Trade Liberalization, Wage Rigidity, and Labor Market Dynamics with Heterogeneous Firms*

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Abstract

Adjustment to trade liberalization is associated with substantial reallocation of labor across firms within sectors. This salient feature of the data is well captured by models of international trade with heterogeneous firms. In this paper, we reconsider the adjustment of firms and workers to changes in trade costs, explicitly accounting for labor market frictions and the entire adjustment path from an initial to a final steady-state. Transition dynamics exhibit rich patterns, varying across firms that differ in productivity levels and across workers attached to these firms. High-productivity exporters expand employment on impact. However, among lower-productivity firms some close shop on impact, others fire some workers on impact and close shop at a later date, and still, other firms gradually reduce their labor force and stay in the industry. In these circumstances, jobs that pay similar wages ex ante are not equally desirable ex post, because after the trade shock, high-productivity incumbents pay higher wages and provide more job security than low-productivity incumbents. We calibrate the model and provide a quantitative assessment of the importance of various channels of adjustment. We find that gains from trade due to a decline in the consumer price index overwhelm losses from wage cuts, job destruction, and capital losses of incumbent firms, and that these losses increase with the extent of labor market frictions. Furthermore, we find that downward wage rigidity can be welfare-enhancing while generating a trade-off between the workers' displacement rates and the labor income loss. Decomposing dynamic gains from trade, we show that while firm profits decrease in the minimum wage as firms have to fire more workers on impact, the total gains from trade can increase due to a reduction in the labor income loss.

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1 Introduction

Trade liberalization leads to reallocation of labor across sectors, but even more so across firms within sectors, as pointed out by Balassa (1967) and more recently by Levinson (1999). Similar patterns of reallocation are precipitated by declining transport costs, which have been remarkable in recent decades (see Hummels 2007). Variation in trade policies or shipping costs changes relative prices, setting in motion an adjustment process that benefits some firms and workers, but harms other firms and workers.

Following Melitz (2003), a new generation of trade models have addressed the heterogeneous response of firms within industries to declines in fixed and variable trade costs, capturing salient features of the data, such as the reallocation of workers from shrinking non-exporting firms to exporters (e.g., Eaton, Kortum, and Kramarz 2011). Much of this work is confined, however, to static models, or steady states, or to models with frictionless labor markets. Needless to say, a full assessment of the costs and benefits of lower trade costs has to account for the role of labor market rigidities and the resulting dynamic adjustment of workers and firms. On the adjustment path unemployment may increase, wages and productivity may decline, and firms may close shop. Of particular concern is the extent to which labor market frictions slow down reallocation and how the resulting costs vary with the size of these frictions.[1]

Heterogeneity of labor market outcomes for workers employed by different firms raises another concern. As we shall see, reductions in trade costs, which are equivalent to a positive productivity shock, produce winners and losers among ex ante homogenous workers, who are paid similar wages in the pre-trade-liberalization state. Among these workers some are employed by high-productivity exporters while others are employed by low-productivity domestic firms. The former jobs prove to be good jobs while the latter prove to be bad when the economy is hit by a reduction in trade costs. Employees of high-productivity firms suffer *no* decline in (nominal) wages and experience no worsening of employment prospects, while employees of low-productivity firms suffer wage cuts and worsened employment prospects. Therefore, trade liberalization results in undesirable labor market outcomes for some job holders in non-exporting firms.

To address these issues we develop a dynamic version of the Helpman and Itskhoki (2010) model in which there are two sectors producing traded and nontraded goods. The non-traded good is homogeneous while the traded product is differentiated. In the traded sector heterogeneous firms specialize in brands of the differentiated product and engage in monopolistic competition. In the nontraded sector the product market is competitive. In both sectors there are frictions in the labor market. Firms post vacancies and workers search for jobs. Matching takes place via a constant returns to scale matching function and wages are negotiated ex post. That is, we introduce Diamond-Mortensen-Pissarides (DMP) type search and matching frictions into the labor markets (see, for example, Pissarides 2000). This enables us to develop a sufficient statistic for the level of labor market frictions that we can vary in order to study the impact of these frictions on the dynamics of adjustment. Moreover, firms in both

[1] Papers that emphasize the importance of the transition to the new steady-state equilibrium when analyzing the gains from trade include Burstein and Melitz (2011), Alessandria, Choi, and Ruhl (2021), Dix-Carneiro (2014), and more recently Caliendo, Dvorkin, and Parro (2019) and Rodríguez-Clare, Ulate, and Vasquez (2022).

sectors face downward wage rigidity, which is non-binding in the long run but affects firms' decisions along the transition. Following the shock, firms cannot flexibly adjust the wages which causes displacement rates of workers in low productivity firms. Firms enter and exit the two industries, driven by profit considerations and exogenous shocks to incumbents. We characterize shifts in steady states and transition dynamics between them that result from a lowering of variable trade costs.[2] In addition, we use a calibrated version of the model to quantify the costs and benefits of lower trade impediments.

The model features two symmetric countries. It gives rise to a steady state that is similar to Helpman and Itskhoki (2010). In response to a decline in variable trade costs the new steady state features exit of the least productive firms, an expansion of the most productive firms, and a contraction of firms with intermediate productivity levels who serve only the domestic market. As a result, productivity rises in the traded sector and so does welfare.

We find rich patterns of transition dynamics to the new steady state. First, the transition may take a long time. Second, while exporting incumbents adjust quickly, many non-exporters adjust slowly. Some contract on impact, partially or fully (by exiting), others contract gradually via attrition of their labor force. Among those who stay and gradually reduce their labor force, some close shop when their employment reaches a threshold that depends on productivity, while others never leave the industry. Third, while wages paid by exporters and non-exporters at the upper end of the productivity spectrum do not change in terms of the nontraded good, lower-productivity firms that do not exit slash wages on impact and raise them gradually subsequently, as their employment shrinks. Additionally, because of the downward wage rigidity, such firms are forced to fire more workers on impact, which potentially speeds up the transition, but results in a higher transitional worker displacement and unemployment. The resulting heterogeneous outcomes for observationally identical workers, tied to the fates of their employers, are consistent with evidence in Verhoogen (2008) and Amiti and Davis (2011). Fourth, the temporary survival of low-productivity incumbents slows down the adjustment process and crowds out higher-productivity entrants, thereby temporarily reducing aggregate productivity in the traded sector. While productivity declines in the short run, it overshoots its long-run value in the medium term.[3] Lastly, exports increase on impact but undershoot their long-run value, which is only gradually attained, thereby resulting in a time-varying trade elasticity that increases over time.

Despite these turbulent responses, there are welfare gains from lower trade costs that result from a decline in the price index of traded goods. These gains are partially offset by lower labor income and lower firm profits, the former resulting from temporarily lower wages and employment. We find in the quantitative analysis that this offset is small in present value terms relative to the long-run welfare gains from lower prices of tradables. While income losses from wages and profits occur over a finite interval of time, the gains from lower prices are permanent and instantaneously realized.[4]

[2]We focus on reductions of variable trade costs, such as transport or tariffs, although reductions in fixed costs of exporting can be similarly analyzed.

[3]The second force that affects sectoral productivity is due to product varieties of surviving incumbents in the short run (see e.g. Alessandria, Choi, and Ruhl 2021). We show that misallocation dominates in the short run while the variety effect dominates in the medium run: over time, unproductive incumbents shrink and become less important relative to new entrants.

[4]The *proportional* rise in the real consumption index of traded goods, which is the source of gains in consumer surplus in our model, does not depend on labor market frictions, although these frictions are important determinants of the long-run levels of productivity and welfare (as well as comparative advantage, as in the static model of Helpman and Itskhoki 2010).

As pointed out above, we extend Helpman and Itskhoki (2010), where we study the long-run effects of labor market and trade reforms in countries with heterogeneous firms and asymmetric labor market institutions.[5] The consequences of labor market frictions for transitional dynamics in otherwise neoclassical trade models are studied by Davidson, Martin, and Matusz (1999), Kambourov (2009) and Cosar (2013).[6] Other scholars, such as Coşar, Guner, and Tybout (2011), Fajgelbaum (2013), Danziger (2013), Cacciatore (2014) and Felbermayr, Impullitti, and Prat (2014), also discuss labor market dynamics with heterogeneous firms. Some papers highlight that different types of labor market frictions like restricted job-to-job mobility, costly search or wage rigidities impact the dynamic gains from trade (Kambourov 2009, Cosar 2013, Fajgelbaum 2020, Rodríguez-Clare, Ulate, and Vasquez 2022). Our framework is richer in comparison, as it combines both search and matching frictions and wage rigidities in a tractable way, yielding sophisticated responses of firms and workers to the lowering of variable trade costs.[7]

This paper relates to the literature studying the implication of wage rigidities and, in particular, the effects of minimum wage. We find that wage rigidity can be welfare-enhancing. This supports findings in Flinn (2006), who shows that minimum wage can create additional bargaining power for workers and encourages more search activity from their side. Rodríguez-Clare, Ulate, and Vasquez (2022) find that downward nominal wage rigidity accounts for both an increase in unemployment and a decline in labor force participation, which resulted in smaller welfare gains in some states following the China Shock. In this paper, we focus on the effect of minimum wage on unemployment and firms' entry and exit decisions. We show that although firms suffer higher profit losses, workers income loss is significantly mitigated, and this can overweight high temporary unemployment rates.

This work complements Helpman, Itskhoki, and Redding (2010) and Helpman, Itskhoki, Muendler, and Redding (2017). While in the earlier papers we study the long-run impact of trade on heterogeneous firms and workers, the current study is concerned with transition dynamics. For this purpose we assume that workers are homogeneous and only firms are heterogeneous, in order to emphasize the fact that homogeneous workers suffer different fates during the transition, depending on the characteristics of their employers. In particular, we find that more-productive incumbents respond to a decline in variable trade costs by paying higher wages relative to less-productive incumbents.

The remainder of the paper is structured as follows. We present our model in Section 2. In Section 3, we describe the long-run equilibrium, compare steady states with different levels of trade impediments, and discuss how wage rigidities affect the long-run equilibrium in the economy. The main quantitative analysis is developed in Sections 4–6, where we discuss the transition dynamics of the economy. First, we calibrate the model to fit some well-known moments in the data and describe the firms' adjustment patterns following the trade shock in Section 4. In Section 5, we discuss the dynamics of workers'

[5]Felbermayr, Prat, and Schmerer (2011) also study the steady state effects of trade liberalization in an economy with heterogeneous firms and search frictions in the labor market, and Davis and Harrigan (2011) and Egger and Kreickemeier (2009) analyze the effects of other forms of labor market frictions in a trade model with heterogeneous firms.

[6]Labor market dynamics in the aftermath of a trade liberalization in the presence of labor adjustment costs across sectors, occupations, and regions, are studied in Artuç, Chaudhuri, and McLaren (2010), Dix-Carneiro (2014) and Caliendo, Dvorkin, and Parro (2019).

[7]In the macro-labor literature, the dynamics of labor markets with heterogeneous firms are studied in Acemoglu and Hawkins (2013), Elsby and Michaels (2013), Schaal (2012), Kaas and Kircher (2015), Cacciatore and Ghironi (2021), and Bilal, Engbom, Mongey, and Violante (2022).

income and unemployment, as well as the job creation process during the transition. Finally, we discuss in detail the decomposition of the dynamic gains from trade and its comparative statics with respect to various labor market frictions in Seciton 6. In Section 7, we conclude by reflecting on our main assumptions, pointing out their limitations and the roles they play in shaping the results.

2 Setup

Consider a world of two symmetric countries. Every country produces two goods: a non-traded homogenous product and a traded differentiated product. The differentiated product is manufactured by heterogeneous firms that engage in monopolistic competition, and exporting this product requires both variable and fixed trade costs. In the labor market there is search and matching and wage bargaining between firms and workers. Time is discrete, with short time intervals of length Δ. While the appendix provides exact discrete-time expressions, in the main text we occasionally use the continuous-time approximation in order to simplify the exposition.

2.1 Households

Each country is populated by a mass of identical infinitely-lived households, normalized to equal one. Per-period household utility is given by $u\big(q_0, Q\big)$, where q_0 is consumption of the homogeneous good and Q is the consumption index of the differentiated product. The per-unit time discount rate equals r. We suppress the dependence of variables on time, t, whenever it leads to no confusion.

We follow Helpman and Itskhoki (2010) in assuming that the utility function is quasi-linear:

$$u(q_0, Q) = q_0 + \frac{1}{\zeta} Q^{\zeta}, \qquad \zeta > 0. \tag{1}$$

This type of the utility function focuses the analysis on dynamics that are driven by labor market frictions, not confounded by effects that can arise from the curvature of the utility function. Moreover, as is well known, this utility function leads to outcomes that have a partial equilibrium flavor. This is a reasonable feature in our context, because only a fraction of consumer spending is on (a given) tradable sector, and we can therefore think about the homogenous-good sector as the rest of the economy that is large in comparison.

Consumption of the differentiated product, Q, is a CES aggregator of individual varieties:

$$Q = \left(\int_{\omega \in \Omega} q(\omega)^{\beta} \mathrm{d}\omega \right)^{1/\beta}, \qquad \zeta < \beta < 1, \tag{2}$$

where ω denotes a variety and Ω is the set of varieties available for consumption. The elasticity of substitution between varieties is $\varepsilon = 1/\left(1-\beta\right)$. The parameter restriction $\beta > \zeta$ implies that the differentiated varieties are better substitutes for each other than for the homogenous good.

The non-traded homogenous good serves as numeraire and we set its price, p_0, to equal one in all time periods, i.e., $p_0 \equiv 1$.[8] With preferences given by (2) and (1), consumer optimization yields a utility flow per unit time that can be expressed as:[9]

$$u = I + \frac{1-\zeta}{\zeta} Q^{\zeta}, \tag{3}$$

where I is income and the second term is consumer surplus from the differentiated product. It follows that lifetime utility equals the discounted present value of household income and consumer surplus.

Every household has a measure L of workers whom it allocates between the two sectors. A worker can be either employed or unemployed. Unemployed workers search for jobs and they can frictionlessly reallocate between the two sectors, while employed workers need to first separate into unemployment in order to search for a new job. Unemployed workers in both sectors face Diamond-Mortensen-Pissarides type search frictions, as we describe below. Every unemployed worker receives a per unit time unemployment benefit b_u (in units of the homogenous good). The unemployment benefits are financed with a lump-sum tax on all households.

2.2 Non-traded sector

In the non-traded sector output per worker equals one per unit time. As a result, a match between a firm and a worker produces a constant flow Δ of output per period. Since Δ is small (e.g., $\Delta = 1/12$, corresponding to one month), we can use approximations around $\Delta = 0$, and the appendix lays out all of the derivations with a finite $\Delta > 0$.

The market for homogeneous goods is competitive and all firms are single-worker firms (or jobs). A firm can enter this sector freely and post a costly vacancy in order to attract a worker. Denote by b_0 the expected (instantaneous) cost of attracting a worker in the non-traded sector, which equals the flow cost of a vacancy divided by the vacancy-filling rate. We show in Appendix A.1 that a Cobb-Douglas matching function yields the following relationship between b_0 and the job finding rate x_0:

$$b_0 = a_0 x_0^{\alpha}, \qquad \alpha > 0, \tag{4}$$

where a_0 is a derived parameter that rises with the cost of a vacancy and declines with the productivity of the matching technology (see also Helpman and Itskhoki 2010). The job finding rate x_0 is a measure of the sector's labor market tightness.[10] When matched with a worker, the firm and the worker bargain over the wage rate with no long-term commitment and, for simplicity, with equal weights. A match is exogenously dissolved with a constant hazard rate s_0 per unit time.

We make the following assumption:

[8]This can be interpreted in the context of a monetary model where the monetary authority chooses to stabilize the price of the non-traded good. Note that even if the homogenous good were *tradable*, in an equilibrium of symmetric countries it is *not traded*.

[9]Due to the constant marginal utility of the non-traded good, the market clearing interest rate equals the discount rate r, and therefore, without loss of generality, expenditure can be assumed to equal income in every period. Let P be the price index of the differentiated product (i.e., the price of Q) in units of the numeraire. Then expenditure equals $q_0 + PQ = I$. The optimal choice of Q satisfies $PQ = Q^{\zeta}$, and therefore the utility flow is $I - PQ + Q^{\zeta}/\zeta = I + \frac{1-\zeta}{\zeta} Q^{\zeta}$.

[10]With a Cobb-Douglas matching function the job finding rate is a power function of the vacancy-unemployment ratio, a conventional measure of labor market tightness.

Assumption 1 *L is large enough so that along the equilibrium path the stock of unemployed searching for jobs in the homogenous-good sector, $U_{0,t}$, is positive. That is, $U_{0,t} > 0$ in every period t.*

Note that our quasi-linear utility function implies that all income effects are absorbed in the non-traded sector. As a result, raising L expands the non-traded sector but does not impact demand or supply in the differentiated sector. For this reason a large enough L ensures unemployment in the homogeneous sectors. With this assumption, we prove in Appendix A.2:

Lemma 1 *Under Assumption 1:* (a) *The job finding rate x_0 and the hiring cost b_0 are positive, finite, constant over time, and given* (4) *solve:*

$$\left[2(r + s_0) + x_0\right]b_0 = 1 - b_u. \tag{5}$$

(b) *The economy-wide value to an unemployed worker is constant over time and given by:*

$$rJ_0^U = b_u + x_0 b_0. \tag{6}$$

Indeed, given the parameters of the model, equations (4) and (5) provide a unique solution for (b_0, x_0). Importantly, this solution does not depend on trade frictions. Intuitively, free entry of firms in the homogenous sector leads to equalization of the value of a filled vacancy, J_0^F, with the hiring cost b_0. This, in turn, yields a single value of labor market tightness, x_0, which is consistent with free entry, and is the solution of (4)–(5), as we show in the appendix.

Furthermore, the present value of income of an unemployed worker, J_0^U, is also constant over time, as it is pinned down by the non-changing labor market tightness in the outside sector. Indeed, Nash bargaining equalizes the surplus from the employment relationship between the firm and the worker. As a result, an unemployed worker receives b_u in unemployment benefits and expects to find employment at rate x_0, with the surplus from employment given by b_0, as shown in (6). The resulting wage rate in the homogenous sector then equals:

$$w_0 = b_u + (r + s_0 + x_0)b_0. \tag{7}$$

Evidently, since (b_0, x_0) does not depend on trade frictions, neither does the wage rate w_0 nor the capital value of unemployment J_0^U. In short, the vector $\left(b_0, x_0, w_0, J_0^U\right)$ is constant over time and independent of trade shocks.

Since unemployed workers can freely move across sectors, J_0^U is also the outside option of unemployed workers in the differentiated product sector. Therefore, in an equilibrium with unemployment in both sectors, J_0^U is the common value of unemployment for all workers, and this value does not depend on trade frictions.

Assumption 1 is critical for this result: a large enough labor force L ensures that there are unemployed workers in the non-traded sector in every time period. This means that the trade shocks considered below are not big enough to eliminate unemployment in the non-traded sector, not even temporarily. Since employment shares of tradable sectors are modest in most countries, due to the dominant size of non-traded services, considering changes in trade costs that do not eliminate unemployment in the non-traded sector is reasonably realistic. Under these circumstances, we obtain a block-recursive structure of the model. That is, we obtain a solution for $\left(b_0, x_0, w_0, J_0^U\right)$ that is independent of time and also independent of trade shocks. This solution is, in turn, used as an input into the analysis of the traded sector. In Section 4, we quantitatively evaluate the leeway obtained from unemployment in the non-traded sector and discuss in Section 7 what happens when Assumption 1 is violated.

2.3 Traded sector

In the differentiated sector a firm pays a sunk cost f_e in order to enter the industry. Upon entry it acquires a technology with productivity θ that enables it to produce a unique variety ω of the differentiated product. The firm's productivity is drawn from a known distribution $G(\theta)$, which for simplicity we assume to be Pareto, with CDF $G(\theta) = 1 - \theta^{-k}$ and shape parameter $k > \varepsilon - 1$. The firm's production function is therefore $y = \theta h$ per unit time, where h is employment.

In addition to a flow of labor costs the firm faces a flow of fixed operating costs per unit time, f_d, and an additional flow of fixed export costs per unit time, f_x, in case it chooses to export. All these costs are in terms of the numeraire. An exporter also faces melting iceberg variable trade costs, represented by $\tau \geq 1$, where τ units of a good must be shipped in order for one unit to arrive in the foreign market.

We show in Appendix A.3 that, given the CES preference aggregator (2), the revenue per unit time of a firm with productivity θ and employment h is:

$$R(h, \iota; \theta) = \Theta(\iota; \theta)^{1-\beta} h^{\beta}, \qquad \Theta(\iota; \theta) \equiv \left[1 + \iota \tau^{-\frac{\beta}{1-\beta}} \left(\frac{Q^*}{Q}\right)^{-\frac{\beta-\zeta}{1-\beta}}\right] Q^{-\frac{\beta-\zeta}{1-\beta}} \theta^{\frac{\beta}{1-\beta}}, \tag{8}$$

where $\iota \in \{0, 1\}$ is an indicator variable that equals 1 if the firm exports and zero otherwise, while Q and Q^* are the real consumption indexes of the differentiated product at home and abroad, respectively. The revenue of a non-exporting firm is $Q^{\zeta-\beta} y^{\beta}$, while an exporting firm (with $\iota = 1$) optimally splits its output y between the domestic and foreign markets, which results in $\Theta(1; \theta) > \Theta(0; \theta)$. When the two countries are symmetric, $Q = Q^*$, the expression for $\Theta(\iota; \theta)$ simplifies to:

$$\Theta(\iota; \theta) \equiv \left(1 + \iota \tau^{-\frac{\beta}{1-\beta}}\right) Q^{-\frac{\beta-\zeta}{1-\beta}} \theta^{\frac{\beta}{1-\beta}}. \tag{9}$$

As in the homogeneous sector, firms in the differentiated sector post costly vacancies in order to be matched with workers. However, unlike in the homogeneous sector, a firm in the differentiated sector hires a continuum of workers, h. The hiring cost per worker, b, equals again the flow cost of a vacancy divided by the vacancy-filling rate, where the latter is determined in industry equilibrium. Assuming a Cobb-Douglas matching function yields a relationship between the cost of hiring and the job-finding rate, x, similar to (4):

$$b = a x^{\alpha}, \tag{10}$$

where a is a derived parameter equal to the cost of a vacancy divided by the productivity of the matching function (see the appendix for details). We assume that the exponents of the Cobb-Douglas matching functions are the same in both sectors and therefore α is the same in (4) and (10).

Upon matching, the firm engages in a bilateral Nash bargaining with equal weights with each one of its workers, taking into account that the departure of a worker causes a renegotiation of wages with all the remaining workers. The bargaining is over the revenues of the firm once the employment decision and the per-period fixed production and exporting costs are sunk. The outcome of this bargaining is described in the following lemma (the proof is in Appendix A.3):

Lemma 2 *Wage bargaining between a firm and its h workers results in the wage schedule:*

$$w(h,\iota;\theta) = \frac{\beta}{1+\beta}\frac{R(h,\iota;\theta)}{h} + \frac{1}{2}rJ^U, \tag{11}$$

where the value of being unemployed, $J^U = J_0^U$, is determined in the homogeneous sector by (6).

Importantly, the wage schedule in (11) applies in every period whether a firm expands or contracts its labor force, and independently of whether the firm's employment level is optimal or not. Note that the wage rate equals a share $\beta/(1-\beta)$ of revenue per worker plus half the forgone flow value of unemployment, rJ^U. Combining (11) with (8), we can express the operating profits of a firm per unit time, gross of hiring costs, as:

$$\varphi(h,\iota;\theta) \equiv \frac{1}{1+\beta}R(h,\iota;\theta) - \frac{1}{2}rJ^U h - f_d - \iota f_x. \tag{12}$$

A firm exogenously separates with a fraction of its workforce at rate σ per unit time and it dies at rate δ per unit time. As a result, $s = \sigma + \delta$ represents the *exogenous* rate of employment loss per unit time for workers employed in the differentiated sector. In addition, employment losses can also arise from endogenous decisions by firms to fire workers. We assume that a firm can fire some or all of its workers at no direct cost. Therefore, a firm that seeks to change its workforce from h to h' in the course of one period bears the hiring cost:

$$C(h',h) = b[h' - (1-\sigma\Delta)h]^+, \tag{13}$$

where $[\cdot]^+ \equiv \max\{\cdot, 0\}$. When $h' > h$, a part of the hiring cost, $bh\sigma\Delta$, is borne to replace the exogenous labor force attrition, while the remaining part, $b\,(h'-h)$, is paid to increase the size of the labor force. A firm with $h' \leq (1-\sigma\Delta)h$ does not hire new workers and bears no hiring (or firing) costs.[11]

We prove in Appendix A.3 the following results:[12]

[11] A generalization with a per-worker firing costs z features $C(h',h) = b[h' - (1-\sigma\Delta)h]^+ + z[(1-\sigma\Delta)h - h']^+$.

