professionals n.e.c.	.1	1
30 Accountants, statisticians and admin. technicians	.1	2
31 Agricultural, medical and related technicians	.09	2
32 Mining, metallurgy and geology technicians	.08	2
33 Civil construction, sanitation and related technicians	.03	1
34 Electrical, electronic and telecommunications technicians	.3	2
35 Mechanical technicians	.3	2
36 Chemical technicians and related workers	.4	2
37 Textile technicians	.03	2
38 Design technicians	.4	2
39 Technicians and related workers n.e.c.	1.1	2
41 Commercial pilots, aviation mechanics and related workers	.005	2
42 Naval commanders, crew officials and related workers	.007	2
43 Naval engine-controlling officers	.002	2
51 Biologists and associated professionals	.006	1
52 Bacteriologists, pharmacologists and similar professionals	.007	1
61 Doctors and medical specialists	.05	1
63 Oral surgeons	.01	1
65 Veterinary doctors and related professionals	.02	1
67 Pharmacists	.01	1
68 Nutritionists and related professionals	.04	2
71 Nurses	.05	1
72 Nursing technicians and related workers	.0000165	2
73 Social workers	.02	1
74 Psychologists	.01	1



Table I.1: Occupations and Their Employment Shares, 1994, Continued

CBO Occupation	Employment share (percent)	Aggreg. categ.
75 Opticians	.005	2
76 Therapists	.0008273	2
77 Medical and dental equipment operators	.004	2
79 Medical and nursing professionals n.e.c.	.007	2
81 Statisticians	.006	1
82 Mathematicians and actuaries	.004	1
83 Systems analysts	.2	1
84 Computer programmers	.1	1
91 Economists	.07	1
92 Administrators	.1	1
93 Accountants	.1	1
99 Auditors and related professionals n.e.c.	.04	1
121 Lawyers	.04	1
129 Legal counsellors and other legal professionals	.003	1
131 Didactics and education professors	.0009431	1
132 Chemistry and physics professors	.000364	1
133 Engineering and architecture professors	.003	1
134 Mathematics, statistics and related professors	.0003144	1
135 Economics and business administration professors	.0003144	1
136 Law and humanities professors	.0002813	1
137 Biology and medical sciences professors	.0001158	1
138 Language and literature professors	.0000165	1
139 College and university professors n.e.c.	.0004964	1
141 Secondary-education teachers	.002	1
142 Primary-education teachers	.03	1
143 Pre-primary educators	.006	1
144 Professors and instructors in professional education	.04	1
145 Special education teachers	.000364	1
149 Principals, supervisors and teachers n.e.c.	.009	2
151 Writers and critics	.002	1
152 Journalists and writers	.1	1
153 Radio and television announcers and commentators	.003	2
159 Publishers, agents, and related professionals	.04	1
161 Sculptors, painters and related professionals	.006	1
163 Photographers, camera operators and related professionals	.03	2
171 Composers, musicians and singers	.0007777	1
172 Choreographers and dancers	.001	1
173 Actors and directors	.0004302	1
174 Entertainment industry managers and and producers	.003	1
175 Clowns, acrobats and other circus artists	.0001158	2
179 Presenters and performers n.e.c.	.001	2
181 Sports coaches and related professionals	.007	2
182 Professional athletes	.0003144	2
189 Referees, and sports professionals n.e.c.	.0003144	2
191 Librarians, archivists and museum curators	.007	1
192 Social scientists and related professionals	.001	1
195 Philologists, translators and interpreters	.003	1