[12] The proof uses the recursive Bellman equation for a firm with productivity θ:

$$J^F(h) = \max_{h'}\left\{\varphi(h)\Delta - C(h',h) + \frac{1-\delta\Delta}{1+r\Delta}J_+^F(h')\right\}$$

where $J^F(\cdot)$ and $J_+^F(\cdot)$ denote the current and next period value functions of the firm. The first order condition for the choice

Lemma 3 (a) *The job finding rate x and the hiring cost b satisfy:*

$$xb = x_0 b_0. \tag{14}$$

(b) *The optimal employment of a hiring firm satisfies $\varphi'(h) = (r+s)b$, and is given by:*

$$h(\iota;\theta) = \Phi^{1/\beta}\Theta(\iota;\theta), \qquad \textit{where} \qquad \Phi \equiv \left(\frac{2\beta}{1+\beta}\frac{1}{b_u + \big[2(r+s)+x\big]b}\right)^{\frac{\beta}{1-\beta}} \tag{15}$$

and $\Theta(\iota;\theta)$ is defined in (8).

The intuition behind these results is as follows. A hiring firm equalizes the value of a marginal worker with the cost of hiring, b. The splitting of the surplus in the bargaining game ensures, in turn, that the employment value to the worker equals the employment value to the firm. Therefore, the unemployed workers in the differentiated sector have the job finding rate x and the gain in value b upon employment. In the homogeneous sector workers have a job finding rate x_0 and a gain in value b_0. The indifference of unemployed workers between the two sectors then requires $xb = x_0 b_0$.[13]

The optimal employment rule (15) results from the equalization of the flow value from a marginal worker, $\varphi'(h)$, characterized by (12), with the flow cost of hiring an extra worker, $(r+s)b$. The derived parameter Φ represents the extent of labor market imperfections, and it decreases in the hiring cost b. More productive firms and exporters have larger optimal employment levels due to higher $\Theta(\iota;\theta)$, while all firms are smaller in a more frictional labor market due to lower Φ.[14]

Since the employment level of a firm is a jump variable, linearity of the hiring cost implies that a firm with employment below the optimal level immediately raises its employment to the optimal level. Moreover, in a stationary environment a firm with the optimal level of employment maintains this level in every period by hiring workers that just offset the exogenous attrition of the workforce. On the other side, if a firm has more workers than the optimal level, then three possibilities arise. First, if a firm cannot cover the flow of fixed costs by firing some of its workers, it fires all of them and exits

of h' is:

$$\frac{1-\delta\Delta}{1+r\Delta}J^F_{h,+}(h') = \begin{cases} 0, & \text{if} \quad h' < (1-\sigma\Delta)h, \\ \in [0,b], & \text{if} \quad h' = (1-\sigma\Delta)h, \\ b, & \text{if} \quad h' > (1-\sigma\Delta)h, \end{cases}$$

and the Envelope Theorem yields:

$$J^F_h(h) = \varphi'(h)\Delta + \frac{1-s\Delta}{1+r\Delta}J^F_{h,+}(h'),$$

where the subscript h indicates the partial derivative with respect to employment. The inequalities in the first order condition reflect the sS nature of the labor force adjustment in this model. Given a constant b, we have $J^F_h = J^F_{h,+} = \frac{1+r\Delta}{1-\delta\Delta}b$ for a hiring firm, which together with the Envelope Theorem characterizes optimal employment, $\varphi'(h) = \frac{r+s}{1-\delta\Delta}b$. The approximation with $\Delta \approx 0$ yields $J^F_h = b$ and $\varphi'(h) = (r+s)b$.

[13] Given (10), this results in the following solution for x and b: $x = x_0\big(a/a_0\big)^{-\frac{1}{1+\alpha}}$ and $b = b_0\big(a/a_0\big)^{\frac{1}{1+\alpha}}$, where x_0 and b_0 are characterized in Lemma 1.

[14] Helpman and Itskhoki (2010) focus on the effects of cross-country differences in labor market frictions (Φ) on the steady state comparative advantage and asymmetric gains from trade. Note that using (5) and part (a) of Lemma 3, one can show that Φ is decreasing in $(sb - s_0 b_0)$. Therefore, countries with higher overall levels of labor market frictions (high $b = b_0$) have comparative advantage in sectors with greater labor market turnover (higher relative separation rates s/s_0), consistent with the evidence in Cuñat and Melitz (2011).

the industry. Second, if a firm has a positive optimal labor force and its workforce is only a little larger than the optimal employment level, its best strategy is to allow its workforce to gradually decline via exogenous attrition until it reaches the optimal level. Finally, if a firm has a positive optimal labor force and its workforce exceeds the optimal employment level by a large amount, its best strategy is to instantly fire some of its workers and let the remaining stock decline gradually via exogenous attrition until it reaches the optimal level. Which of these cases applies depends on a firm's productivity level $\Theta(\iota;\theta)$ and the initial size of its workforce h, as we describe below.

Recall that under Assumption 1, x_0 and b_0 are constants that do not vary over time and do not depend on trade frictions. Also note that, given x_0 and b_0, equations (10) and (14) provide a unique solution for x and b, which also are constants that do not vary over time. It follows that under Assumption 1 sectoral job-finding rates and sectoral hiring costs are independent of trade frictions and they do not vary over time. Substituting the optimal employment level (15) into the wage schedule (11) yields an expression for the equilibrium wage rate paid to all employed workers by hiring firms:

$$w = b_u + (r + s + x)b, \tag{16}$$

which parallels (7) for the wage rate in the homogeneous sector. Evidently, all hiring firms pay the same wage rate in the differentiated sector and this wage rate does not depend on trade frictions, nor does it change over time. In contrast, it can be seen from (8) and (11) that non-hiring firms and firing firms pay lower wages, because their employment is above the optimal level.[15]

Lastly, let $J^V(\theta)$ be the value function of a firm with productivity θ and zero employees. Then the free-entry condition can be expressed as

$$\int J^V(\theta)\mathrm{d}G(\theta) \leq f_e, \tag{17}$$

with equality holding when entry takes place. A firm with a positive value, $J^V(\theta) > 0$, that enters at time t hires workers and produces in period $t + \Delta$, while firms with negative values exit immediately. The following lemma, proven in Appendix A.3, provides a characterization of the value of an entering firm with productivity θ in a special case that is relevant for our analysis:

Lemma 4 *The value of a firm with productivity θ and zero employees that hires workers in every period satisfies:*

$$(r + \delta)J^V_{-1}(\theta) - \dot{J}^V(\theta) = \max_{\iota\in\{0,1\}} \left\{ \frac{1-\beta}{1+\beta}\Phi\Theta(\iota;\theta) - f_d - \iota f_x \right\}, \tag{18}$$

where the subscript (-1) represents the previous period, $\dot{J}^V(\theta) \equiv (J^V(\theta) - J^V_{-}(\theta))/\Delta$, and $\Theta(\iota;\theta)$ and Φ are as defined in (8) and (15), respectively.

Lemma 4 applies to every time path of aggregate state variables. Furthermore, we will see that

[15] Indeed, from (8) and (11), the wage rate is a decreasing function of employment, and a firm chooses to reduce its labor force only if its current employment exceeds the desired level given by (15). Furthermore, the wage rate paid by firing firms equals the flow value of unemployment, $rJ^U = b_u + xb$, as employment in this case yields no surplus. Therefore, wages paid by non-hiring firms fall between $b_u + xb$, paid by firing firms, and $b_u + (r + s + x)b$, paid by hiring firms.

the requirement that a firm hires in every period is satisfied on the equilibrium path following a trade liberalization for high-productivity exporting incumbent firms. Such firms instantly adjust their labor force up to its optimal level and maintain this level of employment by hiring new workers to offset the exogenous attrition of their workforce. The same applies for all the entering firms.

In the quantitative analysis we focus on the case in which not only are the two countries symmetric, but the two sectors have the same ratios of vacancy costs to productivity of matching, $a = a_0$, and the same exogenous separation rates of workers, $s = s_0$. Under these circumstances, labor market frictions are effectively the same in both sectors, in which case the resulting hiring costs are the same, i.e., $b = b_0$, and the job finding rates are the same, i.e., $x = x_0$. Therefore, firms that actively hire workers in the differentiated sector pay the same wages as firms in the homogeneous sector, $w = w_0$, as follows from (7) and (16). Conditions (5) and (15) then imply that

$$\Phi = \left(\frac{2\beta}{1+\beta}\right)^{\frac{\beta}{1-\beta}}, \tag{19}$$

so that Φ depends on neither trade impediments nor labor market frictions.

2.4 Wage rigidity

We now generalize the model to feature wage rigidities that may arise either due to downward wage stickiness or due to minimum wages. While the specific interpretation does not affect the formal analysis, it is perhaps more realistic to consider downward wage rigidity as the relevant friction in the specific application that we consider, namely transition dynamics in response to a trade liberalization shock. In particular, in what follows, we assume that wage rigidities do *not* bind in steady state, but *do* constrain some firms during transition after a trade shock, in line with our focus on transition dynamics.

For simplicity, we describe the friction here as if it arises from a minimum wage requirement, w_m. That is, all firms must pay all their workers h a wage no lower than w_m. This does not constrain the firms with wage rate $w(h, \iota; \theta) > w_m$, where $w(h, \iota; \theta)$ is given by (11). However, if $w(h, \iota; \theta) < w_m$, the firm must pay w_m, and this affects the firm's optimal employment choice in the differentiated sector. In the homogenous sector, a filled vacancy now has a value of $J_0^F = \frac{1-\bar{w}_0}{r+s_0} = b_0$, where $\bar{w}_0 = \max\{w_m, w_0\}$, with w_0 given by (7). Together with (4), this condition determines (x_0, b_0). Finally, the value of the unemployed is now given by $rJ_0^U = b_u + \frac{x_0}{r+s_0+x_0}(\bar{w}_0 - b_u)$. These results generalize Lemma 1, and we provide the derivation in the appendix.

For the differentiated sector, we prove in the appendix:

Lemma 5 (a) *Wages of a non-hiring firm with employment h are given by $\bar{w}(h, \iota; \theta) = \max\{w(h, \iota; \theta), w_m\}$, where $w(h, \iota; \theta)$ is defined in* (11). (b) *Wages of a hiring firm are $\bar{w} = \max\{w, w_m\}$, where w is given by* (16)*, and its employment choice satisfies:*

$$\bar{h}(\iota; \theta) = \bar{\Phi}^{1/\beta}\Theta(\iota; \theta), \qquad \bar{\Phi} \equiv \left(\max\left\{\frac{2\beta}{1+\beta}\frac{1}{2\bar{w} - rJ^U}, \frac{\beta}{\bar{w} + (r+s)b}\right\}\right)^{\frac{\beta}{1-\beta}}, \tag{20}$$

and $rJ^U = b_u + \frac{x}{r+s+x}(\bar{w} - b_u)$.

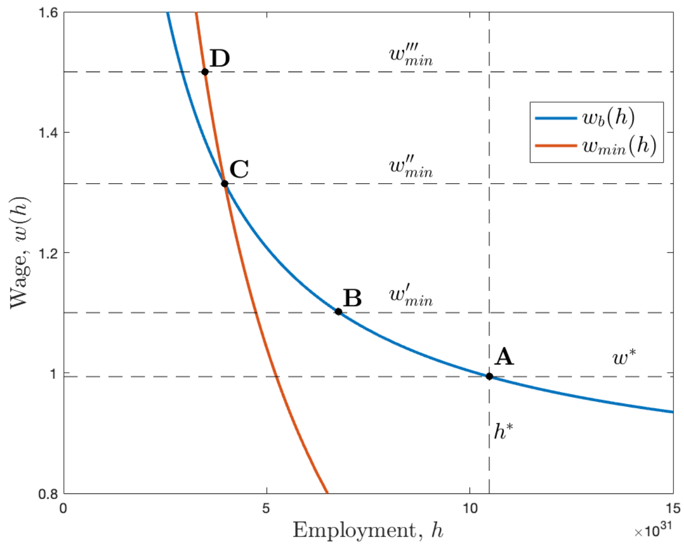

Figure 1: Wage schedule and optimal employment

Note: The blue curve denoted $w_b(h)$ shows the bargaining wage schedule (11) as a function of firm's size, and the red curve denoted $w_m(h)$ shows counterfactual employment schedule as a function of an exogenous wage $w = w_m$ for different values of w_m. Point A is the bargained wage w corresponding to the optimal employment h when $w > w_m$. Point B corresponds to the employment that delivers a bargaining wage outcome of w'_m, when $w'_m > w$. Point C is an indifference point, and at point D firms take the minimum wage w'''_{min} as exogenous, as the red curve is above the blue. For $w_m \leq w$, the minimum wage does not constrain employment, $h = w_b^{-1}(w)$; for all $w_m > w$, the minimum wage is binding and $h = \min\{w_b^{-1}(w_m), w_m^{-1}(w_m)\} < w_b^{-1}(w)$, where w is the equilibrium wage without minimum wages as given by (16).

The employment rule (20) generalizes (15) in part (b) of Lemma 3: the two coincide when $\bar{w} = w > w_m$, and differ otherwise. The maximum operator in the definition of $\bar{\Phi}$ reflects two possible scenarios. First, when $w_m > w$, but sufficiently small, the firm will reduce employment and move up along the bargaining wage schedule (11) until the resulting wage equals w_m. The employment choice in this case is still larger than the optimal employment under an exogenously given wage rate w_m, reflecting the over-hiring incentive under the Nash bargaining solution. Second, as w_m increases sufficiently above w, the firm eventually switches to an employment choice that maximizes operating revenues when taking w_m as given, without bargaining with workers. This is because a sufficiently high w_m dominates the over-hiring incentive: the employment when w_m is taken as given is larger than the employment that would ensure the same wage outcome under bargaining, as we illustrate in Figure 1. In other words, equilibrium employment is the upper envelope of the possible employment outcomes under the two scenarios, i.e., under bargaining and taking minimum wage as given.

Requiring the equalization of the value to unemployed in both sectors, $rJ^U = rJ_0^U$, we characterize labor market tightness x, and thus also the hiring cost $b = ax^\alpha$, as a function of x_0 when the minimum wage is binding. Assuming that the minimum wage is the same in both sectors, $s = s_0$ is now sufficient for $x = x_0$, and $a = a_0$ further implies $b = b_0$. This generalizes part (a) of Lemma 3 for the case with minimum wages.

Finally, there exists a generalizations of the value for a hiring firm, $J^V(\theta)$, which parallels Lemma 4 in the presence of minimum wages. We display the steady-state value of an entrant in this case:

$$\begin{aligned}(r+\delta)J^V(\theta) &= \max_{\iota\in\{0,1\}} \left\{R(\bar{h}(\iota;\theta)\iota;\theta) - [\bar{w}+(r+s)b]\bar{h}(\iota;\theta) - f_d - \iota f_x\right\} \\ &= \max_{\iota\in\{0,1\}} \left\{\left[1-[\bar{w}+(r+s)b]\bar{\Phi}^{\frac{1-\beta}{\beta}}\right]\bar{\Phi}\Theta(\iota;\theta) - f_d - \iota f_x\right\}, \end{aligned} \tag{21}$$

where the second line uses the definition of $\bar{h}(\iota;\theta)$ in (20). Note that the square bracket in (21) is a function of labor market parameters, but does not depend on ι or θ of the firm, and thus $J^V(\theta)$ scales with θ in the same way it does without minimum wages.

2.5 General equilibrium

To close the model we need to characterize aggregate employment, H, and the number of firms, M, in the differentiated sector. M evolves according to:

$$M_{t+1} = (1-\delta\Delta)M_t + m_t^e,$$

where m_t^e is the number of entrants in period t; $m^e \geq 0$ together with (17) satisfy a complementary slackness condition.

Next, let $\mathcal{G}(h,\theta)$ be the joint cumulative distribution function of firm employment and productivity in a give time period among the M currently active firms. The m^e new entrants have zero employment until the following period. Then aggregate employment in the differentiated sector is:

$$H = M\int h\mathrm{d}\mathcal{G}(h,\theta), \tag{22}$$

The evolution of $\mathcal{G}(\cdot)$ is derived from the firms' employment policies (see footnote 12). Given the number of firms and their employment and exporting decisions, we can use (2) to compute the consumption index of the differentiated product, Q. This can in turn be used to recover the aggregate number of vacancies in the differentiated sector, V, and the sectoral unemployment level, U, which secure the equilibrium labor market tightness x. The number of workers attached to the differentiated and homogeneous sectors are $N = H + U$ and $N_0 = L - N$, respectively. Further details are provided in Appendix A.4.

3 Long-run Equilibrium

Before studying transition dynamics in response to a trade shock, we first characterize the long-run equilibrium for different levels of trade frictions, relegating the details of derivations to Appendix A.4. Specifically, we consider a steady state with symmetric countries, in which all variables have the same values at home and in foreign, and in particular $Q = Q^*$. We start by characterizing the long-run equilibrium of the model in case of non-binding minimum wages and then discuss the effects of binding minimum wages on the long-run gains from trade liberalization.

3.1 Long-run equilibrium without wage rigidities

In steady state, due to the positive exogenous workforce attrition rate, $\sigma > 0$, all producing firms hire workers in order to offset the exogenous attrition of their workforce and, therefore, their employment satisfies (15). Due to the positive firm death rate, $\delta > 0$, there is constant firm entry and, therefore, the free-entry condition (17) is satisfied with equality. We show in the appendix that the free-entry condition can be expressed as:

$$f_d \int_{\theta \geq \theta_d} \left[(\theta/\theta_d)^{\varepsilon-1} - 1\right] \mathrm{d}G(\theta) + f_x \int_{\theta \geq \theta_x} \left[(\theta/\theta_x)^{\varepsilon-1} - 1\right] \mathrm{d}G(\theta) = (r+\delta) f_e,$$

which under the Pareto distributed productivity θ simplifies to:

$$\left[f_d \theta_d^{-k} + f_x \theta_x^{-k}\right] = \left(\frac{k}{\varepsilon - 1} - 1\right)(r+\delta) f_e. \tag{23}$$

Now consider an entrant in such a steady state, where minimum wages are not binding. The conditions of Lemma 4 are satisfied for all firms that do not exit immediately, and, therefore, the value of an entrant with productivity θ and zero employment is:

$$J^V(\theta) = \frac{1}{r+\delta} \max_{\iota \in \{0,1\}} \left\{ \frac{1-\beta}{1+\beta} \Phi \Theta(\iota; \theta) - f_d - \iota f_x \right\}. \tag{24}$$

The solution to this maximization problem identifies the exporting cutoff θ_x as the smallest productivity level for which $\iota = 1$; that is, $\iota(\theta) \equiv \mathbf{1}_{\{\theta \geq \theta_x\}}$. The cutoff below which firms exit the industry, θ_d, is solved from $J^V(\theta_d) = 0$; all entrants with $\theta \geq \theta_d$ hire workers and produce in the long run.

Using the definition of $\Theta(\iota; \theta)$ in (9), the two long-run cutoff conditions can be expressed as:

$$\frac{1-\beta}{1+\beta} \Phi Q^{-\frac{\beta-\zeta}{1-\beta}} \theta_d^{\varepsilon-1} = f_d, \tag{25}$$

$$\frac{\theta_x}{\theta_d} = \tau \left(\frac{f_x}{f_d}\right)^{\frac{1}{\varepsilon-1}}, \tag{26}$$

and we choose the parameter values so that $\tau (f_x/f_d)^{1/(\varepsilon-1)} > 1$, to ensure $\theta_x > \theta_d$. These cutoff conditions together with the free-entry condition (23) allow us to solve for the long-run values of (θ_d, θ_x, Q):

$$\theta_d = \left[\frac{f_d}{f_e} \cdot \frac{1 + (f_d/f_x)^{k/(\varepsilon-1)-1} \tau^{-k}}{[k/(\varepsilon-1) - 1](r+\delta)}\right]^{1/k}, \tag{27}$$

$$\theta_x = \tau \left(\frac{f_x}{f_d}\right)^{\frac{1}{\varepsilon-1}} \left[\frac{f_d}{f_e} \cdot \frac{1 + (f_d/f_x)^{k/(\varepsilon-1)-1} \tau^{-k}}{[k/(\varepsilon-1) - 1](r+\delta)}\right]^{1/k}, \tag{28}$$

$$Q = \left(f_d^{-1} \frac{1-\beta}{1+\beta} \Phi\right)^{\frac{1-\beta}{\beta-\zeta}} \left[\frac{f_d}{f_e} \cdot \frac{1 + (f_d/f_x)^{k/(\varepsilon-1)-1} \tau^{-k}}{[k/(\varepsilon-1) - 1](r+\delta)}\right]^{\frac{1-\beta}{k(\beta-\zeta)}}. \tag{29}$$

It is evident from this solution that in steady state neither θ_d nor θ_x depend on labor market frictions, but both depend on trade impediments. For this reason the share of exporting firms is also independent of labor market frictions. On the other hand, the real consumption index, Q, depends on labor market frictions through Φ, as long as labor market frictions vary across sectors (see (19) and the ensuing discussion), as well as on trade costs. However, with similar labor market frictions in both sectors Q does not depend on them. We therefore have:

Lemma 6 *In steady state, the cutoffs θ_d and θ_x depend on trade costs, but not on labor market frictions, while Q depends on both trade costs and labor market frictions, unless labor market frictions are similar in both sectors. That is, if $a = a_0$ and $s = s_0$, then Q is independent of labor market frictions, which in this case only affect real income and consumption of the homogeneous good, q_0.*

Next, from (2) and (22) we obtain the employment level in the differentiated sector:

$$H = \Phi^{\frac{1-\beta}{\beta}} Q^{\zeta}, \tag{30}$$

and the number of firms in the differentiated sector:

$$M = \frac{1}{k(r+\delta)f_e} \frac{\beta}{1+\beta} Q^{\zeta}, \tag{31}$$

where we used properties of the Pareto distribution for productivity. Both sectoral variables H and M are proportional to Q^{ζ}, which equals aggregate revenue PQ in the differentiated sector.

The flow of hires in the differentiated sector equals sH. The flows in and out of unemployment are equalized in steady state, and therefore $sH = xU$. Given the job-finding rate x, we can write that the number of workers in the differentiated sector, $H + U$, equals $N = (1 + s/x)H$. By similar argument, the number of workers in the homogeneous sector is $N_0 = (1 + s_0/x_0)H_0$. Moreover, labor market clearing requires $N_0 + N = L$, which together with (30) yields:

$$H_0 = \frac{x_0}{x_0 + s_0} \left[L - \frac{x+s}{x} \Phi^{\frac{1-\beta}{\beta}} Q^{\zeta} \right]. \tag{32}$$

Using these values we can recover the remaining variables of interest.

In steady state, aggregate profits equal zero in both sectors. Therefore, welfare, which consists of spending plus consumer surplus in the differentiated sector, can be expressed as the sum of wage income, $w_0 H_0 + wH$, and consumer surplus, $(1-\zeta)Q^{\zeta}/\zeta$. While unemployment benefits are part of household income, households pay lump-sum taxes to finance these benefits and, therefore, the net contribution of UI to income is nil. Under these conditions the utility flow is:

$$W = \frac{w_0 x_0}{x_0 + s_0} L + \left[\frac{wx}{x+s} - \frac{w_0 x_0}{x_0 + s_0} \right] \frac{x+s}{x} \Phi^{\frac{1-\beta}{\beta}} Q^{\zeta} + \frac{1-\zeta}{\zeta} Q^{\zeta}, \tag{33}$$

where the first two terms are net income and the last term is consumer surplus. This is a convenient representation of welfare for the following analysis. It shows clearly the two channels through which steady-state welfare changes when Q rises: an increase in consumer surplus and an increase in income when $wx/\left(x+s\right) > w_0 x_0/\left(x_0+s_0\right)$, or a decrease in income otherwise. In an economy with symmetric labor market frictions, the income effect of an increase in Q is nil. At the same time, a reduction in labor market frictions that raises $w_0 x_0/(x_0+s_0)$ and $wx/(x+s)$ proportionally, increases aggregate income without changing Q.

3.2 Long-run equilibrium with rigid wages

When minimum wage is binding in steady state, firms adjust their employment decisions. Nonetheless, the productivity cutoffs for entering the domestic and export markets, θ_d and θ_x respectively, do not depend on the level of the minimum wage, w_m. Indeed, this is the case because the value of a firm, $J^V(\theta)$, provided in (21) has the same structure as under non-binding minimum wages.

However, each firm is now facing higher cost of labor and, therefore, all firms reduce their employment level, for any given level of trade costs, with the optimal employment given by (20). Figure 2a shows the steady-state employment with and without minimum wage. Both before and after the trade shock, all firms find it optimal to hire less workers when the minimum wage is binding.

(a) Employment across firms (b) Differentiated sector aggregates

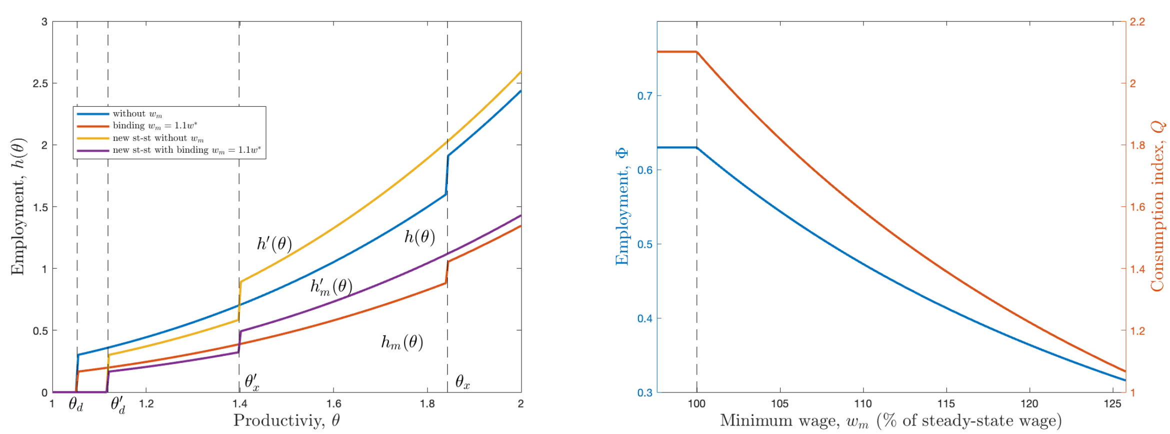

Figure 2: Steady state employment and output

Note: Left panel plots firm employment in the differentiated sector as a function of productivity θ. The blue line and the red line show optimal employment before trade shock as a function of firm's productivity level, in case of non-binding and binding minimum wage respectively. Yellow and purple lines show optimal employment after trade liberalization, in case of non-binding and binding minimum wage respectively. Both before and after the shock, binding minimum wage is set to be 10% higher than the unconstrained equilibrium wage w. Right panel plots aggregate employment H and output Q in the differentiated sector as the function of minimum wage w_m. The vertical line indicates the level of minimum wage such that $w_m = w$. The graph is plotted for the steady state before trade liberalization.

As the minimum wage increases, firms become more constrained, and reduce their employment further. Although productivity cutoffs stay constant, aggregate production Q depends on the level of minimum wage according to:

$$\left[1-\left[\bar{w}+(r+s)b\right]\bar{\Phi}^{\frac{1-\beta}{\beta}}\right]\bar{\Phi}Q^{-\frac{\beta-\zeta}{1-\beta}}\theta_d^{\frac{\beta}{1-\beta}}=f_d, \tag{34}$$

which parallels condition (25). We can show that the aggregate consumption index, Q, and, therefore, aggregate employment and number of firms in the tradable sector decrease with $\bar{w}$, as we show in Figure 2b.

Lemma 7 *When the minimum wage is binding, i.e., $w_m > w$, (a) productivity cutoffs θ_d and θ_x do not change; (b) the optimal employment of each firm decreases; and (c) aggregate employment, number of firms, and consumer surplus in differentiated sector (H, M, and Q, respectively) decrease.*

For the rest of the paper, we concentrate on the case when the minimum wage is not binding in the steady state (and optimal employment is given by equation (15)), but becomes a constraint when some firms need to reduce wages following the trade shock.

3.3 Long-run gains from trade

Steady state just described yields the following comparative statics result:

Proposition 1 *Comparing steady states, a bilateral reduction in variable trade costs τ leads to:* (a) *the same proportional increase in $PQ = Q^{\zeta}$, H and M, which does not depend on labor market frictions (including wage rigidity); and if $a_0 = a$ and $s_0 = s$ then* (b) *aggregate unemployment and labor income do not change;* (c) *welfare rises due to an increase in consumer surplus only and the rise in consumer surplus does not depend on labor market frictions (including wage rigidity).*

Proof: Equations (27)-(29) imply that θ_d rises, θ_x declines and Q rises in response to a decline in τ. Let a hat over a variable represent a proportional rate of change, e.g., $\hat{Q} = \mathrm{d}Q/Q$. Then (30) and (31) imply:[16]

$$\zeta\hat{Q} = \hat{H} = \hat{M}. \tag{35}$$

This proves part (a) of the proposition.

When $a_0 = a$ and $s_0 = s$, we have $x = x_0$, $b = b_0$ and $w = w_0$, as discussed in the end of Section 2.3. Therefore, aggregate employment, $H_0 + H$, equals $xL/(x+s)$ and labor income equals $wxL/(x+s)$, which are independent of trade costs. This proves part (b).

Finally, note that $\hat{Q} > 0$ implies an increase in consumer surplus. Moreover, since similar labor market frictions in both sectors imply that Φ depends only on β, it follows from (29) that Q does not depend on labor market frictions and neither does $\hat{Q}$ when variable trade costs decline. It, therefore, follows that the rise in consumer surplus is independent of labor market frictions. When the same labor

[16]The only result in this equation that depends on the assumption that productivity is distributed Pareto is the characterization of $\hat{M}$. Without Pareto $\hat{M}$ may not equal $\hat{H}$.

market frictions exist in both sectors, (33) can be expressed as

$$W = \frac{wxL}{x+s} + \frac{1-\zeta}{\zeta} Q^{\zeta}, \tag{36}$$

and the welfare change as $\mathrm{d}W = (1-\zeta)\, Q\hat{Q}$, which does not depend on labor market frictions. This proves part (c) of the proposition. ■

The result in Proposition 1 that the long-run welfare gain from a reduction in variable trade costs does not depend on labor market frictions is interesting and intriguing. Hoever, it is important to note that it is still possible for labor market frictions to impact transition dynamics and, consequently, welfare. In particular, labor market frictions may delay the adjustment process and thereby delay the rise in Q, or they may cause transitional unemployment that reduces market income. Furthermore, although average employment in the differentiated sector, H/M, is constant across steady states, firms with different productivity levels change their employment differently. As we shall see, after the decline of trade costs, the less productive, non-exporting firms shrink, while the more productive exporting firms expand employment. Indeed, labor market frictions slow down this reallocation, leading to misallocation of employment across firms, with negative welfare consequences. We evaluate the strength of these forces below.

4 Dynamics of Firms' Adjustment to Trade

In this section, we establish the key property of the model concerning the dynamics of the real consumption index, and then describe in detail the dynamic adjustment of entrants and incumbent firms to the shock of trade costs. As we show below, the transitional dinamics is very rich and varies across firms with different productivity levels. We discuss how firms adjust their employment and export strategies, as well as the implications trade liberalization has for entry and exit.

4.1 Dynamics of consumer surplus

As a preliminary analysis of transition dynamics, we study the dynamics of consumer surplus, which we have shown to be the only source of long-run gains from trade when labor market frictions are similar in both sectors. We consider an unexpected and permanent reduction of variable trade costs τ to $\tau' < \tau$. We denote steady-state variables without subscripts and, as usual, the value of a variable at time t with a time subscript. Initial endogenous steady-state variables have no primes while new endogenous steady-state variables are denoted with a prime (e.g., Q is the initial steady-state real consumption index of the differentiated product while Q' is its value in the new steady-state).

Recall that consumer surplus equals $(1-\zeta)\, Q^{\zeta}/\zeta$. Therefore, the dynamics of consumer surplus are fully characterized by the dynamics of the real consumption index Q. Our main results are that on the transition path real consumption Q is at least as large as in the new steady-state (i.e., there can be short run overshooting of Q) and that it jumps immediately to its steady-state value when new firms enter the differentiated-product industry in every time period. These findings are summarized in (Appendix B.1 provides the proof):

Proposition 2 *On impact, at time $t = 0$, a bilateral reduction in variable trade costs from τ to $\tau' < \tau$ leads to:* (a) $Q_t \geq Q' > Q$ *for all* $t > 0$*; and* (b) $Q_t = Q'$ *in all periods* $t > 0$ *if there is entry of firms in the differentiated sector in all periods* $t \geq 0$*.*

Although a reduction in variable trade costs can induce transition dynamics in which no new firms initially enter the differentiated-product sector (in which case the number of firms declines due to exogenous attrition), our numerical analysis in the next section suggests that this is an unlikely outcome. In the opposite case, when there is entry of new firms in all periods, Proposition 2 states that Q rises immediately to its steady-state level, which means that the long-run gains in consumer surplus from lower trade frictions are instantly realized.

Another source of welfare changes is the time path of household income, which consists of wage income and profits net of entry costs (again, unemployment insurance has no net effect on aggregate household income). Moreover, wage income and net profits do change on the transition path, despite the fact that wages are constant in the homogeneous sector and the same wages are paid by new entrants and growing firms in the differentiated sector, and hiring costs and job-finding rates are constant in both sectors. The reason is that wages paid by contracting firms in the differentiated sector are lower and changing over time, in which case wage income and profits net of entry costs vary over time. We quantify these effects in the next section.

Free entry plays a key role in part (b) of Proposition 2. When firms enter the industry in every time period, condition (23) has to be satisfied at all times, and, therefore, the real consumption index Q is given by (29). Evidently, in this case the solution to Q is the same in every period and, therefore, a reduction in variable trade costs leads to an immediate upward adjustment of Q to its new steady-state value. Moreover, the upward adjustment $\hat{Q}$ does not depend on labor market frictions and, therefore, the proportional rise in consumer surplus is also independent of labor market frictions. Yet labor market frictions impact transition dynamics through their effects on wage income and profits net of entry costs, as we discuss below.

When the minimum wage starts binding after the shock, it may trigger additional exit of firms, and, therefore, reinforces the need for entry of new firms. If the economy without minimum wage features entry upon trade liberalization, it will still be the case with the binding minimum wage. Hence, the consumer surplus will jump to its new long-run level right after the shock, although firms' profits and labor income along the transition will depend on the level of the minimum wage.

Proposition 2 focuses on the dynamics of an important macroeconomic variable, the real consumption index Q, in response to a reduction in variable trade costs. In addition to consumer surplus, discussed above, this macro variable determines the *demand level* for a variety ω in the differentiated-product sector, $q(\omega) = Q^{-\frac{\beta-\zeta}{1-\beta}} p(\omega)^{-\frac{1}{1-\beta}}$. Evidently, the demand for every variety declines with Q. In this sense Q also represents a *measure of competition*; the larger Q is the more competitive the differentiated product market is. Indeed, other things equal, Q is larger the larger the number of incumbents in the industry. Viewed from this angle, Proposition 2 states that a reduction in variable trade costs raises the competitive pressure, and that the competitive pressure may initially overshoot its long-run level.

4.2 Calibration

We discuss here the choice of the parameter values used for the quantitative analysis. The rest of our analysis proceeds under:

Assumption 2 $a = a_0$ *and* $s = s_0$.

In words, labor market frictions are the same in both sectors and so is the rate of job loss per unit time. Under this assumption the job-finding rate is the same in both sectors, i.e., $x = x_0$, and so is the cost of hiring, i.e., $b = b_0$. Moreover, these variables are constant, they depend on labor market frictions, but they are independent of trade frictions. It then follows that Φ defined in (15) is independent of both labor market and trade frictions, when the wage rigidity constraint is not binding.

The possible lack of aggregate dynamics in Q masks rich micro-level dynamics of labor reallocation, both across firms within the differentiated sector and across sectors. These dynamics shape in turn the evolution of employment, productivity and trade flows. We characterize these micro-dynamics in the next section. While complete analytical results are provided in the appendix, in the main text we report numerical simulations based on calibrated values of key parameters. This enables us to trace out alternative dynamic patterns on the one hand and to evaluate the sensitivity of outcomes to alternative parameter values on the other. This analysis helps in identifying features of the model that are quantitatively important as opposed to features with a modest quantitative impact.

Table 1 summarizes the values of parameters and the corresponding empirical moments that we use in the benchmark analysis. We calibrate the productivity of the matching function — a key parameter controlling the extent of labor market frictions — to match average unemployment duration of 6 month, which corresponds to an annualized job-finding rate of $x = 2$. This feature is chosen to match European rather than U.S. labor markets, and together with $s = 0.2$ it generates an economy-wide unemployment rate of approximately 9%. Because this is an important parameter, we shall discuss the sensitivity of our results to variations in its value.[17]

We choose L to match an employment share of 14% in the traded sector, in line with the evidence on manufacturing employment in developed economies. Our parameters yield an initial steady state in which 16% of the tradable output is exported, which is close to the U.S. data. Starting with this steady state, we examine a 50% reduction in variable trade costs, i.e., $(\tau - \tau')/(\tau - 1) = 0.5$. This large change leads to a new steady state with an export share comparable to the large European countries. Smaller and larger trade shocks are also studied.

The level of rigid wage constraint is chosen to restrict a drop in wages after the shock to no more than 2%. We keep it high enough so that it is binding during the transition, but not so high to remain in the parameter region where all staying firms gradually shrink after the trade liberalization instead of

[17] See the notes to Table 1 for additional details about the calibration. In our model labor market frictions are characterized by the parameters $(a, a_0, \alpha, b_u, s, s_0)$, which fully determine (x, x_0, b, b_0).

Table 1: Benchmark parameters

Moment	Parameter	Value	Comment
Discount rate	r	0.05	
Exogenous separation rate	s	0.2	$s_0 = s$
— Labor force attrition rate	σ	0.175	
— Firm death rate	δ	0.025	
Job finding rate	x	2	$a_0 = a = 0.12$
Relative elasticity of matching	α	1	
Unemployment benefit	b_u	0.4	
Pareto shape parameter	k	4	
CES within sector	ε	4	$\beta = 3/4$
Semi-elasticity across sectors		2	$\zeta = 1/2$
Rigid wage	w_m	0.978	$(w - w_m)/w = 2\%$
Employment share in the traded sector		14%	$L = 10$, $f_d = 0.05$
Fraction of exitors		25%	$(r + \delta)f_e/f_d = 2.7$
Fraction of exporters		11%	$f_x/f_d = 1$
Fraction of output exported		16%	$\tau = 1.75$
Trade liberalization			$\tau' = 1.375$
— Fraction of exporters		28%	
— Fraction of output exported		28%	

Notes: All rates are annualized: for example, the separation rate is 20% per year and the job finding rate of 2 corresponds to an average unemployment duration of $1/x = 0.5$ years. $\alpha = 1$ is the ratio of the two elasticities of the matching function with respect to employment and vacancies, respectively, which sum to one (thereby ensuring constant returns to scale in matching). The unemployment benefit $b_u = 0.4$ corresponds to a 45% replacement ratio. The semi-elasticity across sectors is chosen to b $1/(1 - \zeta) = 2$. The shape parameter of the Pareto distribution of employment and sales in steady state is set to satisfy $k/(\varepsilon - 1) = 1.33$. The middle panel shows the values of fixed costs f_d, f_d, f_e and variable trade cost τ that were chosen to match the moments on exports and exit of firms; "fraction of exitors" is the fraction of entrants that choose not to produce in the initial steady state.

firing workers on impact. In this sense, this constraint during transition dynamics, which takes about a year, corresponds more closely to a downward wage rigidity than a minimum wage.

The dynamic patterns of adjustment depend on the extent of labor market frictions and the size of the trade shock τ/τ'. When either the decline in variable trade costs is large or labor market frictions are small, there is continuous entry of firms in the differentiated sector and, therefore, in view of Proposition 2, $Q_t = Q'$ for all $t > 0$. This is indeed the case in our benchmark calibration displayed in Table 1. We, therefore, begin the analysis with this case and discuss robustness below.

4.3 Employment adjustment

To understand the adjustment of employment and output in response to a reduction in variable trade costs, we need to separately examine new entrants into the differentiated sector and incumbents, because an entrant and an incumbent who have the same productivity level may choose different employment strategies. The reason is that after the trade shock an incumbent may find out that its labor force exceeds the desired value, but it may, nevertheless, not be wise to immediately fire the excess workers because at that point the hiring costs had been sunk. However, unlike the incumbent, a new entrant does not overhire.

Entrants Consider a firm that enters the differentiated sector after the reduction in τ. It stays in the industry if its productivity exceeds the entry cutoff (27) and it exports if its productivity exceeds the export cutoff (28), where τ is replaced with τ'. Then, conditional on staying in the industry, it chooses employment according to (15), where τ is replaced with τ' and Q is replaced with Q'. Using primes to denote the new steady-state variables, this employment level is:

$$h'(\theta) = \Phi^{1/\beta}\left[1 + \iota'(\theta)\left(\tau'\right)^{1-\varepsilon}\right]\left(Q'\right)^{-\frac{\beta-\zeta}{1-\beta}}\theta^{\frac{\beta}{1-\beta}} \qquad \text{for} \quad \theta \geq \theta'_d, \tag{37}$$

where $\iota'(\theta) = \mathbf{1}_{\{\theta \geq \theta'_x\}}$, and $h'(\theta) = 0$ for $\theta < \theta'_d$, as illustrated in Figure 2a. The new entrants attain this level of employment at the end of the initial period, during which they post vacancies and match with workers (recall that the length of a period is Δ). Comparing this employment level to the employment level of a firm with the same productivity $\theta \geq \theta'_d$ before the decline in trade costs, it is easy to see that $h'(\theta)$ is smaller than $h(\theta)$ for firms with productivity $\theta \in [\theta'_d, \theta'_x)$, who serve only the domestic market in the new equilibrium, because $Q' > Q$.[18] On the other side, comparing firms with higher productivity levels, who choose to export, i.e., whose productivity satisfies $\theta \geq \theta'_x$, we find that $h'(\theta)$ exceeds $h(\theta)$ and that $h'(\theta) / h(\theta)$ is the largest for productivity levels $\theta \in [\theta'_x, \theta_x)$ that made exports unprofitable before trade liberalization but makes them profitable after.[19]

Incumbents Now consider an incumbent with productivity θ. It starts with an employment level given by (15). Its optimal response to the decline in variable trade costs depends on its productivity and the size of the trade shock. We show in Appendix B.2 that similarly to new entrants, there exist two cutoffs, denoted by $\bar{\theta}'_d$ and $\bar{\theta}'_x > \bar{\theta}'_d$, such that incumbents with productivity $\theta \geq \bar{\theta}'_d$ stay indefinitely in the industry while incumbents with productivity $\theta \geq \bar{\theta}'_x$ stay and export in all time periods. Moreover, $\bar{\theta}'_d \leq \theta'_d$ and $\bar{\theta}'_x \leq \theta'_x$, with equality in the absence of labor market frictions, when the cutoffs for new entrants and incumbents coincide, and otherwise with a strict inequality.

Evidently, the range of productivity levels that make it optimal to stay in the traded sector is larger for incumbents than for new entrants, and similarly for the range of productivity levels that make it optimal to export. These differences arise from the fact that an incumbent with productivity θ has

[18] Since $\theta'_x < \theta_x$, firms with productivities $\theta \in [\theta'_d, \theta'_x)$ served only the domestic market in the old steady state too.

[19] Indeed, we have:

$$\frac{h'(\theta)}{h(\theta)} = \frac{1 + \mathbf{1}_{\{\theta \geq \theta'_x\}}\left(\tau'\right)^{1-\varepsilon}}{1 + \mathbf{1}_{\{\theta \geq \theta_x\}}\tau^{1-\varepsilon}}\left(\frac{Q'}{Q}\right)^{-\frac{\beta-\zeta}{1-\beta}} \qquad \text{for } \theta \geq \theta'_d.$$

Since $Q' > Q$, the employment of non-exporting firms ($\theta'_d \leq \theta < \theta_x$) shrinks. However, for old exporters (with $\theta > \theta_x$) the effect of a reduction in trade costs, $\tau' < \tau$, dominates, and their employment rises (see appendix). The employment of new exporters (with $\theta \in [\theta'_x, \theta_x)$) then increases *a fortiori*. Note from (27) and (28) that $\theta'_d > \theta_d$ and $\theta'_x < \theta_x$.

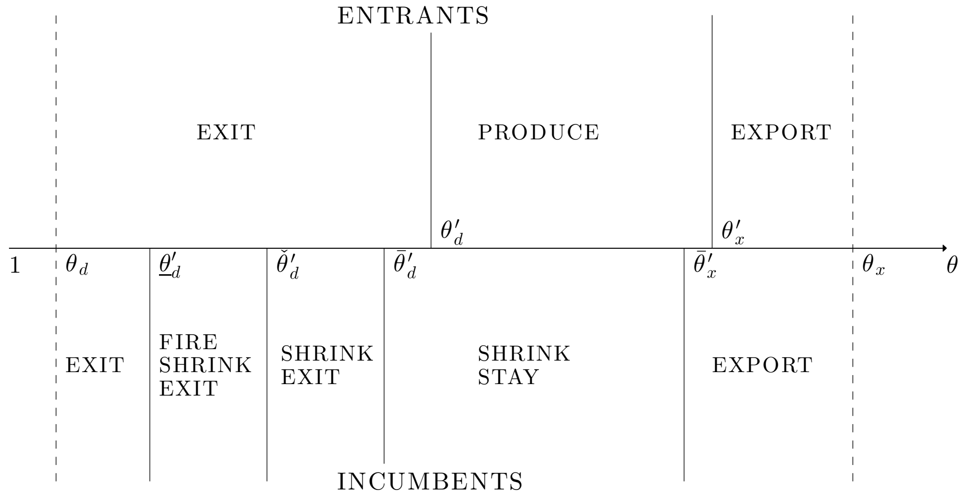

Figure 3: Production and export cutoffs for entrants and incumbents

Note: Illustration of the cutoff patterns for the case $\check{\theta}'_d \in [\underline{\theta}'_d, \bar{\theta}'_d]$. In the alternative case, $\check{\theta}'_d = \bar{\theta}'_x$, all exiting and staying firms with $\theta'_d \leq \bar{\theta}'_x$ fire a fraction of their workers on impact.

already sunk the hiring cost of $h(\theta)$ workers. For such an incumbent the long-run optimal employment level, $\bar{h}'(\theta)$, is similar to (15) and given by:

$$\bar{h}'(\theta) = \Phi^{1/\beta}\left[1 + \bar{\iota}'(\theta)\left(\tau'\right)^{1-\varepsilon}\right]\left(Q'\right)^{-\frac{\beta-\zeta}{1-\beta}} \theta^{\frac{\beta}{1-\beta}} \qquad \text{for} \quad \theta \geq \bar{\theta}'_d, \tag{38}$$

where $\bar{\iota}'(\theta) \equiv \mathbf{1}_{\{\theta \geq \bar{\theta}'_x\}}$ and $\bar{h}'(\theta) = 0$ for $\theta < \bar{\theta}'_d$. Note that $\bar{h}'(\theta) > h'(\theta)$ for $\theta \in [\bar{\theta}'_d, \theta'_d) \cup [\bar{\theta}'_x, \theta'_x)$ and $\bar{h}'(\theta) = h'(\theta)$ otherwise. In other words, while new entrants with $\theta \in [\bar{\theta}'_d, \theta'_d)$ do not stay and, therefore, have zero employment, incumbents with productivity in this range stay in the industry and serve only the domestic market. On the other side, while new entrants with $\theta \in [\bar{\theta}'_x, \theta'_x)$ serve only the domestic market, incumbents with similar productivity levels export as well. Finally, incumbents and new entrants with comparable productivity levels $\theta \in [\theta'_d, \bar{\theta}'_x)$ or $\theta \geq \theta'_x$ have similar long-run employment levels and similar long-run business strategies concerning sales in the foreign market; the former serve only the home market, while the latter also export. As pointed out above, the sunk costs of hiring drive these long-run differences between incumbents and new entrants.[20]

Figure 3 shows the initial steady-state cutoffs (θ_d, θ_x), as well as the new steady-state cutoffs (θ'_d, θ'_x) and $\left(\bar{\theta}'_d, \bar{\theta}'_x\right)$ for new entrants and incumbents, respectively. It also shows additional cutoffs to be discussed below. Figure 4 shows two panels with initial steady-state employment levels $h(\theta)$, as well as new steady-state employment levels $h'(\theta)$ and $\bar{h}'(\theta)$ for new entrants and incumbents, respectively. It also depicts additional employment functions to be discussed below. The left panel describes our benchmark case with $x = 2$ (unemployment duration of 6 months and an initial steady-state unemployment rate of about 9%) and a drop of τ from 1.75 to 1.375. The right panel uses the same parameters, except for two: a smaller labor market friction that yields $x = 24$ (unemployment duration of 1/2 month and

[20]The downward wage rigidity may appear as a constraint for incumbents only, since entrants do not over-hire, and doesn't affect the long-run employment level of any firm due to the natural attrition of the labor force.

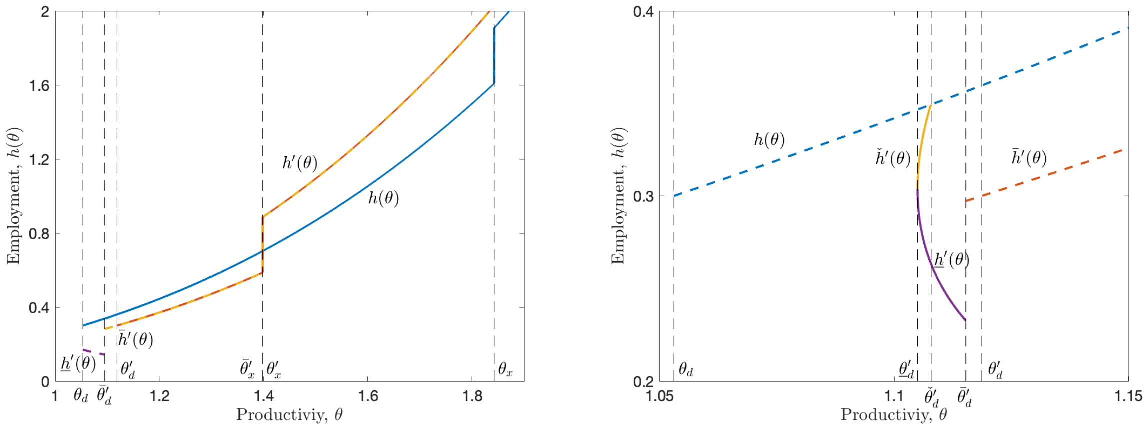

Figure 4: Employment as a function of productivity for incumbents and entrants

Note: Benchmark parameterization with $x = 2$ (unemployment duration of 6 months and unemployment rate of 9%) in the left panel and $x = 24$ (unemployment duration of 1/2 months and unemployment rate of 1%) and a larger trade shock ($\tau' = 1.25$) in the right panel. With $x = 2$, no firm exits or fires workers on impact (i.e., $\underline{\theta}'_d = \check{\theta}'_d = \theta_d$), while with $x = 24$ some firms exit on impact, and some firms that stay in the short run fire workers on impact ($\theta_d < \underline{\theta}'_d < \check{\theta}'_d < \bar{\theta}'_d$, as in Figure 3). The right panel zooms in on the range $[\theta_d, \theta'_d]$.

an initial steady-state unemployment rate of 1%) and a larger reduction of τ, from 1.75 to 1.25. Moreover, the right panel zooms in on low productivity levels that are of special interest to illustrate the model's mechanism (θ smaller than θ'_d as well as somewhat higher productivity levels).