Table I.1: Occupations and Their Employment Shares, 1994, Continued

CBO Occupation	Employment share (percent)	Aggreg. categ.
196 Religious organization representatives and related professionals	.0001324	1
197 Occupational analysts	.09	1
198 Business analysts and technicians and related workers	.02	2
199 Commercial agents and analysts and artistic professionals	.04	2
211 Senior legislative office holders	.000182	1
212 Senior members of the executive branch of government	.0000331	1
213 Members of the judiciary	.0001489	1
214 Senior public servants	.01	2
221 Diplomats	.0008769	1
231 Directors of manufacturing enterprises	.07	1
232 Directors of agriculture, fishing and mining enterprises	.004	1
233 Directors of utilities enterprises	.002	1
234 Directors of civil construction enterprises	.001	1
235 Directors of commerce, hotel and similar enterprises	.004	1
236 Directors of transport and communications enterprises	.005	1
237 Directors of financial and similar enterprises	.003	1
238 Directors of medical and social services enterprises	.0005626	1
239 Directors of enterprises n.e.c.	.03	1
241 Administrative and similar managers	.3	1
242 Production and research managers	.4	1
243 Financial and commercial managers	.5	1
249 Operations managers and managers n.e.c.	.1	1
301 Office supervisors	.7	1
302 Finance office supervisors	.2	1
309 Administrative supervisors n.e.c.	.3	1
311 Administrative agents	.2	2
312 Tax officials	.006	2
313 Senior police officers	.003	2
314 Legal agents and related professionals	.003	2
319 Public and private business administration agents n.e.c.	.007	2
321 Secretaries and business assistants	.6	3
323 Stenographers and related workers	.03	3
331 Accounting assistants, cashiers and related workers	.7	3
332 Counter attendants, ticket clerks and related workers	.009	3
339 Accounting services workers, cashiers and related workers n.e.c.	.5	3
341 Accounting equipment and calculator operators	.008	3
342 Data processing operators	.3	2
343 Punchers and checkers	.009	3
344 Production controllers, computer operators and related technicians	.04	1
351 Railway agents	.004	1
352 Postal and telecommunications services managers	.001	1
353 Aviation services managers, traffic controllers and rel. professionals	.003	1

Table I.1: Occupations and Their Employment Shares, 1994, Continued

CBO Occupation	Employment share (percent)	Aggreg. categ.
354 Road transportation managers and inspectors	.03	1
355 Naval transportation services managers	.002	1
360 Transportation supervisors, collectors and forwarders n.e.c.	.005	3
370 Mail classifiers, mailers and messengers	.05	3
380 Telephone, telegraph and related communications operators	.2	3
391 Stock keeping and conveyance operators	2.0	3
393 Office assistants and related workers	4.0	2
394 Receptionists	.2	3
395 Archivists	.04	3
399 Administrative services workers n.e.c.	1.2	3
410 Commercial salespersons	.03	2
421 Sales supervisors	.8	1
422 Purchasing supervisors	.3	2
431 Technical sales agents and supervisors	.1	2
432 Vendors, salespersons and related workers	.7	2
441 Insurance, real estate and securities brokers	.006	2
442 Sales agents	.06	2
443 Appraisers, auctioneers, and related workers	.0008273	2
451 Wholesale and retail sellers and related workers	1.5	2
452 Street vendors, door-to-door vendors and newspaper vendors	.1	3
453 Demonstrators and fashion models	.2	3
454 Decorators and related workers	.009	2
490 Butchers, market salespersons and trade workers n.e.c.	.5	4
500 Hotel, restaurant, bar and similar establishment workers	.04	1
520 Butlers, housekeepers and related workers	.006	4
531 Cooks and meal packers	.8	3
532 Waiters, bartenders and related workers	.5	3
540 Domestic servants, nannies and other attendants	.09	3
541 Passenger transportation attendants	.002	3
551 Building administrators and related services workers	.1	1
552 Maintenance and cleaning workers	1.1	5
560 Laundry, dry-cleaning and related workers	.2	4
570 Hairdressers, beauticians and related workers	.003	3
572 Nursing, midwifery and laboratory assistants	.1	2
581 Fire fighters	.04	3
582 Police officers	.0009431	2
583 Security guards	1.5	3
584 Traffic guards	.0004798	3
589 Prison, life and security guards n.e.c.	.03	3
591 Travel agents and tourist guides	.002	2
592 Undertakers and embalmers	.0004964	3
599 Hospitality workers and cleaners n.e.c.	.2	5

Table I.1: Occupations and Their Employment Shares, 1994, Continued

CBO Occupation	Employment share (percent)	Aggreg. categ.
600 Agriculture and forestry administrators	.04	1
601 Agriculture and forestry foremen	.05	1
611 Agricultural goods producers	.0007611	4
612 Farmers, livestock raisers and specialized plant growers	.003	4
621 Agricultural workers	.3	4
631 Field-crop growers	2.5	4
632 Fiber crop growers	.01	4
633 Vegetable crop growers	.006	4
634 Flower growers	.004	4
635 Fruit growers and plantation workers	.2	4
636 Coffee, tea and spice growers and plantation workers	.02	4
637 Oily plants plantation workers	.03	4
638 Aromatic plants, medical or toxic substances plantation workers	.001	4
639 Specialized agricultural growers and workers	.1	4
641 Large-animal dairy and livestock workers	.03	4
642 Medium-sized animal dairy and livestock workers	.02	4
643 Small-animal dairy and livestock workers	.2	4
644 Insect breeding workers	.01	4
649 Specialized dairy and livestock workers n.e.c.	.01	4
651 Logging and wood extraction workers	.1	4
652 Rubber and resin extraction workers	.006	4
653 Forestry extraction workers n.e.c.	.002	4
654 Specialty foods extraction workers	.01	4
655 Extraction workers for aromatic, medical or toxic substances	.000364	4
659 Forestry workers n.e.c.	.04	5
661 Fishery managers	.002	1
662 Industrial fishery workers	.002	4
663 Artisan fishery workers	.0000496	4
664 Fish and seafood breeders	.001	4
669 Fishery related workers n.e.c.	.002	5
671 Agricultural equipment operators	.3	4
672 Agricultural machine operators	.005	5
673 Forestry equipment operators	.06	4
701 Supervisors at manufacturing enterprises	1.2	1
702 Supervisors at mining enterprises	.009	1
703 Supervisors at utilities	.02	1
704 Textile industries foremen	.2	2
705 Foremen and supervisors of system operations	.2	2
711 Mining and stonemasonry workers	.02	4
712 Ore extraction equipment operators	.02	4
713 Stone and ore processing equipment operators	.1	4
714 Petroleum and gas drillers	.004	4