Incumbent non-exporters with $\theta \in [\theta'_d, \bar{\theta}'_x)$ and incumbent exporters with $\theta > \theta'_x$ have the same long-run employment as the new entrants, that is $\bar{h}'(\theta) = h'(\theta)$. In contrast, incumbents with productivity $\theta \in [\bar{\theta}'_d, \theta'_d)$ stay in the industry and do not exit, and thus their long-run employment differs from entrants with the same productivity who immediately exit without hiring workers, $\bar{h}'(\theta) > h'(\theta) = 0$. Similarly, incumbents with $\theta \in [\bar{\theta}'_x, \theta'_x)$ choose to export, unlike new entrants with the same productivity, and, hence, their long-run employment is higher, $\bar{h}'(\theta) > h'(\theta)$. Furthermore, for incumbents with $\theta > \bar{\theta}'_x$, the new steady-state employment is higher than it was before the decline in trade costs while for lower productivity firms it is lower than before. In the right panel, the employment functions have similar characteristics for firms with productivity levels between $\bar{\theta}'_d$ and $\theta'_d + \varepsilon$. We illustrate these patterns in Figure 4, where the right panel zooms in on the firms around the θ'_d cutoff.[21]

Next consider an incumbent with $\theta \in [\theta_d, \bar{\theta}'_d)$. Such a firm closes shop in the long run, but its optimal strategy may be to stay active in the short run. Indeed, we show in the appendix that there exists a cutoff $\underline{\theta}'_d \in [\theta_d, \bar{\theta}'_d)$ such that the optimal strategy of incumbents with productivity $\theta \in [\underline{\theta}'_d, \bar{\theta}'_d)$ is to temporarily stay in the industry, allow its labor force to decline via firing or exogenous attrition until its employment reaches the level $\underline{h}'(\theta)$, at which operating profits equal zero, i.e., $\underline{h}'(\theta)$ satisfies $\varphi\big(\underline{h}'(\theta); \theta\big) = 0$ (see (12)), and then fire the remaining workers and exit.[22]

[21]Note that although $\bar{\theta}'_x < \theta'_x$, it is not immediately visible in the left panel of the figure without zooming in.

[22]We introduce in the appendix the continuation value function for an incumbent with productivity θ and employment $h(\theta)$ at time $t = 0$ equal to $J^I(\theta) = \max\{0, \underline{J}^I(\theta), \bar{J}^I_d(\theta), \bar{J}^I_x(\theta)\}$, where $\bar{J}^I_x(\theta)$ is the value of staying and exporting in

Figure 3 depicts the location of $\underline{\theta}'_d$ relative to the other cutoffs discussed above, and $\underline{\theta}'_d$ is also displayed in the right panel of Figure 4 together with the employment function $\underline{h}'(\theta)$. Note that in the left panel of Figure 4, labor market frictions are sufficiently large so that $\underline{\theta}'_d = \theta_d$. The right panel of Figure 4 shows that an incumbent with productivity $\theta \in [\underline{\theta}'_d, \bar{\theta}'_d)$ exits the differentiated sector with a lower labor force the higher its productivity level, i.e., $\underline{h}'(\theta)$ is a declining function of productivity. The reason is that a more productive firm can cover the fixed operating costs with a smaller scale of operation, and, therefore, it allows its labor force to shrink more before it exits.

While every incumbent with productivity $\theta \in [\underline{\theta}'_d, \bar{\theta}'_d)$ stays in the differentiated sector in the short run but not the long run, and each one of them exits when its labor force contracts to $\underline{h}'(\theta)$, the contraction process differs between the low- and high-productivity firms in this range. We show in the appendix that there exists a productivity level $\check{\theta}'_d \geq \underline{\theta}'_d$, such that an incumbent with productivity below $\check{\theta}'_d$ fires some workers on impact (in period $t = 0$) and then lets its labor force shrink as a result of exogenous attrition until it reaches $\underline{h}'(\theta)$, at which point the firm fires the remaining workers and closes shop. The firing of workers in the first period takes place whenever initial employment exceeds a threshold $\check{h}'(\theta)$, at which the value of a marginal worker equals zero; that is, $\check{h}'(\theta)$ satisfies $J_h^F\left(\check{h}'(\theta); \theta\right) = 0$. If, however, $h(\theta) \leq \check{h}'(\theta)$, an incumbent does not fire in the first instance and just lets exogenous attrition to shrink its workforce to the critical level $\underline{h}'(\theta)$ that leads to exit.

As it happens, either $\check{\theta}'_d \leq \bar{\theta}'_d$ or it equals the export cutoff $\bar{\theta}'_x$; it cannot take on values in between $\bar{\theta}'_d$ and $\bar{\theta}'_x$, as all such firms choose the same firing strategy. Figure 3 depicts a case in which $\underline{\theta}'_d < \check{\theta}'_d < \bar{\theta}'_d$. Incumbents with productivities $\theta \in [\bar{\theta}'_d, \bar{\theta}'_x)$ – who stay in the differentiated sector in all time periods – never fire in the first period, but rather let their labor force shrink gradually to its steady state value $\bar{h}'(\theta)$. It follows that for incumbents with $\theta \in [\underline{\theta}'_d, \bar{\theta}'_x)$ employment at $t = 0$ is $h_0(\theta) = \min\left\{h(\theta), \check{h}'(\theta)\right\}$ while for incumbents with larger productivity levels, who export in the new steady state, employment jumps up immediately to its steady-state level $\bar{h}'(\theta)$. To summarize, the ranking of the various cutoffs is:

$$\theta_d \leq \underline{\theta}'_d \leq \bar{\theta}'_d < \theta'_d < \bar{\theta}'_x < \theta'_x < \theta_x \qquad \text{and} \qquad \check{\theta}'_d \in [\underline{\theta}'_d, \bar{\theta}'_d) \cup \{\bar{\theta}'_x\},$$

and Figures 3 and 4b illustrate the case in which $\check{\theta}'_d \in [\underline{\theta}'_d, \bar{\theta}'_d)$, while Figures 4a corresponds to $\check{\theta}'_d = \bar{\theta}'_x$.

The extent of downward wage rigidity affects both incumbents' employment decisions and productivity cutoffs. As discussed before, it doesn't have an impact on the long-run employment of stayers (both domestic and exporters), however, it determines how many workers firms choose to fire on impact. It also affects the productivity cutoff $\check{\theta}'_d$. As we show in numerical simulations, there is a level of minimum wage $w'_{\min}$ such that for all $w_m > w'_{\min}$ all firms choose to fire some workers on impact instead of gradually shrinking its workforce (i.e., $\check{\theta}'_d = \bar{\theta}'_x$). Additionally, $\bar{\theta}'_x$ is decreasing with the minimum wage. That is, even more incumbents start exporting compared to the case without wage rigidity. This is intuitive because by becoming exporters and increasing employment these firms are

the long run; $\bar{J}_d^I(\theta)$ is the value of staying and not exporting in the long run; $\underline{J}^I(\theta)$ is the value of producing in the short run but not in the long run; and 0 represents the outside option of firing all workers immediately and not producing at all after the decline in τ. The continuation value is the upper envelope of these individual value functions and their intersections determine the three cutoffs, $\underline{\theta}'_d$, $\bar{\theta}'_d$ and $\bar{\theta}'_x$.

not constrained by the downward rigid wage, which is not binding at the long-run optimal employment $\bar{h}(\theta, \iota)$. In contrast, if they stayed non-exporters, part of their transition dynamics would now be additionally constrained by the rigid wage.

The right panel of Figure 4 shows the employment function $\check{h}'(\theta)$. The difference between $h(\theta)$ and $\check{h}'(\theta)$ represents the extent of firing on impact while the difference between $\check{h}'(\theta)$ and $\underline{h}'(\theta)$ represents the gradual attrition after which a firm fires the remaining workers and exits the industry. For an incumbent with productivity $\theta \in [\underline{\theta}'_d, \check{\theta}'_d]$, the firing on impact is larger the lower its productivity, while the reduction of labor via exogenous attrition is larger the higher its productivity.[23]

In our benchmark case, the new steady-state employment level of non-exporting incumbents is 11% below their initial level, and with the labor force attrition rate of $\sigma = 0.175$ it takes 7.5 months for non-exiting firms to gradually reduce their employment to the new long-run level. However, firms that eventually exit the industry take much longer to reach their terminal level of employment $\underline{h}(\theta)$, between 3.75 and 4.5 years. This long time results from the fact that these incumbents have a long way to go before their labor force shrinks enough to just barely cover the fixed operating costs. As a result, low-productivity firms with no long-term prospects linger on for a long time before closing shop.

Our model captures the rich dynamics of employment at the micro level. Of particular note is the observation that the distribution of employment across firms with varying productivity levels can differ considerably in the short run compared to the long run. Moreover, since the transition to the new steady-state in response to a reduction of variable trade costs can be protracted, the deviation from the Melitz (2003) norm can be substantial during a prolonged time span, and this deviation is sensitive to the size of labor market frictions. Specifically, during the transition, there is heterogeneity in employment and output of firms with similar productivity levels because incumbents and new entrants adopt different business strategies. Naturally, the role of incumbents declines over time as they die out and are replaced by new entrants. Nevertheless, as shown in Proposition 2, the gains in consumer surplus resulting from lower trade impediments are instantaneously realized, and they do not depend on labor market frictions; a surprising result indeed.

4.4 Entry, exit and firing

In the previous section, our discussion centered on economic environments in which there is entry of new firms into the differentiated sector across all time periods. In this case—in response to a reduction in variable trade costs—the real consumption index Q adjusts on impact to its new steady state level, i.e., $Q_t = Q'$ in all time periods $t > 0$ (see Proposition 2). In this section, we examine various labor market

[23] We can also compute the time it takes a low-productivity incumbent to exit the industry, $\underline{T}(\theta)$, and the time it takes a non-exporting incumbent to shrink its labor force to the steady-state level, $\bar{T}$:

$$\underline{T}(\theta) = \frac{1}{\sigma} \log \frac{\min\{h(\theta), \check{h}'(\theta)\}}{\underline{h}'(\theta)} \qquad \text{and} \qquad \bar{T} = \frac{1}{\sigma} \log \frac{\min\{h(\theta), \check{h}'(\theta)\}}{\bar{h}'(\theta)}.$$

The latter does not depend on the firm's productivity because the employment choices of all non-exiting firms scale with θ. In the limit with no labor market frictions all adjustment happens at once, $\bar{T} = \underline{T}(\theta) \equiv 0$, and all cutoffs collapse to $\underline{\theta}'_d = \bar{\theta}'_d = \theta'_d$ and $\check{\theta}'_d = \bar{\theta}'_x = \theta'_x$, so that all firms with $\theta < \theta'_d$ fire all workers at time $t = 0$ and exit on impact, while all firms with $\theta \in [\theta'_d, \theta'_x)$ immediately reduce their forkforce to their new long-run employment level.

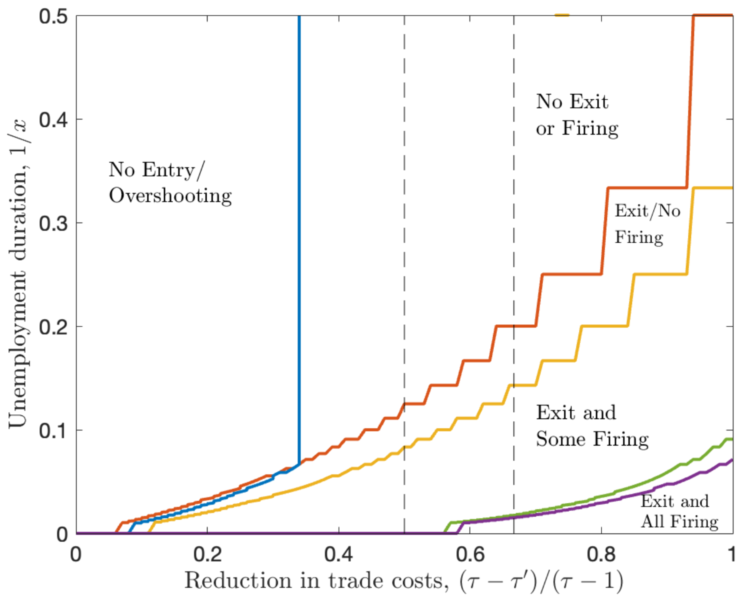

Figure 5: Dynamic adjustment patterns

Note: Patterns of adjustment to lower trade costs ('Exit' refers to some firms exiting on impact). Regions: (0) In the "No Entry/Overshooting" region $Q_t > Q'$ for some $t > 0$ and the characterization of Section 4.1 does not apply (see Section 7); (1) "No Exit or Firing": $\underline{\theta}'_d = \check{\theta}'_d = \theta_d$; (2) "Exit/No Firing": $\theta_d < \underline{\theta}'_d = \check{\theta}'_d < \bar{\theta}'_d$; (3) "Exit and Some Firing": $\theta_d < \underline{\theta}'_d < \check{\theta}'_d < \bar{\theta}'_d$; (4) "Exit and All Firing": $\theta_d < \underline{\theta}'_d < \bar{\theta}'_d$ and $\check{\theta}'_d = \bar{\theta}'_x$. The border between (3) and (4) satisfies $\check{\theta}'_d = \bar{\theta}'_d$, i.e., all exiting firms fire some workers on impact, but all staying firms do not fire on impact. See Figure 3 for the definition of cutoffs. The two vertical dashed lines represent the main trade cost reductions: the benchmark $\tau' = 1.375$ and $\tau' = 1.25$.

frictions and reductions in variable trade costs that map out regions where Q does not overshoot its long-run value and a region in which there is no initial entry of new firms. This analysis also identifies sets of parameters that generate different equilibrium patterns of labor dynamics at the firm level.

Figure 5 plots our regions of interest. We use $(\tau - \tau')/(\tau - 1)$ on the horizontal axis to measure by how much trade costs are reduced. The lower τ' is the larger this measure, which equals zero when $\tau' = \tau$ and one when $\tau' = 1$ (free trade). For measuring the extent of labor market frictions we use the inverse of the job-finding rate, $1/x$, where x is higher the lower the cost of vacancies and the higher the productivity of matching. It follows that lower values on the vertical axis represent lower labor market frictions in the way they map into equilibrium unemployment duration.

The figure shows a (North-Western) region, in which, following a reduction of variable trade costs, there is no entry of new firms on impact. This case is obtained when our index of free trade is below 0.35 and labor market frictions are relatively high. Otherwise new firms enter in all time periods. When the index of free trade exceeds 0.35, new firms enter continuously no matter how severe labor market frictions are.

Next, consider a reduction of trade costs that raises the index of free trade somewhat above 0.35. If labor market frictions are high, no incumbent firm fires workers on impact and none leaves the traded sector. Since the cost of hiring workers is large, the surplus from the employment relationship is large and it remains positive despite the rise in competitive pressure. Low-productivity incumbents adjust employment downward via exogenous attrition while high-productivity incumbents instantly increase their labor force to the new steady-state level.

Alternatively, when labor market frictions are lower, some low-productivity incumbents eventually exit the industry. Some may exit on impact if labor market frictions are low enough. Or some may fire workers on impact and stay for a while in the industry while other, somewhat higher-productivity incumbents, do not fire on impact but exit eventually when their labor force shrinks to $\underline{h}'(\theta)$ as a result of exogenous attrition (recall the discussion in the previous section). In a range of intermediate labor market frictions there is no firing on impact, yet some low productivity incumbents exit the industry after allowing their labor force to shrink to $\underline{h}'(\theta)$ (see the "Exit/No Firing" region).

It is clear from Figure 5 that both firing and exit of incumbents on impact require sufficient labor market flexibility and large enough reductions of variable trade costs. When the reduction of trade costs is of the size postulated in our benchmark case, exit of incumbents becomes an equilibrium phenomenon only when unemployment duration falls short of two months, while firing on impact becomes an equilibrium phenomenon only when unemployment duration drops even lower to around one month. As the size of the free-trade index rises, both exit and firing become more likely, and they arise even under more rigid labor markets. Indeed, a larger reduction of variable trade costs destroys more of the employment surplus for low-productivity incumbents, forcing them to cut employment or exit on impact.

5 Unemployment and Income Dynamics

Now let's zoom in on the dynamics of the workers' employment and wages following the trade shock. As we highlighted earlier, the heterogeneity in wage adjustment is coming entirely from the firms' side, while the workers are ex ante identical. In this section, we will discuss the dynamics of labor income, which depends on firms choosing to shrink wages on impact and/or fire some of its labor force, and the trade-off between temporary unemployment and income losses that arises due to labor market frictions and wage rigidities.

5.1 Good jobs, bad jobs

As in Helpman and Itskhoki (2010), here too in the long-run equilibrium every worker is paid the same wage rate in the differentiated sector, independently of whether she is employed by a low- or high-productivity firm, or whether her firm exports or serves only the domestic market. Nevertheless, wages of many of these workers may differ in the short and medium run following a reduction of trade costs, and since some workers lose their jobs, their income drops to the level of unemployment benefits. Recall that workers bargain with their firm over wages and the resulting wage is (11). Given the firm's productivity and employment, this wage rate declines at time $t = 0$ due to the rise in competitive pressure, reflected in the increase in Q. For non-exporters this spells a decline in wages, as they are hurt by the intensified competition domestically and do not export. As a result, all incumbent non-exporters cut wages.

Wage dynamics differ for exporters whose productivity exceeds $\bar{\theta}'_x$. For them the rise in the competitive pressure is more than compensated for by the lower export costs. As a result, they instantly expand employment to the new steady-state level and do not change the wage settlements with their workers. Their wages also equal the wages of new entrants.

Works, while ex ante identical, can be split into four groups ex post. First group are those who are initially employed by high-productivity firms that export in the new equilibrium (with $\theta \geq \bar{\theta}'_x$). These workers do not experience wage cuts (measured in terms of the numeraire, which is the non-traded good). Moreover, they — as do all other workers — benefit from a lower price index of the differentiated product Q, hence their real wages increase.

The second group consists of workers employed by firms that do no leave the industry, but their productivity is too low to export in the new steady-state, that is firms with $\theta \in [\bar{\theta}'_d, \bar{\theta}'_x)$. These firms have too many workers at time $t = 0$, but they fire no one under the baseline parametrization. Instead, they allow their workforce to gradually decline until it reaches the steady-state level $\bar{h}'(\theta)$ for a firm with productivity θ. It is evident from the wage equation (11) that each one of these firms cuts wages on impact, in response to the rise in Q, but then raises wages gradually as its labor force shrinks. The workers in these firms suffer wage cuts on impact, even if their wage is bounded below by the rigid wage w_m and then recovers gradually over time back to its long-run level. The present value of their income declines, as shown in the three panels of Figure 6. The vertical axis measures the loss in the present value of income in percentage terms (e.g., -0.01 represents a one percent loss). The figure also shows that the present value of nominal income of workers employed by exporters does not change.

For workers initially employed by firms with productivity $\theta \in [\theta_d, \bar{\theta}'_d)$ there are two possibilities: either they are employed by firms that leave the differentiated sector on impact, or they are employed by firms that stay temporarily and leave eventually, since all the incumbents in this productivity range close shop in the long run. The left panel of Figure 6 shows a case in which $\check{\theta}'_d = \theta_d$, i.e., no firm fires workers nor closes shop on impact. Under this condition, employees in a firms with productivity $\theta \in [\theta_d, \bar{\theta}'_d)$ suffer a wage cut on impact but see increase in wages afterwards, as the size of the firm's labor force declines. Once the employment level hits $\underline{h}'(\theta)$, the firm fires all remaining workers and closes shop. In this productivity range the loss in present value of income is larger for workers employed by less productive firms, because they stay less time in business.[24]

In the right panel of Figure 6 we have $\theta_d < \underline{\theta}'_d < \check{\theta}'_d < \bar{\theta}'_d$. This is the case where all incumbents with productivity lower than $\underline{\theta}'_d$ close shop on impact, causing their workers the largest possible income loss. Firms with productivities in the interval $[\underline{\theta}'_d, \bar{\theta}'_d)$ temporarily stay in the industry and each one of them fires all remaining workers and closes shop when its labor force declines to $\underline{h}'(\theta)$. Although firms with productivity above $\check{\theta}'_d$ reach the exit level of employed via attrition only, while those with lower productivity fire some workers on impact and use exogenous attrition to further reduce employment, workers in all these firms suffer a loss of income in present value terms and this loss is larger for employees of less productive firms, as shown in the figure.[25]

[24] The loss in present value income is bounded below by $J^E - J^U = b$, which is not binding in the top left panel of Figure 6 but is binding in the top right panel.

[25] Employees of incumbent firms in the non-traded sector do not suffer income losses, because their wages do not change.

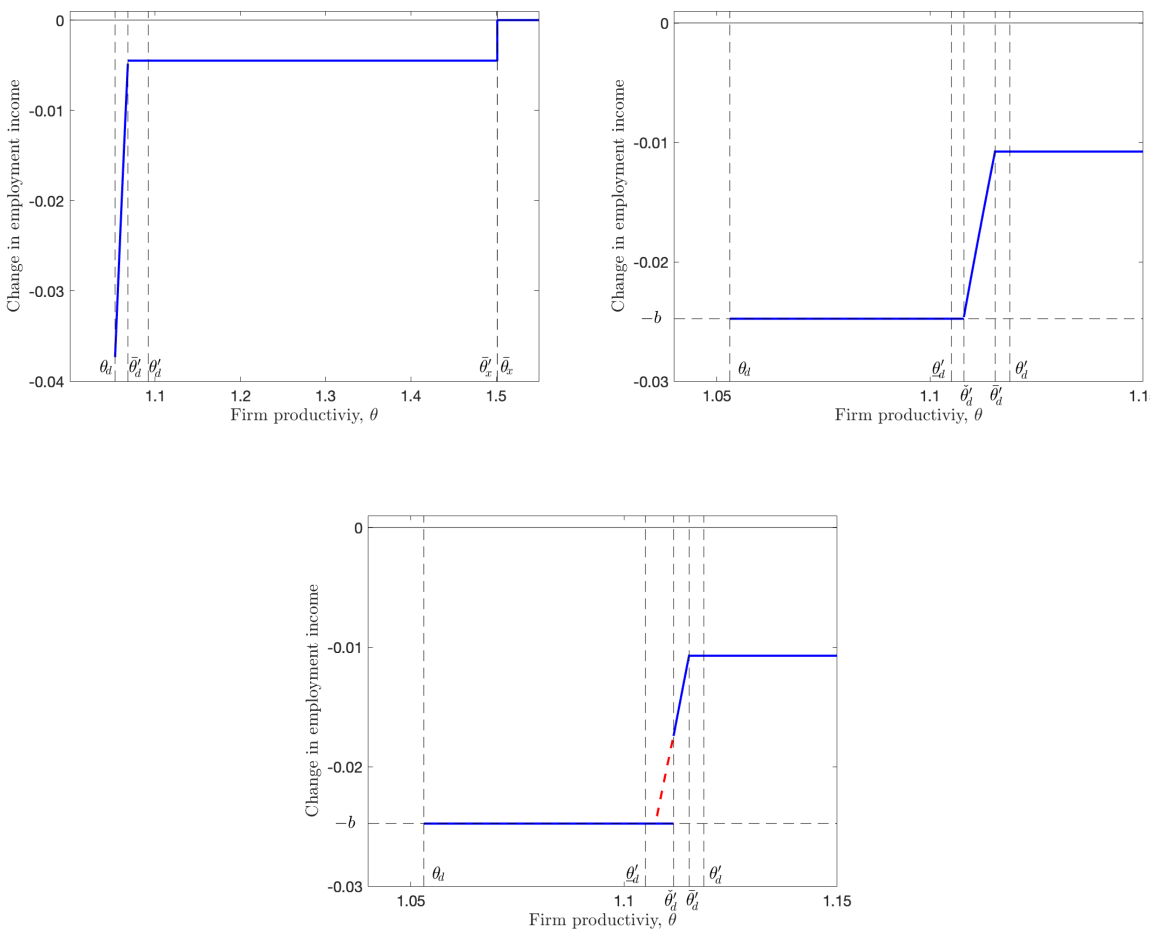

Figure 6: Change in the value of employed workers by firm type

Note: The loss in the value of the employed workers is measured as $(\tilde{J}^E(\theta) - J^U - b)$, which we then normalize by the worker's expected annual income in steady state (i.e., -0.02 corresponds to a loss of 2% of annual income); $\tilde{J}^E(\theta)$ is the continuation value of employment at a firm with productivity θ at the instant of the trade shock, $t = 0$, and b is the steady state worker's surplus from employment. Parameterizations in the first two panels correspond to those in Figure 4: (a) benchmark case in the left panel (with $x = 2$ and $\tau' = 1.375$) and (b) a more flexible labor market ($x = 24$) and a larger trade liberalization ($\tau' = 1.25$) in the right panel. Panel (c) is the case when the minimum wage is binding ($w_m = 0.98$, with $x = 24$ and $\tau' = 1.25$). In the top left panel, no firm exits or fires workers on impact, $\underline{\theta}'_d = \check{\theta}'_d = \theta_d$, while the other two panels $\theta_d < \underline{\theta}'_d < \check{\theta}'_d < \bar{\theta}'_d < \theta'_d$, as in Figure 3.