Table I.1: Occupations and Their Employment Shares, 1994, Continued

CBO Occupation	Employment share (percent)	Aggreg. categ.
715 Well drill operators	.001	4
716 Salt bed workers	.001	4
719 Mining, stonemasonry, drilling and related workers n.e.c.	.02	5
720 Steel mill workers	.009	4
721 Furnace operators and metallurgy workers	.2	4
722 Laminate operators	.2	4
723 Metallurgical furnace operators	.06	4
724 Metal melter operators and workers	.2	4
725 Molders	.2	4
726 Thermal and chemical treatment workers	.09	4
727 Wire drawers and metal extruders	.1	4
728 Galvanizers and metal coating workers	.2	4
729 Finishing operators and iron and steel workers n.e.c.	3.8	4
731 Wood treatment workers	.4	4
732 Wood cutting equipment operators	.4	4
733 Wood grinders and pulp preparers	.08	4
734 Paper- and cardboard-making machine operators	.2	4
735 Setup operators for plywood and agglomerated wood	.3	4
739 Wood treatment and paper- and cardboard-making workers n.e.c.	.5	4
741 Crushing, mixing and grinding machinery operators	.07	4
742 Thermic equipment operators	.08	4
743 Filtering equipment operators	.04	4
744 Distillery and reaction equipment operators	.2	4
745 Petroleum refining operators	.02	4
746 Coke treating equipment operators	.02	4
747 Pharmaceutical production workers	.2	4
749 Chemical processing operators and related workers n.e.c.	.6	4
751 Fibers preparers	.4	4
752 Spinners, twistors and related workers	1.0	4
753 Weaver operators and weaving preparers	.4	4
754 Weavers	.5	4
755 Jersey weavers	.2	4
756 Fabric treating, dying and printing workers	1.1	4
759 Lace makers, weavers, dyers and related workers n.e.c.	.5	5
761 Fur and leather graders and tanners	.5	4
771 Millers	.05	4
772 Sugar refining and production workers	.5	4
773 Slaughterers, boners and similar workers	1.1	4
774 Cooks and food preparation workers	1.5	4
775 Milk, dairy products and similar treatment workers	.4	4
776 Bakers, confectioners and related workers	.9	4

Table I.1: Occupations and Their Employment Shares, 1994, Continued

CBO Occupation	Employment share (percent)	Aggreg. categ.
777 Coffee, cocoa and similar production workers	.3	4
778 Beer, wine and other beverages production workers	.4	4
779 Food and beverage preparation workers n.e.c.	.6	4
781 Tobacco workers	.2	4
782 Cigar makers	.002	4
783 Cigarette makers	.07	4
791 Tailors, dressmakers and sewers	.1	4
793 Hatters	.01	4
794 Clothes designers and cutters	.4	4
795 Dressmakers	4.6	4
796 Upholsterers and related workers	.2	4
797 Embroiderers and darners	.1	4
799 Sewers, upholsterers and related workers n.e.c.	1.1	4
801 Shoe-makers	.1	4
802 Shoe-makers n.e.c.	4.5	4
803 Leather workers	.4	4
811 Cabinet makers and similar workers	1.1	4
812 Wood carving operators	.4	4
819 Carpenters, wood carving machine operators and sim. workers n.e.c.	.4	4
820 Stone cutters, polishers and engravers	.1	4
831 Furnace operators	.07	4
832 Tool maintenance and metal workers	.4	4
833 Metalwork, rotary-grind, slicing, and similar workers	1.5	4
834 Tool and machine preparers for assembly-line production	.3	4
835 Tool and machine operators in assembly-line production	2.3	4
836 Metal polishers and tool sharpeners	.2	4
837 Computer numeric control tool and machine operators	.01	4
839 Blacksmiths and metal processing workers n.e.c.	.6	4
840 Mechanical adjusters	.4	4
841 Machine assemblers	.6	4
842 Clock, watch and precision instrument makers	.09	4
843 Motor vehicle maintenance mechanics	.3	4
844 Aircraft maintenance mechanics	.01	4
845 Machine and other maintenance mechanics	1.8	4
849 Machine assemblers, maintenance mechanics and sim. workers n.e.c.	.5	4
851 Electrical-equipment assemblers	.4	4
852 Electronic-equipment assemblers	.5	4
854 Electrical and electronic repair technicians	.6	4
855 Electrical line installers	.2	4
856 Installers and servicers of telecommunications equipment	.03	4
857 Installers and servicers of telecommunication lines	.01	4
859 Electricians, electronic and sim. workers n.e.c.	.3	4