In the third (lower) panel of Figure 6, when the minimum wage is binding, workers employed by firms with $[\check{\theta}'_d, \bar{\theta}'_d)$ experience a loss in value of employment, but it is reduced as their wages cannot go below w_m (see below), and none of these workers are fired on impact. In contrast, workers in firms with productivity $[\underline{\theta}'_d, \check{\theta}'_d)$ are facing some probability of being fired, and unlike before (in second panel), firing is now associated with a loss of value — since the wage for those who remain employed cannot go below w_m. Thus, the value of workers with continued employment (in red) exceeds the value of the workers that are fired (blue flat line at $-b$, the maximum loss of value).

However, they gain from the lower price index of the differentiated product.

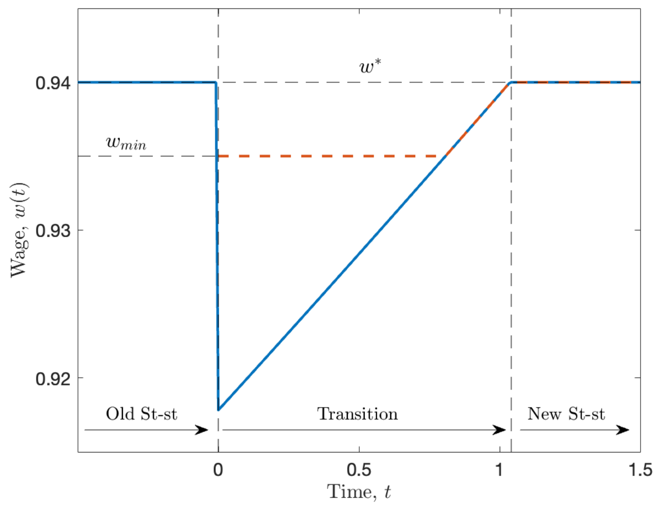

Figure 7: Wage dynamics

Note: The blue line corresponds to the case without wage rigidity, and the red dashed line is the case with the low binding minimum wage. The two vertical dashed lines represent the time period $t = 0$ when the trade cost changes and time period when the new steady state is reached. The graph utilizes the case where $x = 24$ and $\tau' = 1.25$.

It is evident from this analysis that the best jobs in the differentiated sector are provided by incumbents that export in the new equilibrium, while the worst jobs are provided by the least productive incumbents who fire workers and exit the industry (even if they fire only a subset of workers and exit only eventually). In between, there are jobs provided by non-exporting firms that do not leave. A worker's income loss is (weakly) declining with the productivity of her employer, and the losses are largest for workers employed by the low-productivity, non-exporting firms.[26] On the other hand, employees of exporting firms gain. These predictions are consistent with the recent body of work that links the welfare impact of trade on workers to the performance of their employers (see Verhoogen 2008, Amiti and Davis 2011, Helpman, Itskhoki, Muendler, and Redding 2017).[27]

Figure 7 shows the dynamics of wages paid by firms who stay in the domestic market after the shock, i.e., firms with productivity $[\bar{\theta}'_d, \bar{\theta}'_x)$, assuming they do not fire workers on impact. In the absence of wage rigidity, firms cut their wages on impact and then gradually increase them until they reach the steady-state level (see blue line). If the minimum wage is binding, firms pay the minimum wage until it eventually reaches the bargained level above w_m, and then pay according to a blue line schedule again. When the minimum wage becomes high enough, all stayers start firing workers on impact, and the more so the higher is the minimum wage. The wage dynamics then still follows the red curve, but

[26] Eaton, Kortum, and Kramarz (2011) estimate that over 90% of French firms would shrink in response to a reduction in trade costs, emphasizing that the majority of the manufacturing workers may loose in nominal terms. In our benchmark calibration, 76% of the firms, accounting for 23.5% of manufacturing employment, cut their employment and wages after the trade shock.

[27] Note that the predictions of our model about the short-run response of factor income is similar to the Ricardo-Viner specific factor model (see Jones (1971)). Indeed, workers attached to an incumbent become partially "specific" to this firm due to the hiring cost. As a result, the well-being of workers is correlated with the well-being of their employers (see also Davidson, Martin, and Matusz 1999). In the long run this specificity element wears off.

it reaches the bargained level sooner, since some workers are fired, and the labor force is reduced not exclusively via attrition.

5.2 Job destruction versus wage cuts

Employed workers suffer income losses from a reduction of variable trade costs that emanate from two sources: wage cuts and job destruction. The degree to which one or the other play a bigger role depends on labor market frictions. In an economy with low frictions in the labor markets the surplus from employment is small and, as a result, a reduction in variable trade costs destroys many jobs but has a modest impact on wages. On the other side, in an economy with high frictions in labor markets the surplus from employment is large and, therefore, a reduction in variable trade costs generates fewer job losses but reduces wages substantially. This reasoning suggests that the contribution of job losses and wage cuts to the present value of workers' income vary with the degree of labor market rigidity.

This tradeoff is quantitatively explored in the left panel of Figure 8. Labor market frictions are measured on the horizontal axis where we vary unemployment duration, $1/x$; larger values of this statistic represent more rigid labor market. On the vertical axis we measure the fraction of workers fired on impact, fired eventually, and the workers' present value of income loss as a fraction of the present value of income in the traded sector. On this axis, a number such as 0.07 represents a 7% loss of traded-sector jobs but a 0.7% income loss. This scaling is chosen in order to make all the curves visible in the same figure. In this figure, we use the benchmark parameters except for the size of τ', which takes here on a smaller value 1.25. This choice of reduction in variable trade costs is made in order to sharpen the visibility of the curves in the figure; similar patterns arise with a more modest reduction to $\tau' = 1.375$.

First, note that in the absence of labor market frictions workers lose no income, but the fraction of displaced workers is largest, equal to 7.1% of employment in the differentiated sector (see the left panel of Figure 8). Of the 7.1% job destruction 2.5% are caused by staying firms that shed labor and 4.6% by firms that exit and close shop. As labor market frictions rise, both fractions of displaced workers decline, because the value of the match between a firm and its workers rises. However, the two fractions do not decline at the same pace. In particular, the fraction of workers fired on impact declines faster and it reaches zero when unemployment duration is somewhat larger than 0.2 (i.e., a little more than 2.5 months). For higher labor market frictions no firms fire workers on impact and the only source of job destruction is exit of low-productivity firms. When unemployment duration is 3 months no worker is fired on impact and only 3.5% of workers are employed by firms that eventually exit the industry.

Unlike job destruction, income losses are not monotonic in labor market frictions. The share of the present value of income lost rises initially with labor market frictions, reaching a peak of 0.4% when unemployment duration is between 2 and 3 months. It declines afterwards, but stays flat at around 0.3% for the most part. What happens essentially is that once unemployment durations reaches two months, further increases in labor market friction shield workers from income losses (more on this below). Evidently, there is either substantial job destruction or significant labor income loss, but not both simultaneously.

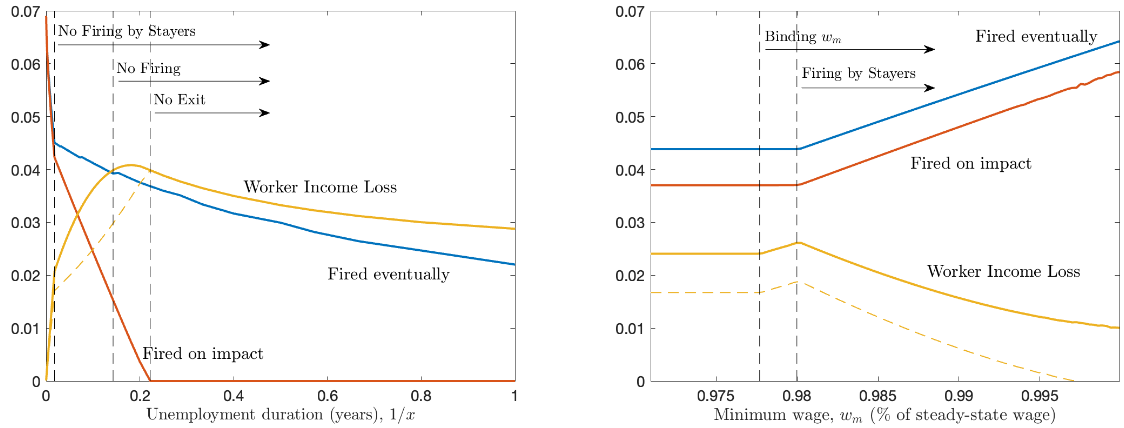

Figure 8: Displacement rates and income losses

Note: This figure uses the parameters from Table 1, with the exception of the matching productivity parameter (left panel), which we vary to span various degrees of labor market rigidity, as captured by unemployment duration on the horizontal axis (similar to Figure 5), and the minimum wage (right panel), which reflects various extent of the downward wage rigidity. This figure also uses the larger trade shock: $\tau' = 1.25$ instead of $\tau' = 1.375$. The red and blue curves depict the fractions of workers fired on impact and those fired eventually (when firms exit), respectively. The yellow curve depicts the aggregate income loss of workers in the traded sector as a share of the sector's income, which we multiply by a factor of 10 for scaling purpose (so that 0.04 corresponds to 0.4% of annual manufacturing income). The dashed yellow curve depicts the aggregate income share loss of workers who are not fired on impact. The vertical dashed lines in the left panel separate the adjustment regions in the parameter space (as in Figure 5), while in the right panel identify regions where minimum wage starts being binding and causes firing on impact, respectively.

An increase in the minimum wage results in a lower labor income loss but comes at the expense of displacement rates. This tradeoff is reflected in the right panel of Figure 8. The initial level of workers' income losses and displacement rates correspond to $x = 24$ (short unemployment spell) and big trade shock ($\tau' = 1.25$). After the minimum wage starts binding (the left dashed line), displacement rates stay constant for a while, as firms do not change their firing strategies at first. As discussed in the previous section, when the minimum wage is high enough all stayers start firing workers on impact (the second dashed line). After that point, even though workers' income loss is monotonically decreasing with the minimum wage, both displacement rates increase mainly due to high rates of workers fired on impact.

5.3 Job creation and unemployment

While the reduction of variable trade costs leads to job destruction by low-productivity firms, high-productivity firms that export and new entrants create jobs. There is, in fact, a spike in job creation in the traded sector in period $t = 0$, which expands its size as Q rises to its new steady-state level. This expansion is accommodated by the reallocation of unemployed workers from the non-traded sector to the traded one. Most of this job creation takes place in the first period. Later job creation is small; it just accommodates the continuous reallocation of workers from shrinking and exiting firms to new entrants. More details are provided in the appendix.[28]

Next consider unemployment. In the benchmark case with unemployment duration of 6 months, the reduction of variable trade costs induces an adjustment process in which no firm fires workers on impact. As a result, aggregate unemployment does not change on impact, at $t = 0$. There is, however, instant labor reallocation across sectors, because job creation by exporters with $\theta \geq \bar{\theta}'_x$ and by new entrants in the traded sector jumps up at $t = 0$. This requires unemployed workers from the non-traded sector to reallocate to the traded sector in order to secure the equilibrium job-finding rate x, which does not change. For this reason, the number of unemployed workers rises in the traded sector at the beginning of period $t = 0$ and declines in equal number in the non-traded sector. Moreover, aggregate unemployment changes little over time.[29]

When labor markets are sufficiently flexible, some workers are fired on impact. As we show in the left panel of Figure 8, up to 7% of workers in the traded sector can be fired on impact when labor markets are close to frictionless. This causes the number of unemployed to rise and, thereby increases aggregate unemployment. However, note that when τ declines to $\tau' = 1.375$, unemployment rises on impact only when labor markets are quite flexible so that unemployment duration falls short of one month. Under these circumstances the spike in aggregate unemployment is very short lived.

6 Dynamic Gains from Trade

In this section, we present the dynamics of exports and the average productivity in the differentiated sector following trade liberalization. Next, we provide the decomposition of the dynamic gains from trade, consisting of changes in consumer surplus, workers' income and firms' value, and quantitatively analyze the comparative statics with respect to various labor market frictions.

6.1 Trade and productivity

As a result of sunk hiring costs, the traded sector's incumbents are reluctant to fire workers or exit the industry in response to a reduction of variable trade costs. This leads to labor misallocation across firms (in comparison to the economies with no labor market frictions) and to a slower entry of new firms. For this reason, selection, which is an important feature of sectors with heterogeneous firms, is less forceful under these circumstances. Taken together, all these elements impact productivity and trade along the transition path. Figure 9 illustrates the dynamics of aggregate productivity, while Figure 10 presents the trade flow dynamics for different levels of labor market frictions and wage rigidities.

Labor productivity in the traded sector, Q_t^{ζ}/H_t, for the benchmark case (see Table 1) is displayed in the left panel of Figure 9, while the case with higher job-finding rate and rigid wages is presented in the right panel. As shown in Proposition 1, this measure of labor productivity is the same in the old

[28] There is another small spike in job creation when low-productivity incumbents start firing their remaining workers and exiting the industry, thereby clearing the way for new entrants.

[29] When the length of a period is one month, the number of unemployed in the traded sector rises sixfold on impact in the benchmark case. In order to satisfy Assumption 1, the traded sector needs to account for less than 15% of aggregate employment, which is satisfied in many developed countries. See appendix for further discussion.

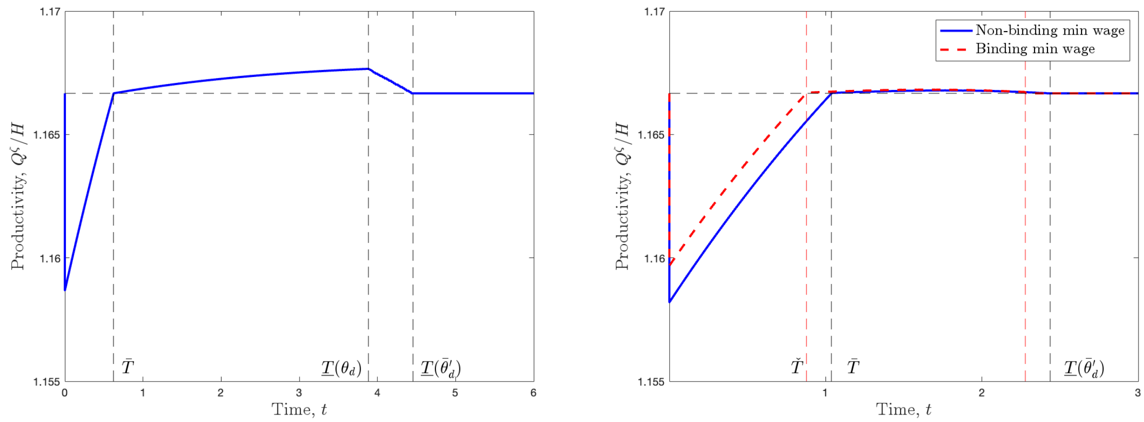

Figure 9: Aggregate productivity dynamics

Note: Both panels plot aggregate productivity in the traded sector, measured as an average sectoral revenue per worker, Q_t^ζ/H_t, starting from the trade shock at $t = 0$. The left panel uses the parameters from Table 1, in particular, the job-finding rate is $x = 2$ and $\tau' = 1.375$. The right panel uses $x = 24$ and $\tau' = 1.25$, and the red dashed line corresponds to the case with binding minimum wage, $w_m = 0.98w$. In both panels, the vertical dashed lines represent times it takes for incumbents to shrink employment to the new steady-state levels. Horizontal dashed lines represent the steady-state value of labor productivity, common for both the pre- and post-shock steady states. See the text describing vertical dashed lines.

and new steady states, represented by the horizontal dashed line in the figure. However, in the short run, labor productivity declines as a result of labor misallocation across firms. After the initial decline, labor productivity rises over time, until it exceeds its long-run value, and then it gradually declines to reach the new steady-state level. These swings along the transition path are caused by incumbents. On impact, no firm leaves the industry, but a large number of new entrants sets up shop and incumbent exporters hire new workers. As a result, the value of output increases, except that due to labor misallocation revenue per worker declines. Over time all non-exporting incumbents gradually contract due to exogenous attrition and new entrants keep setting up shop. Therefore, as over-employment of incumbents declines, labor productivity rises. At time $\bar{T}$ all incumbents that stay in the industry attain their steady-state level of employment and in subsequent periods replace workers that separate for exogenous reasons. In the meantime, lower-productivity incumbents that stay in the industry only temporarily keep shrinking their employment, thereby contributing to rising labor productivity. At this stage, labor productivity overshoots its long-run level. At time $\underline{T}(\theta_d)$ the least productive firms, with productivity θ_d, exit, and as time goes by, firms with higher productivity exit too. These exits lead to the labor productivity decline. At time $\underline{T}(\bar{\theta}'_d)$ firms with productivity $\bar{\theta}'_d$ exit, bringing to a halt the voluntary exit process; in future periods no remaining incumbent voluntarily closes shop. Under these conditions, the steady-state level of productivity is attained at time $\underline{T}(\bar{\theta}'_d)$. With the higher job-finding rate (i.e., shorter unemployment duration), the transition happens faster as the hiring costs paid by the firms pre shock are lower and imply faster exit of low-productivity incumbents. Wage rigidity further speeds up reallocation and reduces the initial drop in productivity by forcing more exit on impact.

The model features no labor misallocation in the long-run, measured as the dispersion of an average revenue per worker across firms (Hsieh and Klenow 2009), which can be seen by substituting the

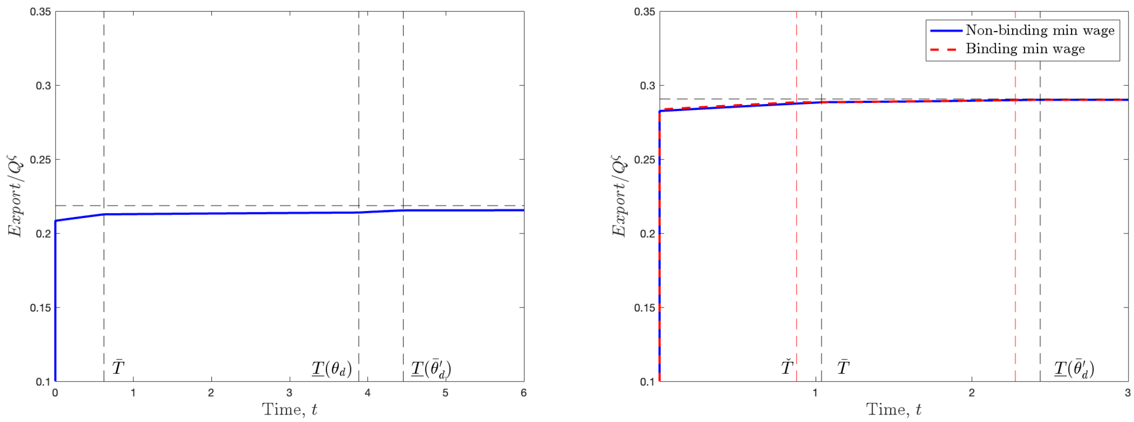

Figure 10: Aggregate trade flow dynamics

Note: Both panels plot aggregate exports relative to the revenue in the traded sector. The left panel uses the parameters from Table 1, in particular, the job-finding rate is $x = 2$ and $\tau' = 1.375$. The right panel uses $x = 24$ and $\tau' = 1.25$, and the red dashed line corresponds to the case with binding minimum wage ($w_m = 0.98w$). Horizontal dashed line display the new (asymptotic) steady-state value.

optimal employment choice (15) into the revenue function (8). Yet there is misallocation during the transition, when non-exporting incumbents are too large and have lower marginal product of labor. This outcome is consistent with the empirical finding that large firms and exporters appear to be too small in relative terms. The model predicts that misallocation is particularly severe among the less productive incumbents.

Exports relative to total revenues are displayed in Figure 10. The old steady-state level is represented by the horizontal axis. Trade rises on impact and keeps increasing gradually afterwards, until it reaches the new steady state. The upward jump on impact is due to the growth of exports at the intensive margin, by firms that exported in the old steady state, as well as due to the growth of exports at the extensive margin, as firms that were just below the export cutoff θ_x start exporting after the decline of variable trade costs. The continued growth of exports is due to the entry of new firms as non-exporting incumbents shrink and exit. Over time, employment is reallocated from incumbents to new entrants and the trade flows converge towards their long-run level.[30] Evidently, our model predicts a lower trade elasticity in the short-run than in the long-run, and the gap between them rises with labor market frictions. The larger trade shock in the right panel implies larger trade flows, while introducing wage rigidity speeds up the transition (see the red dashed line in the same panel) .

6.2 Decomposition of gains from trade

We now examine the total size and composition of welfare changes that result from lower variable trade costs. There are three sources of welfare changes: the present value of consumer surplus, the present

[30]The full convergence does not happen in finite time, before all incumbents die, since the incumbent production and exporting cutoffs are different from the cutoffs of new entrants (see Figure 3). Taking into account the endogenous selection forces, the incumbents are on average less productive and less likely to export relative to the surviving entrants.

value of labor income, and the value of incumbent firms. Since unemployment benefits are financed with lump-sum taxes, their present value equals the present value of taxes and these flows have no direct effect on welfare (they do have, of course, indirect effects through their impact on wages).

We discussed the dynamics of consumer surplus in Section 4.1, where we have identified circumstances in which consumer surplus jumps up on impact to its new steady-state value and the complementary circumstances in which it overshoots the long-run value in the short run. Be it as it may, tracing out the time path of Q_t enables us to compute the new present value of consumer surplus. In the following analysis, we convert all present values into their annualized *constant flow* equivalent values by multiplying the stock with the interest rate r. When the real consumption index Q jumps up on impact to its new steady state level, as it does under our quantitative assumptions, the annual flow equivalent of the rise in consumer surplus is $\left[(Q')^{\zeta} - Q^{\zeta} \right] (1-\zeta)/\zeta$, capturing the increase in real purchasing power of aggregate nominal income.

In Section 5.1, we described the dynamics of wages and employment, where we have seen that workers employed by exporting firms do not experience a change in wages following the decline in variable trade costs, while all other workers employed by incumbent firms in the differentiated sector suffer a decline in the present value of their wage income. Employees of new entrants are paid the steady-state wage rate, which does not change. The employees of low-productivity incumbent firms that do not export, experience wage cuts and job losses, although the wages do not drop all the way due to wage rigidity and then gradually recover over time in the continuing firms. Further, some employees of low-productivity incumbents may be fired on impact, while other workers may see their wages erode and they may eventually also lose their jobs due to deliberate closure of the firm. While these complex dynamics take place in the traded sector, wages in the non-traded sector remain constant. However, employment levels in the traded and non-traded sectors change as do employment levels of individual incumbents. In the aggregate, this leads to a change in the present value of wages from W to W_0, where W is the present value of (nominal) wages in the old steady-state and $W_0 < W$ is the present value of wages at time $t = 0$. As a result, the annualized flow loss of wage income is $r(W - W_0)$.

Lastly, consider the change in the value of incumbent firms. The value of an incumbent with productivity θ is equal to the present value of its operating profits. After the decline in variable trade costs, all incumbents with productivity larger than $\bar{\theta}'_x$, who export in the new steady state, gain in value, while all lower-productivity firms lose value. For some of them, who close shop on impact, the value drops to zero. For new entrants as a group, the net value is zero due to the free-entry condition. For this reason, the annualized flow loss of dividends (profits) from incumbent firms is $r\left(J_I^F - J_{I0}^F\right)$, where J_I^F is the value of incumbents in the old steady state and J_{I0}^F is the value of these same firms at time $t = 0$.

As is evident from this discussion, the annualized change in aggregate welfare is $\frac{1-\zeta}{\zeta}\left[(Q')^{\zeta} - Q^{\zeta}\right] - r(W - W_0) - r(J_I^F - J_{I0}^F)$. It is convenient to report these welfare changes in some normalized fashion rather than in absolute value. For this reason, we report them relative to the size of the traded sector, $PQ = Q^{\zeta}$, in the old steady state. That is, the dynamic gains from trade as a fraction of the size of the

Table 2: Decomposition of the dynamic gains from trade

Wages	Flexible				Rigid downwards			
Trade shock, τ' (% change)	1.375 (50%)		1.25 (66.7%)		1.375 (50%)		1.25 (66.7%)	
Job finding rate, x	2	24	2	24	2	24	2	24
(1) Gains in consumer surplus	5.60	5.60	9.50	9.50	5.60	5.60	9.50	9.50
(2) Capital value of profit losses	0.24	0.03	0.51	0.10	0.25	0.04	0.60	0.08
(3) Capital value of wage losses	0.09	0.12	0.27	0.23	0.06	0.10	0.16	0.23
(3a) — due to firing on impact	—	0.03	—	0.06	—	0.03	—	0.06
(4) Full gains $(=(1)-r[(2)+(3)])$	5.585	5.595	9.46	9.48	5.585	5.592	9.46	9.48

Note: All numbers are percentages of the initial size of the traded sector, $PQ = Q^{\zeta}$: (1) annualized gains in consumer surplus; (2) capital value of profit (dividend) losses; (3) capital value of wage losses, including due to firing into unemployment on impact (3a); and (4) total annualized flow of the dynamic gains from trade according to (39), which subtracts the flow values of (2) and (3) from (1). We consider two trade shocks (50% and 66.7% reduction in τ), two values of labor market frictions ($x = 2$ and $x = 24$, corresponding to 6 months and 0.5 months unemployment duration), and two cases — with and without wage rigidity (in the latter case, we use. For columns 5-7, we use the level of minimum wage which corresponds to 99% of the equilibrium wage, and column 8 represents the benchmark case with the minimum wage which is 97.8% of the equilibrium wage to make it binding.

trade sector, are:

$$GT \equiv \frac{1-\zeta}{\zeta}\frac{\left[(Q')^{\zeta} - Q^{\zeta}\right]}{Q^{\zeta}} - \frac{r\,(W - W_0)}{Q^{\zeta}} - \frac{r\left(J_I^F - J_{I0}^F\right)}{Q^{\zeta}}, \tag{39}$$

where the first term on the right-hand side represents the gains in consumer surplus while the second and third terms represent the losses in wage and dividend incomes, respectively.[31]

Table 2 reports each one of the three components of the dynamic gains from trade for $\tau' = 1.375$ and for a larger decline in trade costs, $\tau' = 1.25$. In each case we report results for two levels of labor market frictions, a high level with $x = 2$ and a low level with $x = 24$, using the benchmark parameters from Table 1. We report results for all cases with and without downward wage rigidity.

A striking result is that gains in consumer surplus dominate the welfare calculus, amounting to annualized gains of 5.6% of output in the traded sector when the decline in trade costs is small and to 9.5% when the decline in trade costs is large. And since these gains do not depend on labor market frictions when these frictions are the same in both sectors, they are the same independently of whether the wage rigidity is present or not and the job-finding rate is low or high. In comparison, when labor market frictions are high, the loss of the *present value* of firm profits amounts to about a quarter of a percent of the value of output when $\tau' = 1.375$ and about half a percent when $\tau' = 1.25$. When labor market frictions are low, the present value of profit loss is even smaller, equal to 0.06 percent of the value of output when when $\tau' = 1.375$ and to 0.10 percent when $\tau' = 1.25$. Evidently, these values vary with labor market frictions, with the losses being larger in economies with more rigid labor markets.

[31] In case Q does not jump on impact to its new steady-state value, $(Q')^{\zeta}$, the first term needs to be replaced with r times the present value of the path of Q_t^{ζ}.

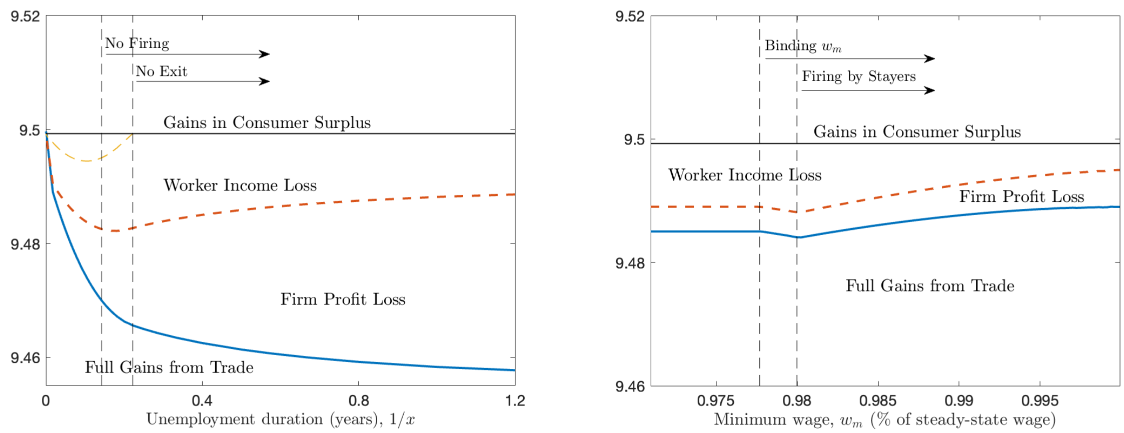

Figure 11: Decomposition of dynamic gains from trade

Note: The figure plots the full dynamic gains from trade (solid blue line) against the extent of labor market frictions, measured by unemployment duration, $1/x$, (left panel), and against the extent of wage rigidity, measured by the level of the minimum wage, w_m (right panel). Full gains are decomposed into three main components: gains in consumer surplus (solid flat black line), wage income losses (the distance between the black line and the red dashed line), and profit losses (the distance between the dashed red line and the solid blue line). The distance between the solid black line and the yellow dashed line in the left panel depicts the component due to job losses on impact. Variable trade costs decline by 66.7%, to $\tau' = 1.25$, while the other parameters are as in Table 1.

The present value of wage losses are also small in comparison to the gains in consumer surplus. However, in contrast to profit losses, they are smaller under rigid wages. In row (3a), we report the contribution of the rise of unemployment on impact to the loss of present value of wages. These losses are small in comparison to the size of the traded sector. Interestingly, when we compare the respective columns for the cases with and without wage rigidity, the total gains from trade are almost the same. However, looking at decomposition of these gains we can notice that there is reallocation between wage losses of workers and profit losses of firms. In the last row, all three elements of the welfare change are added up, annualizing the loss in the present value of profits and wages. These numbers are obviously very close to the numbers in row (1), which reports gains in consumer surplus. In short, capital losses of firm profits on one hand and losses of worker wage income on the other, that happen along the transition, detract little from welfare in comparison to the long-run gains in consumer surplus. However, in the short run, during the first year of transition, these losses are non-negligible, yet still only a fraction of the gains in consumer surplus (compare rows (2) and (3) to row (1) in Table 2).

Figure 11 depicts the gains from trade and their decomposition into components of the dynamic welfare gains formula (39), for a wide range of labor market frictions; the latter is measured by unemployment duration (the inverse of the job-finding rate), $1/x$, in the left panel, and the extent of wage rigidity, w_m, in the right panel. In this figure, $\tau' = 1.25$ and all other parameters are taken from Table 1. The gains in consumer surplus are constant, while the contribution of dividend losses rises with labor market frictions (see left panel). On the other side, the contribution of wage income losses rises initially and declines eventually as unemployment duration increases. Moreover, wage losses start shrinking as unemployment duration exceeds 1.5 months. The reason is that labor market frictions shield workers from separation into unemployment due to the firms' sunk hiring costs. As a result, losses from the trade shock are borne primarily by firms when labor markets are rigid. However, the ultimate residual claimants on firms' income are households, and aggregate households income losses rise with labor market frictions. Surprisingly, the total dynamic gains from trade increase with the minimum wage (see right panel). Although the firms' profit losses increase due to higher cost of labor, the decline in labor income losses decrease dramatically and dominate in the overall welfare calculation.[32]

7 Conclusion

In this paper, we have explored the dynamics of adjustment to a reduction in variable trade costs. These costs may arise from tariffs, transport, or insurance. While the source of these costs is not important for the path of adjustment, welfare consequences of a decline in these costs may depend on the source. Our welfare calculations are based on the assumption that these costs are real, in the sense that they use up resources, which is the case with transport and insurance. But if instead the variable trade costs were due to tariffs, our welfare analysis would still apply if the tariff revenues were wasted rather than used to provide valuable services. The alternative is to assume that tariff revenue is rebated to the public in a lump-sum fashion. Under these circumstances, only the welfare analysis needs to be modified in order to include changes in transfers of tariff revenue from the government to the public.

Our main aim has been to appraise the role of labor market frictions in this adjustment process. We, therefore, have developed a tractable extension of the two-sector model from Helpman and Itskhoki (2010), in which one sector produces a homogeneous non-traded good and the other produces tradable brands of a differentiated product, and the differentiated sector is populated by heterogeneous firms. In each one of these sectors firms post vacancies and workers search for jobs, and they match in the fashion now familiar from the work of Diamond, Mortensen and Pissarides. The cost of vacancies and the efficiency of the matching technology jointly determine the extent of labor market frictions. Using a sufficient statistic for these frictions enables us to explore the impact of variations in these frictions on the entire trajectory of adjustment to a trade shock.

While the most interesting dynamics take place in the traded sector, intersectoral adjustment plays an important role too. Due to costly hiring, labor market frictions impede the adjustment process that encompasses heterogeneous responses of firms with different productivity levels. In particular, low-productivity incumbents — who would have exited the differentiated product industry on impact in a world with frictionless labor markets — do not exit at all or keep operating temporarily and exit subsequently. This response is driven by the sunk cost of hiring, which is fully determined by labor market frictions and unemployment insurance. As a result, during the transition to the new long-run equilibrium labor is misallocated among incumbents within the traded sector, with too many workers employed by low-productivity firms. As a result, labor productivity declines during the transition and

[32]Note that our framework features many departures from a neoclassical model — including non-convexities, frictional labor and product markets — and hence we do not expect general efficiency properties to hold (see e.g. Helpman, Itskhoki, and Redding 2013). Furthermore, we analyze a response to an unexpected shock with sunk ex ante hiring decision resulting in over-employment by a subset of firms. Therefore, labor market frictions may result not only in income redistribution across heterogenous firms and workers, but also in general efficiency gains along the transition.

exports do not rise enough. Nevertheless, the gains from lower trade impediments are substantial and most of them are instantly realized if the level of unemployment in the non-traded sector is high enough to accommodate the rising labor demand in the traded sector. These gains are due to the rise in the consumer surplus, which overwhelms cumulative losses from transitory declines in wage incomes of some workers and profits of some firms.

The tractability of our model relies on a number of assumptions that we consider plausible for the aim of this study. These assumptions enable us to isolate the effects of labor market frictions unencumbered by convexity considerations that arise in more general settings. In this sense, our analysis provides a benchmark for answering the questions at hand. We now discussed some of these assumptions.

First, we use a quasi-linear utility function in which the marginal utility of consuming non-traded good is constant. As is well-known, this helps to focus the analysis on the traded sector, in which most of the interesting action takes place. Moreover, the curvature of the utility function is not directly related to labor market frictions and, therefore, the effects emphasized by our analysis, such as the heterogeneous response of different incumbents to a decline in variable trade costs, will be similar in models with other types of preferences.

Second, our assumptions ensure that the hiring costs are proportional to the number of hires, and that this factor of proportionality does not depend on variable trade costs. This requires a large non-traded sector, so that in response to the trade shock a sufficient number of unemployed workers from the non-traded sector can change search strategies on impact and seek jobs in the traded sector, thereby accommodating the rise in labor demand by new entrants in the differentiated sector. As an alternative, one could imagine an environment in which the pool of unemployed that seek jobs in the traded sector is limited, which will slow down entry of new firms into this sector and spread the adjustment process over longer periods of time. Evidently, this effect is distinct from the direct effects of labor market frictions emphasized in this paper, but it can be incorporated into the quantitative analysis.

Third, the proportionality of the hiring costs also relies on the assumption that the matching function exhibits constant returns to scale. While not being general, this type of a matching function is a natural benchmark; alternative cases, with increasing or decreasing returns in matching, can be explored numerically. Firing costs could also be incorporated into the model; note that this does not affect the qualitative properties of the sS inaction region for individual firms, but it may change certain quantitative conclusions.

Finally, we do not allow idiosyncratic productivity shocks to incumbents, although these type of shocks help in matching the empirical patterns of firm growth (see Luttmer 2010). Our assumption greatly simplifies the analysis, because with idiosyncratic productivity shocks the state-space would be much larger. Moreover, it is not clear whether allowing for these shocks will moderate or amplify the impact of labor market frictions on the response of the economy to changes in variable trade costs. In summary, our assumptions provide tractability and focus. Future research will expand the analysis to more general frameworks.

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APPENDIX

A Setup and Steady State

A.1 Matching function

Matching of unemployed and vacancies in *each sector* is governed by a Cobb-Douglas matching function, i.e. the (annualized) matching rate is given by:

$$m = m(U,V) = \frac{1}{\tilde{a}} U^{\chi} V^{1-\chi}, \tag{A1}$$

where U is the stock of unemployed searching for a job and V is the stock of open vacancies, both in a given sector, and $\tilde{a}$ is the inverse of the productivity of the matching function. Therefore, $m\Delta$ is the flow of new vacancies during the period of length Δ. The job-finding rate is given by

$$x = \frac{m}{U} = \frac{1}{\tilde{a}} \left(\frac{V}{U}\right)^{1-\chi},$$

while the vacancy-filling rate is

$$g = \frac{m}{V} = \frac{1}{\tilde{a}} \left(\frac{V}{U}\right)^{-\chi} = \left(\tilde{a} x^{-\chi}\right)^{-\frac{1}{1-\chi}}.$$

We assume that the time period length Δ is short enough so that the probabilities of finding a job ($x\Delta$) and filling a vacancy ($g\Delta$) are both well-defined (i.e., less than one). We denote by $\gamma\Delta$ the flow cost of posting a vacancy during period Δ. Then the expected cost of filling a vacancy is

$$b \equiv \frac{\gamma\Delta}{g\Delta} = \frac{\gamma}{g} = a x^{\alpha}, \tag{A2}$$

where $a \equiv \gamma \tilde{a}^{1+\alpha}$ and $\alpha \equiv \chi/(1-\chi)$, and this equation corresponds to (4) and (10) in the text. In other words, b is a per-worker (expected) cost of a match: by paying b instantaneously, the firm fills (with certainty, by the law of large numbers) a measure one of vacancies by the end of the period Δ. [33] We assume that the two sectors share the same α, but may differ in terms of γ and/or $\tilde{a}$, and hence in terms of a.

[33] Consider a firm posting v vacancies for n periods, where $t = n\Delta$ is the corresponding length of time (in years). Assuming t is small, so no discounting is needed, the total cost of this action is $v(\gamma\Delta)n = v\gamma t$. The expected yield of matches from this action is $v(g\Delta)n = vgt$. Now if v is the measure of vacancies posted, then by law of large numbers vgt is the measure of workers met. Therefore, by paying $(v\gamma t)/(vgt) = \gamma/g = b$, a firm can meet a measure one of workers during a period of arbitrary small time length t; a corresponding flow payment per period is $(b/t) \cdot \Delta$, which equals b if all hiring is done in one period (i.e, if $t = \Delta$).

A.2 Labor market in the outside sector (Lemma 1)

We denote by J_0^U and J_0^E the values to unemployed and employed workers in the outside sector, which satisfy the following Bellman equations:[34]

$$J_0^U = b_u\Delta + \frac{x_0\Delta}{1+r\Delta}J_{0,+}^E + \frac{1-x_0\Delta}{1+r\Delta}J_{0,+}^U, \tag{A3}$$

$$J_0^E = w_0\Delta + \frac{s_0\Delta}{1+r\Delta}J_{0,+}^U + \frac{1-s_0\Delta}{1+r\Delta}J_{0,+}^E, \tag{A4}$$

where the $+$ subscript denotes the next period's variables.

We denote by J_0^V and J_0^F the values to a vacant and a filled job in the outside sector, which satisfy the following Bellman equations:

$$J_0^V = -\gamma\Delta + \frac{1-\delta_0\Delta}{1+r\Delta}\left[g_0\Delta\cdot J_{0,+}^F + (1-g_0\Delta)J_{0,+}^V\right], \tag{A5}$$

$$J_0^F = (1-w_0)\Delta + \frac{1-\delta_0\Delta}{1+r\Delta}\left[\sigma_0\Delta\cdot J_{0,+}^V + (1-\sigma_0\Delta)J_{0,+}^F\right], \tag{A6}$$

where δ_0 is the death rate of firms and σ_0 is the exogenous separation rate with workers, so that from the point of view of workers the overall exogenous separation rate is s_0 defined by $(1-s_0\Delta)\equiv(1-\delta_0\Delta)(1-\sigma_0\Delta)$. All our results hold in the special case of $\delta_0=0$ and $s_0=\sigma_0$, however, we introduce $\delta_0=\delta$ for symmetry with the differentiated sector to simplify some discrete-time expressions, and the differences between the two cases disappear altogether as $\Delta\to 0$.

Given free entry and unbounded pool of potential entrants in the homogenous sector, we must have $J_0^V\le 0$ in all periods, which holds with equality in all periods when firms post positive vacancies, $V_0>0$. Note that $U_{0,t}>0$ and $V_{0,t}=0$ is inconsistent with equilibrium, since in this case the vacancy is filled instantaneously (and costlessly in the limit as $\Delta\to 0$). Therefore, Assumption 1 implies $V_{0,t}>0$ in all periods along the equilibrium path, and, therefore,

$$J_0^V \equiv 0 \tag{A7}$$

Then (A5) implies that the present value of a filled job in the homogenous sector next period equals the current period hiring cost:

$$\frac{1-\delta\Delta}{1+r\Delta}J_{0,+}^F = b_0, \tag{A8}$$

along the whole equilibrium path.

Upon matching, the firm and the worker determine wages according to Nash bargaining with equal weights and without commitment. This means that in each period when the match is not exogenously destroyed, we have:

$$J_0^E - J_0^U = J_0^F - J_0^V. \tag{A9}$$

We also combine (A3)–(A4) and (A5)– (A6) to obtain:

$$J_0^E - J_0^U = (w_0-b_u)\Delta + \frac{1-\big(s_0+x_0\big)\Delta}{1+r\Delta}\big(J_{0,+}^E - J_{0,+}^U\big),$$

$$J_0^F - J_0^V = (1-w_0)\Delta + \frac{1-s_0\Delta}{1+r\Delta}\big(J_{0,+}^F - J_{0,+}^V\big) + \left(\frac{1-\delta\Delta}{1+r\Delta}J_{0,+}^V - J_0^V\right).$$

[34] When $\Delta\approx 0$, the following approximation is accurate:

$$rJ_0^U = b_u + x_0\big(J_0^E - J_0^U\big) + \dot{J}_0^U,$$

where $\dot{J}_0^U \equiv \big(J_{0,+}^U - J_0^U\big)/\Delta$. This equation is a generalized version of (6) in the text. By analogy, similar approximations can be used for other value functions.

We can sum these two equations to eliminate the wage rate, w_0, and, using (A7)–(A9), obtain a dynamic equation for b_0:

$$\frac{1+r\Delta}{1-\delta\Delta}2b_{0,-1} = (1-b_u)\Delta + \frac{\big[2(1-s_0\Delta) - x_0\Delta\big]b_0}{1-\delta\Delta},$$

or equivalently:

$$\frac{\big[2(r+s_0)+x_0\big]b_0}{1-\delta\Delta} = 1 - b_u + 2\frac{1+r\Delta}{1-\delta\Delta}\frac{b_0 - b_{0,-1}}{\Delta}, \tag{A10}$$

where the (-1)-subscript indicates the previous period.[35] Given (A2), the unique stationary solution of this difference equation has b_0 constant, and therefore:

$$\frac{\big[2(r+s_0)+x_0\big]b_0}{1-\delta\Delta} = 1 - b_u, \tag{A11}$$

which corresponds to (5) in the text, after taking the $\Delta \approx 0$ approximation. Along the explosive solutions of (A10), b_0 converges to zero or becomes unbounded in finite time, both of which are inconsistent with the optimizing behavior (of either workers or firms).

With (x_0, b_0) pinned down by (A2) and (A11), the rest of the equilibrium in the homogenous sector is characterized from (A3)–(A9). In particular, we have:

$$\begin{aligned} w_0 &= b_u + \frac{(r+s_0+x_0)\,b_0}{1-\delta\Delta}, \\ \pi_0 = 1 - w_0 &= \frac{(r+s_0)b_0}{1-\delta\Delta}, \\ \frac{rJ_0^U}{1+r\Delta} &= b_u + \frac{x_0 b_0}{1-\delta\Delta} + \frac{1}{1+r\Delta}\frac{J_{0,+}^U - J_0^U}{\Delta}. \end{aligned}$$

The last equation also has a unique stationary solution

$$\frac{rJ_0^U}{1+r\Delta} = b_u + \frac{x_0 b_0}{1-\delta\Delta}, \tag{A12}$$

with non-stationary solutions violating the no-bubble condition. This equation corresponds to (6) in the text, where we use the approximation that $\Delta \approx 0$. These derivations provide a proof of Lemma 1.

A.3 Differentiated sector

A.3.1 Product market

A firm splits its output between domestic and foreign markets:

$$y = q + \iota\tau q^*,$$

where τ is the iceberg trade costs and $\iota \in \{0,1\}$ is the indicator of whether the firm exports. Given the CES aggregator (2) and the utility function (1), the demand for the demand for a good in a given market satisfies

$$q = Q\left(\frac{p}{P}\right)^{-\frac{1}{1-\beta}} \qquad \text{and} \qquad P = Q^{-(1-\zeta)},$$

[35] In the limit with $\Delta \to 0$, this becomes an ordinary differential equation in b_0 (provided the relationship $b_0 = a_0 x_0^\alpha$):

$$\big[2(r+s_0)+x_0\big]b_0 = 1 - b_u + 2\dot{b}_0,$$

which has a unique non-explosive path with a constant b_0.

which results in a revenue

$$pq = Q^{-(\beta-\zeta)} q^{\beta}.$$

Therefore, the firm's optimal revenue from serving the two markets given output y is given by:

$$R(y,\iota) = \max_{\substack{(q,q^*):\\ q+\iota\tau q^*=y}} \Big\{ Q^{-(\beta-\zeta)} q^{\beta} + \iota (Q^*)^{-(\beta-\zeta)} (q^*)^{\beta} \Big\} = \left[1 + \iota \tau^{-\frac{\beta}{1-\beta}} \left(\frac{Q^*}{Q}\right)^{-\frac{\beta-\zeta}{1-\beta}} \right]^{1-\beta} Q^{-(\beta-\zeta)} y^{\beta},$$

which corresponds to (8) after we substitute $y = \theta h$ in. The quantities supplied to each market are:

$$q = \frac{1}{1 + \iota\tau^{-\frac{\beta}{1-\beta}} \left(\frac{Q^*}{Q}\right)^{-\frac{\beta-\zeta}{1-\beta}}} y \qquad \text{and} \qquad q^* = \frac{\iota\tau^{-\frac{1}{1-\beta}} \left(\frac{Q^*}{Q}\right)^{-\frac{\beta-\zeta}{1-\beta}}}{1 + \iota\tau^{-\frac{\beta}{1-\beta}} \left(\frac{Q^*}{Q}\right)^{-\frac{\beta-\zeta}{1-\beta}}} y. \tag{A13}$$

A.3.2 Firm's problem

The Bellman equation for the problem of the firm is given by:

$$J^F(h) = \max_{h'} \left\{ \varphi(h)\Delta - b\big[h' - (1-\sigma\Delta)h\big]^+ + \frac{1-\delta\Delta}{1+r\Delta} J^F_+(h') \right\}, \tag{A14}$$

where $J^F(h)$ and $J^F_+(h')$ are the values of the firm in this and next periods with employment h and h' respectively; the flow per-period operating revenue gross of hiring cost is denoted by $\varphi(h)\Delta$; we have substituted in the hiring cost $C(h', h)$ from (13); and $[\cdot]^+ \equiv \max\{\cdot, 0\}$. A firm dies at rate δ, losses labor at an exogenous rate σ, and discounts at rate r. We assume $(1 - s\Delta) \equiv (1 - \delta\Delta)(1 - \sigma\Delta)$, which for small Δ is approximately equivalent to $s = \delta + \sigma$, as we state in the text.

The first order condition for the choice of h' is given by:

$$\frac{1-\delta\Delta}{1+r\Delta} J^F_{h,+}(h') = \begin{bmatrix} b, & \text{when } h' > (1-\sigma\Delta)h, \\ \in [0,b], & \text{when } h' = (1-\sigma\Delta)h, \\ 0, & \text{when } h' < (1-\sigma\Delta)h, \end{bmatrix} \tag{A15}$$

where the h-subscript denotes a partial derivative with respect to employment. The Envelope theorem is given by:

$$J^F_h(h) = \varphi'(h)\Delta + \begin{bmatrix} (1-\sigma\Delta)b, & \text{when } h' > (1-\sigma\Delta)h, \\ \frac{1-s\Delta}{1+r\Delta} J^F_{h,+}(h'), & \text{when } h' = (1-\sigma\Delta)h, \\ 0, & \text{when } h' < (1-\sigma\Delta)h, \end{bmatrix}$$

where the intermediate case corresponds to the inaction region in which $h' = (1-\sigma\Delta)h$ and the hiring cost $C(h', h) = 0$; in this region, the continuation value is $\frac{1-\delta\Delta}{1+r\Delta} J^F_+\big((1-\sigma\Delta)h\big)$, and its derivative with respect to h is $\frac{1-s\Delta}{1+r\Delta} J^F_{h,+}\big(h'\big)$. Combining the Envelope theorem with the first order condition, we obtain

$$J^F_h(h) = \varphi'(h)\Delta + \frac{1-s\Delta}{1+r\Delta} J^F_{h,+}(h'), \tag{A16}$$

which holds for all possible firm's hiring decisions (hiring, firing, inaction).