Table I.1: Occupations and Their Employment Shares, 1994, Continued

CBO Occupation	Employment share (percent)	Aggreg. categ.
861 Radio and television station operators	.007	2
862 Sound, scenery, and projection operators	.009	2
871 Plumbers and piping installers	.1	4
872 Welders and oxygen cutters	1.1	4
873 Sheet metal workers and furnace operators	.4	4
874 Metal structure and similar workers	.3	4
880 Jewellers, goldsmiths and engravers	.1	4
890 Glass blowers and molders and similar workers	.07	4
891 Glass and crystal cutters and polishers	.04	4
892 Ceramists and similar workers	1.2	4
893 Oven operators for glass and ceramics	.1	4
894 Glass engravers	.004	4
895 Glass and ceramic decorators and painters	.05	4
899 Glass, ceramic and sim. workers n.e.c.	.3	4
901 Bamboo and rubber products workers	.7	4
902 Tire vulcanization and repair workers	.4	4
903 Plastic products workers	2.3	4
910 Paper and cardboard assembly workers	.4	4
921 Printing and similar workers	.2	4
922 Distributors and printers	.5	4
923 Stereotypers and electrotypers	.007	4
924 Engravers and cliché makers for printing	.04	4
925 Photo reproduction printers and related workers	.1	4
926 Binders and similar workers	.3	4
927 Photographic film developers and similar workers	.02	4
929 Graphic arts workers n.e.c.	.7	4
931 Construction and metal structure painters	.1	4
939 Painters n.e.c.	.4	4
941 Musical instrument makers and similar workers	.02	4
942 Basket weavers and straw product manufacturers	.04	4
943 Non-metallic mineral products manufacturing workers	.2	4
949 Non-metallic, mineral-derivative product makers n.e.c.	.02	4
951 Bricklayers and stucco makers	.4	4
952 Concrete structure setters, pavers, and similar workers	.09	4
953 Roofers	.009	4
954 Carpenters	.2	4
955 Plasterers, tile and wood floor layers, and similar workers	.05	4
956 Insulation workers	.009	4
957 Glaziers	.008	4
959 Civil construction and sim. workers n.e.c.	.4	5
961 Nuclear and electrical power plant operators	.04	4

Table I.1: Occupations and Their Employment Shares, 1994, Continued

CBO Occupation	Employment share (percent)	Aggreg. categ.
969 Immobile machinery and similar equipment operators	1.5	4
971 Cargo loading and packing workers	1.3	5
972 Riggers	.003	4
973 Crane and lift operators	.2	4
974 Construction and earth-moving equipment operators	.2	4
979 Construction, earth-moving, and similar equipment operators n.e.c.	.4	4
981 Onboard naval supervisors, sailors and motorboat drivers	.007	4
982 Ship machinists and stokers	.002	4
983 Train machinists, stokers and similar workers	.03	4
984 Railway shunting agents and maintenance workers	.01	4
985 Car, bus, truck and similar vehicle drivers	2.1	4
986 Animal-drawn vehicle drivers and cattle herders	.002	5
989 Transport vehicle drivers and sim. workers n.e.c.	.2	5
991 Other unskilled workers n.e.c.	6.3	5

*Note:* Share in total formal manufacturing-sector employment in 1994. We map the 348 disaggregated CBO occupations into five aggregate categories as shown in the final column: 5 *Professional and Managerial occupations*, 4 *Technical and Supervisory occupations*, 3 *Other White Collar occupations*, 2 *Skill Intensive Blue Collar occupations*, 1 *Other Blue Collar occupations*.



Table I.2: Industries and Their Employment Shares, 1994

CNAE Industry	Employment share (perc.)	Subs. IBGE
1414 Farming of cattle	.0002317	11
1457 Farming of poultry	.006	13
1503 Growing of crops combined with farming of animals (mixed farming)	.0000827	7
1619 Service activities in agriculture	.03	13
1627 Service activities in animal husbandry, except veterinary activities	.002	13
15113 Slaughtering and processing of animals	1.1	13
15121 Slaughtering and processing of poultry	1.5	13
15130 Rendering and processing of meat products	.5	13
15148 Processing and canning of seafood	.4	13
15210 Processing and canning of fruit	.3	13
15229 Processing and canning of vegetable and other plants	.2	13
15237 Manufacture of fruit and vegetable juice	.4	13
15318 Milling of oilseeds	.3	13
15326 Refining of plant oils	.09	13
15334 Processing of fats, oils and margarine from plants and animals	.1	13
15415 Manufacture of fluid milk	.2	13
15423 Manufacture of dairy product	1.2	13
15431 Manufacture of ice cream	.3	13
15512 Processing and manufacture of rice and rice products	.3	13
15520 Milling and manufacture of wheat and wheat products	.2	13
15539 Manufacture of manioc and derivative products	.08	13
15547 Manufacture of corn flour	.08	13
15555 Manufacture of corn oils and other cereals derivatives	.03	13
15563 Manufacture of feeds for animals	.2	13
15598 Processing, milling, and manufacture of plant products n.e.c.	.7	13
15610 Distillery of sugar	4.4	13
15628 Refinery and milling of sugar	.2	13
15717 Roasting and grinding of coffee	.3	13
15725 Manufacture of soluble coffee	.06	13
15814 Manufacture of bakery products, cakes and other pastries	1.3	13
15822 Manufacture of cookies and crackers	.6	13
15830 Manufacture of cocoa products and chocolate and confectionery prep.	.5	13
15849 Manufacture of flour mixes and dough	.5	13
15857 Manufacture of dressings and prep. sauces, spices and concentrates	.1	13
15865 Manufacture of diet, infant and other canned foods	.2	13
15890 Manufacture of foods n.e.c.	.8	13
15911 Distillery, purification and bottling of liquor	.2	13
15920 Manufacture of wines	.1	13
15938 Manufacture of stout and beer	.5	13
15946 Bottling of mineral water	.1	13
15954 Manufacture of soft drinks	1.0	13
16004 Manufacture of tobacco products	.6	9

Table I.2: Industries and Their Employment Shares, 1994, Continued

CNAE Industry	Employment share (perc.)	Subs. IBGE
17116 Processing of cotton	.2	11
17191 Processing of natural fabrics n.e.c.	.2	11
17213 Weaving and processing of cotton	.8	11
17221 Weaving, knitting and processing of natural fabrics n.e.c.	.2	11
17230 Weaving, knitting and processing of artificial and synthetic fabrics	.3	11
17248 Weaving, knitting and processing of embroidery fabrics	.1	11
17310 Finishing and coating of cotton	1.0	11
17329 Finishing and coating of natural fabrics n.e.c.	.3	11
17337 Finishing and coating of artificial and synthetic fabrics	.6	11
17418 Manufacture of household textiles	.4	11
17493 Manufacture of textile goods n.e.c.	1.3	11
17507 Manufacture of services related to textiles	.3	11
17612 Manufacture of textile products from fabrics	.1	11
17620 Manufacture of carpets and rugs	.09	11
17639 Manufacture of rope and cordage	.04	11
17647 Manufacture of special fabrics and textile products	.08	11
17698 Manufacture of textile products (except apparel) n.e.c.	.3	11
17710 Knitting of fabrics	.4	11
17728 Knitting of hosiery and socks	.2	11
17795 Knitting of textile products n.e.c.	.2	11
18112 Manufacture of cut-and-sew underwear and nightwear	1.1	11
18120 Manufacture of cut-and-sew apparel n.e.c.	6.0	11
18139 Manufacture of cut-and-sew professional gear	.3	11
18210 Manufacture of apparel accessories	.4	11
18228 Manufacture of industrial and personal security accessories	.08	11
19100 Cutting, tanning and finishing of leather and hides	.8	9
19216 Manufacture of suitcases, bags, purses and other luggage	.3	9
19291 Manufacture of leather products n.e.c.	.3	9
19313 Manufacture of leather footwear	3.6	12
19321 Manufacture of athletic footwear	.6	12
19330 Manufacture of rubber and plastics footwear	.3	12
19399 Manufacture of footwear from other materials (except athl. footwear)	.9	12
20109 Cutting of wood	1.5	7
20214 Manufacture of plywood and engineered wood products	1.4	7
20222 Manufacture of cut wood, wood structures and prefab. wood buildings	.5	7
20230 Manufacture of wood containers and packaging material	.1	7
20290 Manufacture of wood products (except furniture) n.e.c.	.5	7
21105 Manufacture of pulp	.2	8
21210 Manufacture of paper	.8	8
21229 Manufacture of paperboard	.2	8
21318 Manufacture of paper bags and containers	.3	8
21326 Manufacture of paperboard containers and coated paperboard	.4	8