A.3.3 Wage schedule (Lemma 2)

The firm simultaniously bargains with all its workers each period as in bilateral Nash bargaining with equal bargaining weights, internalizing the effect of potential worker's departure on the wage rebargaining with all other workers. The Bellman equation characterizing the value to the worker of employment at a given firm with labor force h is given by:

$$J^E(h) - J^U = w(h)\Delta + \frac{1-s\Delta}{1+r\Delta}\big(J^E_+(h') - J^U_+\big) + \left(\frac{1}{1+r\Delta}J^U_+ - J^U\right),$$

and the surplus division implies:

$$J^E(h) - J^U = J^F_h(h) \tag{A17}$$

for all h and in each period of time (that is, including next period for workers that stay with the firm, independently of firm's hiring decision h'). Combining the above two expressions with (A16), we obtain a static differential equation in h for $w(h)$:

$$\varphi'(h) = w(h) - \Delta^U, \qquad \text{where} \qquad \Delta^U \equiv \frac{1}{\Delta}\left(J^U - \frac{1}{1+r\Delta}J^U_+\right).$$

Note that the values of continued employment within the firm have dropped out from both sides and, therefore, the wage determination is effectively static given the dynamics of the value of unemployment. The flow revenue function is defined as:

$$\varphi(h) \equiv R(h) - w(h)h - f_d - \iota f_x,$$

where we suppressed $(\iota;\theta)$ notation inside the revenue function defined in (8), as well as inside the wage schedule. Taking the derivative and substituting in the differential equation, we have:

$$R'(h) - w'(h)h - w(h) = w(h) - \Delta^U.$$

Rearranging, we have $2w(h) + w'(h)h = R'(h) + \Delta^U$, which we can integrate analytically by multiplying both sides by h:

$$w(h)h^2 = \int_0^h \left(R'(\tilde h) + \Delta^U\right)\tilde h \mathrm{d}\tilde h = \frac{\beta}{1+\beta}R(h)h + \frac{1}{2}\Delta^U h^2,$$

where we have used the power function structure of, $R(h) = \Theta^{1-\beta}h^\beta$, which implies $R'(h) = \beta\Theta^{1-\beta}h^{\beta-1}$. Note that we have set the constant of integration to zero to ensure that the wage bill with zero workers is zero, $w(h)h\big|_{h=0} = 0$. Dividing by h^2, we obtain:

$$w(h) = \frac{\beta}{1+\beta}\frac{R(h)}{h} + \frac{1}{2}\Delta^U. \tag{A18}$$

This corresponds to (16) in the text after we impose $J^U_+ = J^U$, so that $\Delta^U = rJ^U/(1+r\Delta)$, and use the approximation with $\Delta \approx 0$. This derivation provides a proof of Lemma 2. Substituting (A18) into the definition of $\varphi(h)$, we have:

$$\varphi(h) = \frac{1}{1+\beta}R(h) - \frac{1}{2}\Delta^U h - f_d - \iota f_x, \tag{A19}$$

which corresponds to (12) in the text, again using approximation $\Delta \approx 0$ for Δ^U.

A.3.4 Value to unemployed (Lemma 3a)

Value to unemployed worker is characterized by a Bellman equation similar to (A3):

$$J^U - \frac{1}{1+r\Delta}J^U_+ = b_u\Delta + \frac{x\Delta}{1+r\Delta}\mathbb{E}\big(J^E_+(h') - J^U_+\big), \tag{A20}$$

where the expectation $\mathbb{E}$ is taken across all potential employers. However, for all hiring firms (i.e., all potential employers) we have from (A17) together with firm optimization (A15):

$$J_+^E(h') - J_+^U = J_{h,+}^F(h') = \frac{1+r\Delta}{1-\delta\Delta}b.$$

Substituting this into the Bellman equation (A20), we have:

$$J^U - \frac{1}{1+r\Delta}J_+^U = \left(b_u + \frac{xb}{1-\delta\Delta}\right)\Delta. \tag{A21}$$

Since there always are some of the unemployed in the homogenous sector (Assumption 1), the indifference condition implies that $J^U = J_0^U$, which is constant over time (Lemma 1). Therefore, we have, using (A12):

$$J^U - \frac{1}{1+r\Delta}J_+^U = J_0^U - \frac{1}{1+r\Delta}J_{0,+}^U = \frac{rJ_0^U\Delta}{1+r\Delta} = \left(b_u + \frac{x_0 b_0}{1-\delta\Delta}\right)\Delta.$$

Combining the two expressions above results in $xb = x_0 b_0$. In view of Lemma 1 and relationship (10), this implies that (x, b) must be constant over time, just like (x_0, b_0). This derivation provides a proof for the first part of Lemma 3.

A.3.5 Optimal hiring (Lemma 3b)

Consider a firm hiring in a current period, as well as in the next period.[36] Then the first order condition (A15) together with the Envelope theorem (A16) can be written as:

$$\varphi'(h')\Delta = \frac{1+r\Delta}{1-\delta\Delta}b - (1-\sigma\Delta)b_+, \tag{A22}$$

and $h' > (1-\sigma\Delta)h$. From the first part of Lemma 3, we know that b is constant over time, and therefore we can rewrite:

$$\varphi'(h') = \frac{r+s}{1-\delta\Delta}b,$$

where we have used $(1-\sigma\Delta)(1-\delta\Delta) = (1-s\Delta)$, and similarly for h. Using the function form of $\varphi(h)$ defined in (A19), we have the optimal employment given by:

$$\frac{\beta}{1+\beta}\Theta^{1-\beta}h^{\beta-1} = \frac{1}{2}\Delta^U + \frac{r+s}{1-\delta\Delta}b = \frac{1}{2}\left[b_u + \frac{2(r+s)+x}{1-\delta\Delta}b\right],$$

where we used the definition of Δ^U and (A21) according to which:

$$\Delta^U = \frac{rJ^U}{1+r\Delta} = b_u + \frac{xb}{1-\delta\Delta}.$$

Solving for optimal employment results in:

$$h = \left(\frac{2\beta}{1+\beta}\left[b_u + \frac{2(r+s)+x}{1-\delta\Delta}b\right]^{-1}\right)^{\frac{1}{1-\beta}}\Theta, \tag{A23}$$

[36] Note that hiring in the consecutive periods of time is a typical outcome for firms, unless there is a sharp movement in aggregate variables. This is because the optimal employment evolves continuously in the aggregate variables, and a hiring firm in one period will likely need to replace, at least, partly the attrition next period, unless its optimal employment declines sharply with some aggregate variable.

which corresponds to (15), under the approximation $\Delta \approx 0$. This derivation completes the proof of Lemma 3. Finally, substituting (A23) into the wage schedule (A18) and using $R(h) = \Theta^{1-\beta}h^{\beta}$ from (8), we obtain the wage rate paid by the hiring firms:

$$w = b_u + \frac{(r+s+x)b}{1-\delta\Delta}, \tag{A24}$$

which corresponds to (16) in the text under the approximation $\Delta \approx 0$.

A.3.6 Value of a hiring firm (Lemma 4)

Now consider a firm which enters at time period t and hires in every period after entry, $h' > (1-\sigma\Delta)h$. Specializing the Bellman equation (A14) for this case, we have:

$$J^F(h) = \varphi(h)\Delta - b\big[h' - (1-\sigma\Delta)h\big] + \frac{1-\delta\Delta}{1+r\Delta}J^F_+(h'),$$

where h and h' satisfy the optimality condition (A15) for the hiring case, that is $\frac{1-\delta\Delta}{1+r\Delta}J^F_{h,+}(h') = b$. The Envelope theorem (A16) for this case can be written:

$$\frac{1+r\Delta}{1-\delta\Delta}b_{-1} = \varphi'(h)\Delta + (1-\sigma\Delta)b,$$

where the (-1)-subscript corresponds to the previous period. Multiplying this expression by h and subtracting from the Bellman equation above, we have:

$$J^F(h) - \frac{1+r\Delta}{1-\delta\Delta}b_{-1}h = \big[\varphi(h) - \varphi'(h)h\big]\Delta + \frac{1-\delta\Delta}{1+r\Delta}J^F_+(h') - bh', \tag{A25}$$

which defines a difference equation for $J^F(h) - \frac{1+r\Delta}{1-\delta\Delta}b_{-1}h$. Using the functional form for $\varphi(h)$ defined in (A19) and substituting optimal employment from (A23), we have:

$$\varphi(h) - \varphi'(h)h = \frac{1-\beta}{1+\beta}\Phi\Theta - f_d - \iota f_x,$$

where $\Phi \equiv \left(\frac{2\beta}{1+\beta}\left[b_u + \frac{2(r+s)+x}{1-\delta\Delta}b\right]^{-1}\right)^{\frac{1}{1-\beta}}$ and corresponds to the definition of Φ (15) in the text under the approximation $\Delta \approx 0$. Finally, we rewrite (A25) as:

$$J^V_{-1} = \frac{1-\delta\Delta}{1+r\Delta}\left[\frac{1-\beta}{1+\beta}\Phi\Theta - f_d - \iota f_x\right]\Delta + \frac{1-\delta\Delta}{1+r\Delta}J^V, \tag{A26}$$

where the value of a vacant firm equals

$$J^V \equiv \frac{1-\delta\Delta}{1+r\Delta}J^F_+(h') - bh',$$

since a firm with zero employees can pay a cost of bh' in the current period to obtain the value $J^F_+(h')$ next period, conditional on surviving with probability $(1-\delta\Delta)$. After an algebraic manipulation, and using approximation $\Delta \approx 0$, (A26) correspond to (18) in the text.[37] This completes the proof of Lemma 4.

[37] As $\Delta \to 0$, the following approximation is accurate (for optimal employment h):

$$J^F(h) = bh + J^V, \qquad (r+\delta)J^V - \dot{J}^V = \frac{1-\beta}{1+\beta}\Phi\Theta - f_d - \iota f_x.$$

A.4 Steady state

A.4.1 Productivity cutoffs and consumption index

In steady state both Φ and Θ are constant and therefore θ_d and θ_x are constant. Firms have natural attrition of labor force and hire in every period to offset it. Additionally, firms die at an exogenous rate δ and are replaced by new entrants, i.e. there is positive entry every period. As a result, Lemma 4 applies and the value of an entrant with productivity θ and zero employment is given by

$$J^V(\theta) = \max\left\{0, \frac{1-\delta\Delta}{r+\delta} \max_{\iota\in\{0,1\}} \left[\frac{1-\beta}{1+\beta}\Phi\Theta(\iota;\theta) - f_d - \iota f_x\right]\right\}. \tag{A27}$$

The two max operators in (A27) define the the entry/exit and the export cutoffs:

$$\max_{\iota\in\{0,1\}} \left[\frac{1-\beta}{1+\beta}\Phi\Theta(\iota;\theta_d) - \iota f_x\right] = f_d,$$
$$\frac{1-\beta}{1+\beta}\Phi\left[\Theta(1;\theta_x) - \Theta(0;\theta_x)\right] = f_x.$$

Additionally, as in Melitz (2003), we assume that $\tau\left(f_x/f_d\right)^{(1-\beta)/\beta} > 1$, which as we show below (see (A31)) ensures $\theta_x > \theta_d$. Under these circumstances, and using the definition of Θ in (8), we can rewrite the two cutoff conditions above as:

$$\frac{1-\beta}{1+\beta}\Phi Q^{-\frac{\beta-\zeta}{1-\beta}}\theta_d^{\frac{\beta}{1-\beta}} = f_d, \tag{A28}$$
$$\frac{1-\beta}{1+\beta}\Phi\tau^{-\frac{\beta}{1-\beta}}Q^{*-\frac{\beta-\zeta}{1-\beta}}\theta_x^{\frac{\beta}{1-\beta}} = f_x. \tag{A29}$$

We can use (A28)–(A29) to rewrite the value function of an entrant in (A27) as:

$$J^V(\theta) = \frac{1-\delta\Delta}{r+\delta}\left[f_d\left[(\theta/\theta_d)^{\frac{\beta}{1-\beta}} - 1\right]^+ + f_x\left[(\theta/\theta_x)^{\frac{\beta}{1-\beta}} - 1\right]^+\right],$$

where $[\cdot]^+ \equiv \max\{\cdot, 0\}$. Given this expression and the positive entry flow of firms, the free entry condition is

$$\int J^V(\theta)\mathrm{d}G(\theta) = f_e,$$

which under the Pareto distributional assumption can be simplified to:

$$\frac{1}{\frac{1-\beta}{\beta}k - 1}\left[f_d\theta_d^{-k} + f_x\theta_x^{-k}\right] = \frac{r+\delta}{1-\delta\Delta}f_e. \tag{A30}$$

Under the symmetry of the two countries (implying $Q = Q^*$), the free entry condition (A30) together with the two cutoff conditions (A28)– (A29) determine (θ_d, θ_x, Q) in steady state, just like in the static model. Taking the ratio of the two cutoff conditions we have:

$$\frac{\theta_x}{\theta_d} = \tau\left(\frac{f_x}{f_d}\right)^{\frac{1-\beta}{\beta}}. \tag{A31}$$

Note that (A30)–(A31) allow us to solve for (θ_d, θ_x) and then recover Q from (A28).

Given Q and the cutoffs, we can characterize individual decisions of every firm, as well as aggregate mass of firms M operating at every instant and aggregate employment H in the differentiated sector. In particular, we can rewrite (22) as:

$$H = M \int_{\theta_d} h(\theta)\mathrm{d}G(\theta)$$
$$= M\Phi^{1/\beta} Q^{-\frac{\beta-\zeta}{1-\beta}} \int_{\theta_d} \left[1 + \mathbf{1}_{\{\theta \geq \theta_x\}} \tau^{-\frac{\beta}{1-\beta}}\right] \theta^{\frac{\beta}{1-\beta}} \mathrm{d}G(\theta)$$

and (2) as:

$$Q^\beta = M \left(\int_{\theta_d}^{\infty} q_d(\theta)^\beta \mathrm{d}G(\theta) + \int_{\theta_x}^{\infty} q_x(\theta)^\beta \mathrm{d}G(\theta) \right)$$
$$= M\Phi Q^{-\beta\frac{\beta-\zeta}{1-\beta}} \int_{\theta_d} \left[1 + \mathbf{1}_{\{\theta \geq \theta_x\}} \tau^{-\frac{\beta}{1-\beta}}\right] \theta^{\frac{\beta}{1-\beta}} \mathrm{d}G(\theta),$$

where we have used (A23) for employment $h(\theta)$, $y(\theta) = \theta h(\theta)$, and the split of $y(\theta)$ into $q_d(\theta)$ and $q_x(\theta)$ from (A13). Combining the expressions for Q and H we have:

$$H = \Phi^{\frac{1-\beta}{\beta}} Q^\zeta,$$

corresponding to (30) in the text. Finally, M can be recovered from either expression. Additionally, under the Pareto distribution θ, we can simplify the integral in the expression for H:

$$H = M\Phi^{1/\beta} Q^{-\frac{\beta-\zeta}{1-\beta}} \theta_d^{\frac{\beta}{1-\beta}} \int_{\theta_d} \left[1 + \mathbf{1}_{\{\theta \geq \theta_x\}} \tau^{-\frac{\beta}{1-\beta}}\right] (\theta/\theta_d)^{\frac{\beta}{1-\beta}} \mathrm{d}G(\theta)$$
$$= M\Phi^{\frac{1-\beta}{\beta}} \frac{1+\beta}{1-\beta} f_d \left[\int_{\theta_d} (\theta/\theta_d)^{\frac{\beta}{1-\beta}} \mathrm{d}G(\theta) + \frac{f_x}{f_d} \int_{\theta_x} (\theta/\theta_x)^{\frac{\beta}{1-\beta}} \mathrm{d}G(\theta) \right]$$
$$= M\Phi^{\frac{1-\beta}{\beta}} \frac{1+\beta}{1-\beta} \frac{k}{k - \frac{\beta}{1-\beta}} \left[f_d \theta_d^{-k} + f_x \theta_x^{-k} \right] = M\Phi^{\frac{1-\beta}{\beta}} \frac{1+\beta}{\beta} k \frac{r+\delta}{1-\delta\Delta} f_e,$$

where the second line uses the cutoff condition (A28) and substitutes in (A31) to split the integral into two parts, the third line integrates using the properties of Pareto and the last equality utilizes the free entry condition (A30). Combining the two above expressions, we obtain (31) in the text.

A.4.2 Comparative statics across steady states

Consider a reduction in τ. We now characterize changes in both aggregate and firm-specific variables across the two steady states. To obtain analytical results for large changes in τ, we adopt the Pareto distribution. Then we can plug in the cutoff ratio (A31) into the free entry (A30) to solve for θ_d explicitly as a function of τ:

$$\theta_d = \left[\frac{\left(\frac{1-\beta}{\beta}k - 1\right)(r+\delta)/(1-\delta\Delta)}{1 + (f_d/f_x)^{\frac{1-\beta}{\beta}k-1} \tau^{-k}} \cdot \frac{f_e}{f_d} \right]^{1/k}, \tag{A32}$$

so that θ_d is an increasing function of τ and each change in θ_d can be linked to a corresponding change in τ. Therefore, we can express changes in other variables as function of the change in θ_d:

$$\frac{Q'}{Q} = \left(\frac{\theta_d'}{\theta_d}\right)^{\frac{\beta}{\beta-\zeta}} \qquad \text{and} \qquad \frac{H'}{H} = \frac{M'}{M} = \left(\frac{\theta_d'}{\theta_d}\right)^{\frac{\beta\zeta}{\beta-\zeta}},$$

which follow from (A28), (30) and (31).[38] Note that from (A32), neither level nor change in θ_d depend on the level of search frictions (reflected in equilibrium values of x and b), and therefore the effect of a change in τ on the

[38]The number of active firms, equal to $M\theta_d^{-k}$, increases when τ decreases if and only if $\beta\zeta > k(\beta - \zeta)$.

change in the aggregate variables such as Q, H and M also does not depend on the frictions in the labor market.

Finally, consider the firm-specific employment outcomes:

$$\begin{aligned}h(\theta) &= \Phi^{1/\beta}\left[1+\mathbf{1}_{\{\theta\geq\theta_x\}}\tau^{-\frac{\beta}{1-\beta}}\right]Q^{-\frac{\beta-\zeta}{1-\beta}}\theta^{\frac{\beta}{1-\beta}} \\ &= \frac{1+\beta}{1-\beta}\Phi^{\frac{1-\beta}{\beta}}\left[f_d\left(\theta/\theta_d\right)^{\frac{\beta}{1-\beta}}+f_x\left(\theta/\theta_x\right)^{\frac{\beta}{1-\beta}}\right], \qquad \text{for} \quad \theta\geq\theta_d,\end{aligned}$$

where the second line substitutes in the cutoff condition (A28) and the uses the cutoff ratio (A31) to split the two terms. For $\theta<\theta_d$, $h(\theta)=0$. Comparing the two steady state with τ and $\tau'<\tau$, we have four types of firms: (i) firms with $\theta\in[\theta_d,\theta_d')$ exit; (ii) firms with $\theta\in[\theta_d',\theta_x')$ reduce employment and stay in the industry as non-exporters; (iii) firms with $\theta\in[\theta_x',\theta_x)$ increase employment and start to export; and (iv) firms with $\theta\geq\theta_x$ continue to export, reduce domestic sales, increase export sales and increase overall employment:

$$\frac{h'(\theta)}{h(\theta)}=\left(\frac{\theta_d}{\theta_d'}\right)^{\frac{\beta}{1-\beta}}\cdot\left[\begin{array}{cll} 0 & =0, & \text{for} \quad \theta\in[\theta_d,\theta_d'), \\ 1 & <1, & \text{for} \quad \theta\in[\theta_d',\theta_x'), \\ \left[1+\tau'^{-\beta/(1-\beta)}\right] & >1, & \text{for} \quad \theta\in[\theta_x',\theta_x), \\ \frac{1+\tau'^{-\beta/(1-\beta)}}{1+\tau^{-\beta/(1-\beta)}} & >1, & \text{for} \quad \theta\geq\theta_x, \end{array}\right. \tag{A33}$$

Note that θ_x decreases when θ_d increases to satisfy the free entry condition (A30), and hence $\theta_x'<\theta_x$. The last inequality is ensured by the parameter restriction $k>\beta/(1-\beta)$ and the free entry condition (A30), which requires that $f_d\theta_d^{-k}+f_x\theta_x^{-k}$ is constant, and hence $f_d\theta_d^{-\beta/(1-\beta)}+f_x\theta_x^{-\beta/(1-\beta)}$ must increase with θ_d, and hence so does $h(\theta)$ for every $\theta\geq\theta_x$. Finally, it is easy to see that the employment increase for the firms with $\theta\in[\theta_x',\theta_x)$ is strictly larger than for the firms with $\theta\geq\theta_x$.

B Transition after Trade Liberalization

B.1 Aggregate Consumption and Entry Dynamics

Consider a reduction in τ in a symmetric two-country world starting from an initial steady state, as described above.

Proposition A1 *Following a reduction in τ, assuming that $U_0>0$ at each instant, there exists a dynamic equilibrium path along which (θ_d,θ_x,Q) jump on impact to their steady state level and have no dynamics thereafter.*

Proof: Conjecture that (θ_d,θ_x,Q) jump on impact and stay at their new steady state level, which we denote by (θ_d',θ_x',Q') and characterized above. We now verify the internal consistency of this conjecture. Further conjecture (and verify later) that there is positive entry in every instance. Under these conjectures, the free entry condition as can be written following (A27) as:

$$\int_{\theta_d'}^{\infty}\int_{t_0}^{\infty}e^{-(r+\delta)(t-t_0)}\left[\frac{1-\beta}{1+\beta}\Phi(b)\left[1+\mathbf{1}_{\{\theta\geq\theta_x'\}}(\tau')^{-\frac{\beta}{1-\beta}}\right](Q')^{-\frac{\beta-\zeta}{1-\beta}}\theta^{\frac{\beta}{1-\beta}}-f_d-\mathbf{1}_{\{\theta\geq\theta_x'\}}f_x\right]\mathrm{d}t\mathrm{d}G(\theta)=f_e,$$

for all $t_0\geq 0$ and where τ' is the new lower level of variable trade costs which drops unexpectedly at $t=0$. Note that b stays unchanged throughout the transition dynamics as follows from Lemma 3 assuming $U_0>0$ at each point in time (which ensures unchanged b_0 and worker indifference then ensures unchanged b).

The cutoff conditions (A28)–(A29) can be written in this case as:

$$\begin{aligned}\Phi(b)(Q')^{-\frac{\beta-\zeta}{1-\beta}}(\theta_d')^{\frac{\beta}{1-\beta}} &= f_d, \\ \frac{\theta_x'}{\theta_d'} &= \tau\left(\frac{f_x}{f_d}\right)^{\frac{1-\beta}{\beta}},\end{aligned}$$

which implicitly defines (θ'_d, θ'_x) as a function of (Q', τ', b), and this solution is unique by IFT. We can rewrite the free entry condition as:

$$\int_{t_0}^{\infty} e^{-(r+\delta)(t-t_0)} \left\{ f_d \int_{\theta'_d}^{\infty} \left[\left(\frac{\theta}{\theta'_d} \right)^{\frac{\beta}{1-\beta}} - 1 \right] \mathrm{d}G(\theta) + \int_{\theta'_x}^{\infty} \left[\left(\frac{\theta}{\theta'_x} \right)^{\frac{\beta}{1-\beta}} - 1 \right] \mathrm{d}G(\theta) \right\} \mathrm{d}t = f_e$$

We, therefore, can rewrite it as:

$$\int_{t_0}^{\infty} e^{-(r+\delta)(t-t_0)} f(Q', \tau'; b) \mathrm{d}t = f_e$$

where

$$f(Q', \tau'; b) \equiv f_d \int_{\theta_d(Q',\tau';b)}^{\infty} \left[\left(\frac{\theta}{\theta_d(Q', \tau'; b)} \right)^{\frac{\beta}{1-\beta}} - 1 \right] \mathrm{d}G(\theta) + \int_{\theta_x(Q',\tau';b)}^{\infty} \left[\left(\frac{\theta}{\theta_x(Q', \tau'; b)} \right)^{\frac{\beta}{1-\beta}} - 1 \right] \mathrm{d}G(\theta).$$

Since an increase in Q' leads to an increase in both θ'_d and θ'_x (for given τ' and b), $f(\cdot)$ is monotonically decreasing in Q'.

Now take the derivative of free entry with respect to t_0:

$$-f(Q', \tau'; b) + (r + \delta) \int_{t_0}^{\infty} e^{-(r+\delta)(t-t_0)} f(Q', \tau'; b) \mathrm{d}t = 0,$$

but not that the integral here is simply f_e. Therefore, for every t_0

$$f(Q', \tau'; b) = (r + \delta) f_e,$$

which implies that indeed there is a unique (due to monotonicity of $f(\cdot)$ in Q') solution Q' for any given (τ', b), and therefore our conjecture that $(\theta'_d, \theta'_x, Q')$ jumps on impact and stays unchanged thereafter is justified.

Finally, we need to verify that there is (1) positive entry in every period and (2) active entrants hire in every period. The second conjecture is easy: given (Q', b), all entrants either exit immediately (for $\theta < \theta'_d$) or stay for good (for $\theta \geq \theta'_d$) and export from the very beginning for good (for $\theta \geq \theta'_x$). Furthermore, they immediately jump to their steady-state size $h_0 = \Phi(b)^{1/\beta}\Theta(Q', \theta)$ and hence only replace the separated workers at rate σh_0.