Table I.2: Industries and Their Employment Shares, 1994, Continued

CNAE Industry	Employment share (perc.)	Subs. IBGE
21415 Manufacture of stationery products from paper and paperboard	.3	8
21423 Manufacture of printed and plain tape and forms	.09	8
21490 Manufacture of pulp, paper and paperboard products n.e.c.	.2	8
22110 Publishing and printing of newspapers	.8	8
22128 Publishing and printing of magazines	.2	8
22136 Publishing and printing of books	.2	8
22144 Publishing of records, tapes, disks and other recording materials	.006	8
22195 Publishing and printing of other products	.3	8
22217 Printing of newspapers, magazines and books	.3	8
22225 Printing services for didactic and commercial materials	.5	8
22292 Printing services n.e.c.	.8	8
22314 Reproduction of records, tapes and disks	.03	9
22322 Reproduction of video tapes	.008	9
22330 Reproduction of motion pictures	.02	9
22349 Reproduction of software	.003	9
23108 Manufacture of coal products	.03	10
23205 Refinery of petroleum	.03	10
23302 Processing of nuclear fuels	.02	2
23400 Manufacture of alcohol	1.3	13
24112 Manufacture of chlorine and alkalies	.05	10
24120 Manufacture of fertilizer ingredients	.02	10
24139 Manufacture of phosphatic, nitrogenous and potassic fertilizer	.2	10
24147 Manufacture of industrial gas	.1	10
24198 Manufacture of inorganic chemicals n.e.c.	.03	10
24210 Manufacture of basic petrochemical products	.2	10
24228 Manufacture of intermediate goods for resins and fibers	.02	10
24295 Manufacture of organic chemicals n.e.c.	.09	10
24317 Manufacture of thermal-forming resins	.1	10
24325 Manufacture of non-thermal-forming resins	.02	10
24333 Manufacture of elastomers	.05	10
24414 Manufacture of artificial fibers and filaments	.02	10
24422 Manufacture of synthetic fibers and filaments	.04	10
24511 Manufacture of medicinal chemicals	.3	10
24520 Manufacture of pharmaceuticals for human use	.7	10
24538 Manufacture of pharmaceuticals for veterinary use	.05	10
24546 Manufacture of supplies for medicinal, diagnostic and odontol. uses	.1	10
24619 Manufacture of insecticides	.02	10
24627 Manufacture of fungicides	.007	10
24635 Manufacture of herbicides	.01	10
24694 Manufacture of agricultural chemicals n.e.c.	.05	10
24716 Manufacture of soap and synthetic detergents	.3	10
24724 Manufacture of sanitation goods and polishes	.2	10

Table I.2: Industries and Their Employment Shares, 1994, Continued

CNAE Industry	Employment share (perc.)	Subs. IBGE
24732 Manufacture of perfumes and cosmetics	.5	10
24813 Manufacture of paint, varnish, lacquer and enamel	.4	10
24821 Manufacture of printing ink	.02	10
24830 Manufacture of coatings, solvents and allied products	.04	10
24910 Manufacture of adhesives and sealants	.06	10
24929 Manufacture of explosives	.1	10
24937 Manufacture of chemical catalysts	.008	10
24945 Manufacture of chemical additives for industrial use	.02	10
24953 Manufacture of photogr. plates, films, paper and photogr. chemicals	.03	10
24961 Manufacture of disks and tapes	.009	10
24996 Manufacture of miscellaneous chemical products n.e.c.	1.3	10
25119 Manufacture of tires and rubber hoses	.4	9
25127 Retreading of tires	.3	9
25194 Manufacture of rubber products n.e.c.	1.0	9
25216 Manufacture of laminated plastics, plates and pipes	.2	10
25224 Manufacture of plastics packaging materials	1.0	10
25291 Manufacture of plastics products n.e.c.	2.4	10
26115 Manufacture of flat and security glass	.09	2
26123 Manufacture of glass containers	.1	2
26190 Manufacture of glass products	.3	2
26204 Manufacture of cement	.3	2
26301 Manufacture of cement and gypsum products and concrete	.7	2
26417 Manufacture of non-refractory structural clay products	1.0	2
26425 Manufacture of refractory products	.3	2
26492 Manufacture of non-refractory other clay products	.8	2
26913 Manufacture of cut stone and stone products	.3	2
26921 Manufacture of lime and gypsum products	.1	2
26999 Manufacture of nonmetallic mineral products n.e.c.	.5	2
27111 Manufacture of plain iron and steel sheets, plates and foils in int. mills	.9	3
27120 Manufacture of formed iron and steel sheets in integrated mills	.5	3
27219 Manufacture of basic iron and steel	.3	3
27227 Manufacture of primary and semi-finished iron and steel	.4	3
27294 Manufacture of iron and steel products and drawing of steel wire	.1	3
27316 Manufacture of iron and steel seamed tubes	.1	3
27391 Manufacture of iron and steel tubes n.e.c.	.2	3
27413 Processing and manufacture of aluminum	.5	3
27421 Processing and manufacture of precious metals	.02	3
27499 Processing and manufacture of nonferrous metals n.e.c.	.5	3
27510 Foundries of iron and steel	.9	3
27529 Foundries of nonferrous metals	.5	3
28118 Manufacture of prefabricated metal structures and components	.6	3
28126 Manufacture of metal structures and plates	.3	3

Table I.2: Industries and Their Employment Shares, 1994, Continued

CNAE Industry	Employment share (perc.)	Subs. IBGE
28134 Manufacture of heavy gauge tanks and heaters	.2	3
28215 Manufacture of metal tanks and central heaters	.2	3
28223 Manufacture of boilers and heat exchangers	.03	3
28312 Forging of iron and steel	.2	3
28320 Forging of nonferrous metals	.2	3
28339 Manufacture of stamped metal products	.7	3
28347 Manufacture of powder metallurgy products	.05	3
28398 Services related to metal coating, heat treating and allied processes	.3	3
28410 Manufacture of cutlery	.2	3
28428 Manufacture of hardware	.2	3
28436 Manufacture of handtools	.1	3
28916 Manufacture of metal containers	.3	3
28924 Manufacture of turned products, springs and wire products	.2	3
28932 Manufacture of domestic appliances	.2	3
28991 Manufacture of metal products n.e.c.	1.5	3
29114 Manufacture of int. comb. engines, turbines and non-electric generators	.07	4
29122 Manufacture of pumps and pumping equipment	.1	4
29130 Manufacture of valves and plumbing fixtures	.1	4
29149 Manufacture of compressors	.2	4
29157 Manufacture of transmissions and gears	.1	4
29211 Manufacture of non-electric industrial ovens and thermic installations	.04	4
29220 Manufacture of electric industrial-process furnaces and ovens	.06	4
29238 Manufacture of elevators, cranes and other handling mach. and equipm.	.3	4
29246 Manufacture of commercial refrigeration and ventilation equipment	.2	4
29254 Manufacture of air-conditioning equipment	.07	4
29297 Manufacture of general-purpose machinery n.e.c.	.7	4
29319 Manufacture of farm machinery and equipment	.6	4
29327 Manufacture of tractors for agriculture	.2	4
29408 Manufacture of machine tools	.2	4
29513 Manufacture of oil and gas extraction machinery and equipment	.03	4
29521 Manufacture of mining and construction machinery and equipment	.08	4
29530 Manufacture of tractors for mining and construction	.09	4
29548 Manufacture of construction machinery and equipment	.03	4
29610 Manufacture of machinery and equipment for metallurgy	.1	4
29629 Manufacture of food production machinery	.2	4
29637 Manufacture of textile machinery	.09	4
29645 Manufacture of apparel, leather and footwear machinery	.07	4
29653 Manufacture of pulp, paper, paperboard and paper prod. mach.	.1	4
29696 Manufacture of specific-use commercial machinery n.e.c.	.6	4
29718 Manufacture of fire arms and ammunition	.04	4
29726 Manufacture of heavy military equipment	.01	4
29815 Manufacture of household cooking, refrig. and laundry appliances	.4	4

Table I.2: Industries and Their Employment Shares, 1994, Continued

CNAE Industry	Employment share (perc.)	Subs. IBGE
29890 Manufacture of household appliances n.e.c.	.3	4
30112 Manufacture of non-electronic office machinery	.03	4
30120 Manufacture of electronic office machinery	.1	4
30210 Manufacture of computers	.08	4
30228 Manufacture of peripherals for data processing equipment	.1	4
31119 Manufacture of electric generators	.03	5
31127 Manufacture of transformers, converters and similar electrical products	.2	5
31135 Manufacture of electric engines	.2	5
31216 Manufacture of controls, switchgears and other app. for energy distrib.	.3	5
31224 Manufacture of electrical equipment for electric wiring	.2	5
31305 Manufacture of electric wire and wiring devices	.3	5
31410 Manufacture of batteries (except for vehicles)	.1	5
31429 Manufacture of batteries for vehicles	.08	5
31518 Manufacture of light bulbs	.06	5
31526 Manufacture of lighting equipment (except for vehicles)	.1	5
31607 Manufacture of electrical equipment for vehicles (except batteries)	.4	5
31917 Manufacture of carbon and graphite products for electrical uses	.03	5
31925 Manufacture of electrical signals and alarm equipment	.04	5
31992 Manufacture of electrical machinery, equipment and supplies n.e.c.	.7	5
32107 Manufacture of basic electronic components	.6	5
32212 Manufacture of broadcasting and telephone exchange equipment	.3	5
32220 Manufacture of telephone and similar communication apparatus	.2	5
32301 Manufacture of audio and video equipment	.4	5
33103 Manufacture of medical and therapeutic apparatus	.3	9
33200 Manufacture of measurement, testing and control instruments	.2	9
33308 Manufacture of instruments and equipment for automation and control	.1	9
33405 Manufacture of optical apparatus and photographic and cinem. equipm.	.2	9
33502 Manufacture of watches and clocks	.07	4
34100 Manufacture of automobiles, light trucks and utility vehicles	1.6	6
34207 Manufacture of heavy duty trucks and buses	.4	6
34312 Manufacture of motor vehicle bodies, interiors and trailers for trucks	.2	6
34320 Manufacture of motor vehicle bodies for buses	.2	6
34398 Manufacture of motor vehicle bodies, interiors and trailers for oth. veh.	.1	6
34410 Manufacture of motor vehicle engine parts	.4	6
34428 Manufacture of motor vehicle transmission and power train parts	.2	6
34436 Manufacture of motor vehicle brake systems	.2	6
34444 Manufacture of motor vehicle steering and suspension components	.1	6
34495 Manufacture of motor vehicle parts and accessories n.e.c.	1.4	6
34509 Rebuilding of engines for motor vehicles	.2	6
35114 Building and repair of ships and floating structures	.3	6
35122 Building and repair of boats for sports and leisure	.04	6
35211 Manufacture of railroad rolling stock	.1	6

Table I.2: Industries and Their Employment Shares, 1994, Continued

CNAE Industry	Employment share (perc.)	Subs. IBGE
35220 Manufacture of railroad rolling stock parts and accessories	.01	6
35238 Repair of railroad rolling stock	.008	6
35319 Manufacture of aircraft	.1	6
35327 Repair of aircraft	.04	6
35912 Manufacture of motorcycles	.05	6
35920 Manufacture of bicycles and tricycles	.2	6
35998 Manufacture of transportation equipment n.e.c.	.08	6
36110 Manufacture of wooden furniture	2.2	7
36129 Manufacture of metal furniture	.3	7
36145 Manufacture of mattresses	.2	7
36919 Etching and engraving of precious stones, metals and jewelry	.2	9
36927 Manufacture of musical instruments	.03	9
36935 Manufacture of hunting, fishing and sporting goods	.02	9
36943 Manufacture of toys and games	.3	9
36951 Manufacture of pens, pencils, marking dev. and other office supplies	.07	9
36960 Manufacture of fasteners, buttons, needles and pins	.01	11
36978 Manufacture of brooms, brushes and mops	.1	7
36994 Manufacture of miscellaneous goods n.e.c.	.4	10
37109 Recycling of metal waste and scrap	.04	3
37206 Recycling of non-metal waste and scrap	.04	9
45128 Drilling and foundation building for civil construction	.0003475	3
51314 Wholesale of dairy products	.02	13
51365 Wholesale of beverages	.003	13
51543 Wholesale of chemical products	.02	10
51926 Wholesale of specific merchandize n.e.c.	.002	13
52124 Retail sale of food and bev. in stores betw. 300 and 5,000 sq meters	.006	13
55247 Catering	.9	13
63126 Cargo warehousing	.007	7
74152 Management activities at headquarters and local administrative units	.09	4
92118 Motion picture and video production	.06	9

Note: Share in total formal manufacturing-sector employment 1994. Subsector IBGE is the reported mode sector (highest-numbered subsector in case of tie).

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