Sufficient condition for an increase in M_0 on impact: We want to derive a sufficient condition for $M_{0+} > M_0$. Under the Pareto productivity assumption, for $t < 0$, the definition of Q can be written as:

$$Q_0^{\zeta} = M_0 \frac{1+\beta}{\beta} k(r + \delta) f_e.$$

Now imagine that $Q_{0+} = Q'$, firms with $\theta \geq \theta'_x$ jump to their new steady-state employment and output, while firms with $\theta < \theta_x$ stay at their old employment. Using the expression for optimal employment (**??**), it must be then that

$$(Q')^{\zeta} \geq M_0 \Phi(b) Q^{-\frac{\beta-\zeta}{1-\beta}} \left[\left(\frac{Q_0}{Q'} \right)^{-\beta \frac{\beta-\zeta}{1-\beta}} \int_{\theta_d}^{\theta'_x} \theta^{\frac{\beta}{1-\beta}} \mathrm{d}G(\theta) + \left(1 + \tau^{-\frac{\beta}{1-\beta}} \right) \int_{\theta'_x}^{\infty} \theta^{\frac{\beta}{1-\beta}} \mathrm{d}G(\theta) \right],$$

otherwise, there is no entry at $t = 0$ and hence the conjecture that $Q_0 = Q'$ is violated. Using the free-entry condition in the old and new steady states, we can rewrite:

$$(Q')^{\zeta} \geq M_0 \frac{1+\beta}{1-\beta} \left[\left(\frac{Q_0}{Q'} \right)^{\beta-\zeta} f_d \int_{\theta_d}^{\theta'_x} \left(\frac{\theta}{\theta_d} \right)^{\frac{\beta}{1-\beta}} \mathrm{d}G(\theta) + \int_{\theta'_x}^{\infty} \left[f_d \left(\frac{\theta}{\theta'_d} \right)^{\frac{\beta}{1-\beta}} + f_x \left(\frac{\theta}{\theta'_x} \right)^{\frac{\beta}{1-\beta}} \right] \mathrm{d}G(\theta) \right],$$

and manipulating further:

$$(Q')^{\zeta} \geq M_0 \frac{1+\beta}{1-\beta} \Bigg[\left(\frac{\theta_d'}{\theta_d}\right)^{-\beta} f_d \theta_d^{-k} \frac{k}{k-\frac{\beta}{1-\beta}} \left[1 - \left(\frac{\theta_d'}{\theta_d}\right)^{\frac{\beta}{1-\beta}-k}\right] + \left(\left(\frac{\theta_d'}{\theta_d}\right)^{\frac{\beta^2}{1-\beta}} - 1\right) f_d (\theta_d')^{-k} \frac{k}{k-\frac{\beta}{1-\beta}} \left[1 - \left(\frac{\theta_x'}{\theta_d'}\right)^{\frac{\beta}{1-\beta}-k}\right] + \frac{1-\beta}{\beta} k(r+\delta) f_e \Bigg]$$

where we evaluated the integrals using the Pareto assumption and used the facts that $Q'/Q_0 = (\theta_d'/\theta_d)^{\beta/(\beta-\zeta)}$ and

$$f_d \int_{\theta_d'}^{\infty} (\theta/\theta_d')^{\frac{\beta}{1-\beta}} \,\mathrm{d}G(\theta) + f_x \int_{\theta_x'}^{\infty} (\theta/\theta_x')^{\frac{\beta}{1-\beta}} \,\mathrm{d}G(\theta) = \frac{k\left[f_d(\theta_d')^{-k} + f_x(\theta_x')^{-k}\right]}{k-\beta/(1-\beta)} = \frac{1-\beta}{\beta} k(r+\delta) f_e$$

from the free-entry condition. We now take this last term outside the bracket, and using $\theta_x'/\theta_d' = \tau'(f_x/f_d)^{(1-\beta)/\beta}$, arrive at:

$$(Q')^{\zeta} \geq M_0 \frac{1+\beta}{\beta} k(r+\delta) f_e \Bigg[\left(\frac{\theta_d'}{\theta_d}\right)^{k-\beta} \frac{1}{1+(\tau')^{-k}\left(f_x/f_d\right)^{1-k\frac{1-\beta}{\beta}}} \left[1 - \left(\frac{\theta_d'}{\theta_d}\right)^{\frac{\beta}{1-\beta}-k}\right] + \left(\left(\frac{\theta_d'}{\theta_d}\right)^{\frac{\beta^2}{1-\beta}} - 1\right) \frac{1}{1+(\tau')^{-k}\left(f_x/f_d\right)^{1-k\frac{1-\beta}{\beta}}} \left[1 - (\tau')^{\frac{\beta}{1-\beta}-k} \left(\frac{f_x}{f_d}\right)^{1-k\frac{1-\beta}{\beta}}\right] + 1 \Bigg]$$

Now, dividing through by the expression for Q_0^{ζ}, and denoting $z \equiv \theta_d'/\theta_d$, we have:

$$z^{\frac{\beta\zeta}{\beta-\zeta}} \geq 1 + \frac{1}{1+(\tau')^{-k}\left(f_x/f_d\right)^{1-k\frac{1-\beta}{\beta}}} \left[z^{k-\beta} - z^{\frac{\beta^2}{1-\beta}}\right] + \frac{1-(\tau')^{\frac{\beta}{1-\beta}-k}\left(f_x/f_d\right)^{1-k\frac{1-\beta}{\beta}}}{1+(\tau')^{-k}\left(f_x/f_d\right)^{1-k\frac{1-\beta}{\beta}}} \left[z^{\frac{\beta^2}{1-\beta}} - 1\right],$$

or equivalently

$$\left(1+(\tau')^{-k}\left(f_x/f_d\right)^{1-k\frac{1-\beta}{\beta}}\right)\left[z^{\frac{\beta\zeta}{\beta-\zeta}} - 1\right] \geq \left[z^{k-\beta} - 1\right] - (\tau')^{\frac{\beta}{1-\beta}-k}\left(f_x/f_d\right)^{1-k\frac{1-\beta}{\beta}} \left[z^{\frac{\beta^2}{1-\beta}} - 1\right],$$

which is the condition in the text. When $z \approx 1$, we can use the approximation $z^{\alpha} - 1 \approx \alpha(z-1)$ to simplify further to:

$$(\tau')^{-k}\left(f_x/f_d\right)^{1-k\frac{1-\beta}{\beta}} \left[\frac{\beta\zeta}{\beta-\zeta} + \frac{\beta^2}{1-\beta}(\tau')^{\frac{\beta}{1-\beta}}\right] \geq k - \frac{\beta^2}{\beta-\zeta},$$

as in the text. ■

In the next section, we can characterize the dynamics of individual firm-level employment during transition. Given the dynamics of employment of all firms we can calculate supply of each variety to each market, and back out the required dynamics of M_t to maintain $Q_t \equiv Q_t'$, taking into account gradual exit of the old firms as time passes. This then also yields the dynamics of aggregate employment H_t.

The last step is to verify under which circumstance $U_0 > 0$ throughout transition. From the dynamics of H_t, we know the dynamics of aggregate sectoral vacancies V_t. Given a constant labor market tightness x, we can recover the corresponding dynamics of U_t (which keeps U_t/V_t constant throughout the transition). We then need to require that $U_0(t)$ is large enough to accommodate the spike in U_t for $t \geq 0$ by reallocation of unemployed across sectors, which is frictionless.

B.2 Firm values and employment after trade liberalization

For the expressions here, we use continues time approximations ($\Delta \to 0$) to reduce notational burden. Upon trade liberalization in a stationary environment, i.e. $b_t \equiv \bar{b}$ and $Q_t \equiv Q_+$ for all $t \geq 0$, a firm has the following non-dominated options:[39]

Table A1: Incumbents Strategies

Name	Productivity	Strategy
X	$\theta \geq \bar{\theta}'_x$	Increase employment from $h(\theta)$ to $\bar{h}'_x(\theta)$ on impact, maintain this level and serve both markets at any $t > 0$.
D	$\theta \in [\bar{\theta}'_d, \bar{\theta}'_x)$	Gradually shrink employment from $h(\theta)$ to $\bar{h}'_d(\theta)$ by means of exogenous attrition only, maintain this level of employment and serve domestic market.
FD	$\theta \in [\bar{\theta}'_d, \bar{\theta}'_x)$	Fire employees on impact until $\check{h}(\theta)$ and then gradually shrink to $\bar{h}'_d(\theta)$, maintain this level and serve the domestic market.
E	$\theta \in [\check{\theta}'_d, \bar{\theta}'_d)$	Gradually shrink employment from $h(\theta)$ to $\underline{h}'(\theta)$ by means of exogenous attrition only, and then exit.
FE	$\theta \in [\underline{\theta}'_d, \check{\theta}'_d)$	Fire employees until $\check{h}_e(\theta)$. Then gradually shrink to $\underline{h}'(\theta)$ and exit.
EI	$\theta \in [\theta_d, \underline{\theta}'_d)$	Exit on impact, which yields value $J_0^F(h(\theta), \theta) = 0$.

X: Increase employment on impact and serve both markets

The new steady-state employment for the firms that serve both domestic and foreign markets is equal to

$$\bar{h}'_x(\theta) = \Phi^{1/\beta}\left[1 + (\tau')^{-\frac{\beta}{1-\beta}}\right] Q'^{-\frac{\beta-\zeta}{1-\beta}} \theta^{\frac{\beta}{1-\beta}} \tag{A34}$$

where τ' is a new level of trade costs. The value of this strategy is equal to (with approximation $\Delta \approx 0$)

$$\begin{aligned} J_X^F(\theta) &= bh(\theta) + \frac{1}{r+\delta}\left[\varphi(\bar{h}'_x(\theta)) - \varphi'(\bar{h}'_x(\theta))\bar{h}'_x(\theta)\right] \\ &= bh(\theta) + \frac{1}{r+\delta}\left[\frac{1-\beta}{1+\beta}\Phi\left(1 + \tau'^{-\frac{\beta}{1-\beta}}\right) Q'^{-\frac{\beta-\zeta}{1-\beta}} \theta^{\frac{\beta}{1-\beta}} - f_d - f_x\right] \end{aligned} \tag{A35}$$

D: Gradually shrink employment and serve domestic market

After facing the shock, the firm shrinks its employment up to new steady-state level $\bar{h}'_d(\theta)$, and it takes time

$$\bar{T} = -\frac{1}{\sigma}\log\left(\frac{\bar{h}'_d(\theta)}{h(\theta)}\right) = \frac{\beta-\zeta}{\sigma(1-\beta)}\log\left(\frac{Q'}{Q}\right) \tag{A36}$$

[39]It is easy to check that all alternative options are strictly dominated by one of the options offered here.

which doesn't depend on productivity θ. This new level of employment is defined as

$$\bar{h}'_d(\theta) = \Phi^{1/\beta} Q'^{-\frac{\beta-\zeta}{1-\beta}} \theta^{\frac{\beta}{1-\beta}} \tag{A37}$$

and employment in each period $t = 1, 2, \ldots, \bar{T}$ is given by

$$h_t(\theta) = e^{-\sigma t} h(\theta) \tag{A38}$$

The value of this strategy is given by

$$J_D^F(\theta) = \int_0^{\bar{T}} e^{-(r+\delta)t}(R(h_t) - w(h_t)h_t - f_d)dt + e^{-(r+\delta)\bar{T}} J_D^F(\bar{h}'_d(\theta)) \tag{A39}$$

The steady-state value for $J_D^F(\bar{h}'_d(\theta))$ is equal to (using equation (A27))

$$J_D^F(\bar{h}'_d(\theta)) = b\bar{h}'_d(\theta) + J^V = b\bar{h}'_d(\theta) + \frac{1}{r+\delta}\left(\frac{1-\beta}{1+\beta}\Phi\Theta'_d - f_d\right)$$

Therefore, the value function of the firm

$$\begin{aligned} J_D^F(\theta) = &\left[\frac{1 - e^{-(r+\delta+\beta\sigma)\bar{T}}}{r+\delta+\beta\sigma}\right] \frac{\Theta_d'^{1-\beta} h(\theta)^\beta}{1+\beta} - \left[\frac{1 - e^{-(r+\delta+\sigma)\bar{T}}}{r+\delta+\sigma}\right] \frac{(b_u + xb)}{2} h(\theta) - \left[\frac{1 - e^{-(r+\delta)\bar{T}}}{r+\delta}\right] f_d \\ &+ e^{-(r+\delta)\bar{T}} \left[b\bar{h}'_d(\theta) + \frac{1}{r+\delta}\left(\frac{1-\beta}{1+\beta}\Phi\Theta'_d - f_d\right)\right] \end{aligned} \tag{A40}$$

From equation (A18), the dynamic wage during the transition is equal to

$$\begin{aligned} w_t(h_t) &= \frac{\beta}{1+\beta}\frac{R(h_t)}{h_t} + \frac{1}{2}\Delta^U \\ &= \frac{\beta}{1+\beta}\Phi^{\frac{\beta-1}{\beta}}\left(\frac{Q'}{Q}\right)^{-(\beta-\zeta)} e^{\sigma(1-\beta)t} + \frac{1}{2}\left(b_u + \frac{xb}{1-\delta\Delta}\right) \end{aligned} \tag{A41}$$

In case, the firm is facing binding minimum wage ($w_t \leq w_m$) after the shock, the value of strategy is different. Let's denote the time T_m such that

$$w_{T_m} = w_m \tag{A42}$$

Then, the value of the strategy is equal to

$$\begin{aligned} J_D^F(\theta) = &\int_0^{T_m} e^{-(r+\delta)t}(R(h_t) - w_m h_t - f_d)dt + \int_{T_m}^{\bar{T}} e^{-(r+\delta)t}(R(h_t) - w(h_t)h_t - f_d)dt \\ &+ e^{-(r+\delta)\bar{T}} J_D^F(\bar{h}'_d(\theta)) \end{aligned} \tag{A43}$$

After integration, it is equal to

$$\begin{aligned} J_D^F(\theta) = &\left[\frac{1 - e^{-(r+\delta+\beta\sigma)T_m}}{r+\delta+\beta\sigma}\right] \Theta_d'^{1-\beta} h(\theta)^\beta - \left[\frac{1 - e^{-(r+\delta+\sigma)T_m}}{r+\delta+\sigma}\right] w_m h(\theta) \\ &+ \left[\frac{e^{-(r+\delta+\beta\sigma)T_m} - e^{-(r+\delta+\beta\sigma)\bar{T}}}{r+\delta+\beta\sigma}\right] \frac{\Theta_d'^{1-\beta} h(\theta)^\beta}{1+\beta} - \left[\frac{e^{-(r+\delta+\sigma)T_m} - e^{-(r+\delta+\sigma)\bar{T}}}{r+\delta+\sigma}\right] \frac{(b_u + xb)}{2} h(\theta) \\ &- \left[\frac{1 - e^{-(r+\delta)\bar{T}}}{r+\delta}\right] f_d + e^{-(r+\delta)\bar{T}} \left[b\bar{h}'_d(\theta) + \frac{1}{r+\delta}\left(\frac{1-\beta}{1+\beta}\Phi\Theta'_d - f_d\right)\right] \end{aligned} \tag{A44}$$

FD: Fire and Gradually shrink

Alternatively, the firm can first fire employees until $\check{h}(\theta)$ and then gradually shrink to $\bar{h}'_d(\theta)$. The time to reach $\bar{h}'_d(\theta)$ is defined as

$$\check{T} = -\frac{1}{\sigma}\log\left(\frac{\bar{h}'_d(\theta)}{\check{h}(\theta)}\right) \tag{A45}$$

from which employment in each period t is given by

$$h_t = e^{-\sigma t}\check{h}(\theta) = e^{-\sigma(t-\check{T})}\bar{h}'_d(\theta) \tag{A46}$$

The value of this strategy is given by

$$\begin{aligned}
J^F_{FD}(\theta) &= \int_0^{\check{T}} e^{-(r+\delta)t}(R(h_t) - w(h_t)h_t - f_d)dt + e^{-(r+\delta)\check{T}} J^F_{FD}(\bar{h}'_d(\theta)) \\
&= \left[\frac{1-e^{-(r+\delta+\beta\sigma)\check{T}}}{r+\delta+\beta\sigma}\right]\frac{\Theta'^{1-\beta}_d \check{h}(\theta)^\beta}{1+\beta} - \left[\frac{1-e^{-(r+\delta+\sigma)\check{T}}}{r+\delta+\sigma}\right]\frac{(b_u+xb)}{2}\check{h}(\theta) - \left[\frac{1-e^{-(r+\delta)\check{T}}}{r+\delta}\right] f_d \\
&+ e^{-(r+\delta)\check{T}}\left[b\bar{h}'_d(\theta) + \frac{1}{r+\delta}\left(\frac{1-\beta}{1+\beta}\Phi\Theta'_d - f_d\right)\right]
\end{aligned}$$

Now we can plug in $\check{h}(\theta) = e^{\sigma\check{T}}\bar{h}'_d(\theta)$

$$\begin{aligned}
J^F_{FD}(\theta) &= \left[\frac{1-e^{-(r+\delta+\beta\sigma)\check{T}}}{r+\delta+\beta\sigma}\right]\frac{\Theta'^{1-\beta}_d \bar{h}'_d(\theta)^\beta}{1+\beta}e^{\beta\sigma\check{T}} - \left[\frac{1-e^{-(r+\delta+\sigma)\check{T}}}{r+\delta+\sigma}\right]\frac{(b_u+xb)}{2}\bar{h}'_d(\theta)e^{\sigma\check{T}} \\
&- \left[\frac{1-e^{-(r+\delta)\check{T}}}{r+\delta}\right] f_d + e^{-(r+\delta)\check{T}}\left[b\bar{h}'_d(\theta) + \frac{1}{r+\delta}\left(\frac{1-\beta}{1+\beta}\Phi\Theta'_d - f_d\right)\right] \\
&= \Phi^{1/\beta}Q'^{-\frac{\beta-\zeta}{1-\beta}}\theta^{\frac{\beta}{1-\beta}}\left[\frac{1}{1+\beta}\Phi^{-\frac{1-\beta}{\beta}}\left(\frac{e^{\beta\sigma\check{T}} - e^{-(r+\delta)\check{T}}}{r+\delta+\beta\sigma} + \frac{e^{-(r+\delta)\check{T}}}{r+\delta}(1-\beta)\right)\right. \\
&\left. + e^{-(r+\delta)\check{T}}b - \frac{b_u+xb}{2}\frac{e^{\sigma\check{T}} - e^{-(r+\delta)\check{T}}}{r+\delta+\sigma}\right] - \frac{f_d}{r+\delta}
\end{aligned} \tag{A47}$$

Since the last term is constant and the multiplier of the first term can be subtracted when doing maximization, $\check{T}$ doesn't depend on firm's productivity θ.

In order for this option to be feasible we require $\check{T} < \overline{T}$, or, equivalently, current level of employment should be $h > \check{h}$. Otherwise, all firms with productivity $\theta \in [\bar{\theta}'_d, \bar{\theta}'_x)$ gradually shrink without firing first.

Analogous to the previous strategy, when the firm faces binding minimum wage, the value of the strategy becomes

$$\begin{aligned}
J^F_{FD}(\theta) &= \left[\frac{1-e^{-(r+\delta+\beta\sigma)T_m}}{r+\delta+\beta\sigma}\right]\Theta'^{1-\beta}_d \bar{h}'_d(\theta)^\beta e^{\beta\sigma\check{T}} - \left[\frac{1-e^{-(r+\delta+\sigma)T_m}}{r+\delta+\sigma}\right] w_m\bar{h}'_d(\theta)e^{\sigma\check{T}} \\
&+ \left[\frac{e^{-(r+\delta+\beta\sigma)T_m} - e^{-(r+\delta+\beta\sigma)\check{T}}}{r+\delta+\beta\sigma}\right]\frac{\Theta'^{1-\beta}_d \bar{h}'_d(\theta)^\beta}{1+\beta}e^{\beta\sigma\check{T}} \\
&- \left[\frac{e^{-(r+\delta+\sigma)T_m} - e^{-(r+\delta+\sigma)\check{T}}}{r+\delta+\sigma}\right]\frac{(b_u+xb)}{2}\bar{h}'_d(\theta)e^{\sigma\check{T}} - \left[\frac{1-e^{-(r+\delta)\check{T}}}{r+\delta}\right] f_d \\
&+ e^{-(r+\delta)\check{T}}\left[b\bar{h}'_d(\theta) + \frac{1}{r+\delta}\left(\frac{1-\beta}{1+\beta}\Phi\Theta'_d - f_d\right)\right]
\end{aligned}$$

or, equivalently

$$J_{FD}^F(\theta) = \bar{h}_d'(\theta)\Bigg(\left[\frac{1-e^{-(r+\delta+\beta\sigma)T_m}}{r+\delta+\beta\sigma}\right]\Phi^{\frac{\beta-1}{\beta}}e^{\beta\sigma\check{T}} - \left[\frac{1-e^{-(r+\delta+\sigma)T_m}}{r+\delta+\sigma}\right]w_m e^{\sigma\check{T}}$$
$$+ \left[\frac{e^{-(r+\delta+\beta\sigma)T_m} - e^{-(r+\delta+\beta\sigma)\check{T}}}{r+\delta+\beta\sigma}\right]\frac{\Phi^{\frac{\beta-1}{\beta}}}{1+\beta}e^{\beta\sigma\check{T}} - \left[\frac{e^{-(r+\delta+\sigma)T_m} - e^{-(r+\delta+\sigma)\check{T}}}{r+\delta+\sigma}\right]\frac{(b_u+xb)}{2}e^{\sigma\check{T}}$$
$$+ e^{-(r+\delta)\check{T}}b + \frac{e^{-(r+\delta)\check{T}}}{r+\delta}\frac{1-\beta}{1+\beta}\Phi^{\frac{\beta-1}{\beta}}\Bigg) - \frac{f_d}{r+\delta} \tag{A48}$$

E: Gradually shrink employment and exit

The firm exits the market at $\underline{h}'(\theta)$ which can be found from

$$\varphi(\underline{h}'(\theta),\theta) = \frac{1}{1+\beta}\Theta_d^{1-\beta}\underline{h}'(\theta)^\beta - \frac{1}{2}(b_u+xb)\underline{h}'(\theta) - f_d = 0 \tag{A49}$$

or, if the firm is facing binding minimum wage

$$\Theta_d^{1-\beta}\underline{h}'(\theta)^\beta - w_m\underline{h}'(\theta) - f_d = 0 \tag{A50}$$

The time to reach this level $\underline{h}'(\theta)$ is defined as

$$\overline{T}_e = -\frac{1}{\sigma}\log\left(\frac{\underline{h}'(\theta)}{h(\theta)}\right) \tag{A51}$$

and the value of the strategy is equal to

$$J_E^F(\theta) = \int_0^{\overline{T}_e} e^{-(r+\delta)t}(R(h_t) - w(h_t)h_t - f_d)dt$$

where $h_t = e^{-\sigma t}h(\theta)$. Hence,

$$J_E^F(\theta) = \int_0^{\overline{T}_e} e^{-(r+\delta+\beta\sigma)t}\left(\frac{\Theta_d^{1-\beta}h(\theta)^\beta}{1+\beta}\right)dt - \int_0^{\overline{T}_e} e^{-(r+\delta+\sigma)t}\frac{(b_u+xb)}{2}h(\theta)dt - \int_0^{\overline{T}_e} e^{-(r+\delta)t}f_d dt$$
$$= \left[\frac{1-e^{-(r+\delta+\beta\sigma)\overline{T}_e}}{r+\delta+\beta\sigma}\right]\frac{\Theta_d^{1-\beta}h(\theta)^\beta}{1+\beta} - \left[\frac{1-e^{-(r+\delta+\sigma)\overline{T}_e}}{r+\delta+\sigma}\right]\frac{(b_u+xb)}{2}h(\theta) - \left[\frac{1-e^{-(r+\delta)\overline{T}_e}}{r+\delta}\right]f_d \tag{A52}$$

With the binding minimum wage, the value of the strategy becomes

$$J_E^F(\theta) = \left[\frac{1-e^{-(r+\delta+\beta\sigma)T_{m,e}}}{r+\delta+\beta\sigma}\right]\Theta_d'^{1-\beta}h(\theta)^\beta - \left[\frac{1-e^{-(r+\delta+\sigma)T_{m,e}}}{r+\delta+\sigma}\right]w_m h(\theta)$$
$$+ \left[\frac{e^{-(r+\delta+\beta\sigma)T_{m,e}} - e^{-(r+\delta+\beta\sigma)\overline{T}_e}}{r+\delta+\beta\sigma}\right]\frac{\Theta_d'^{1-\beta}h(\theta)^\beta}{1+\beta} - \left[\frac{e^{-(r+\delta+\sigma)T_{m,e}} - e^{-(r+\delta+\sigma)\overline{T}_e}}{r+\delta+\sigma}\right]\frac{(b_u+xb)}{2}h(\theta)$$
$$- \left[\frac{1-e^{-(r+\delta)\overline{T}_e}}{r+\delta}\right]f_d \tag{A53}$$

where time $T_{m,e}$ is the time while the minimum wage is binding

$$w_{T_{m,e}} = w_m \tag{A54}$$

FE: Fire and Gradually shrink to exit

First, the firm fires workers up to level $\check{h}_e(\theta)$. Then it gradually shrinks and exits at the level $\underline{h}'(\theta)$ found in the previous section. The time to reach $\underline{h}'(\theta)$ is given by

$$\check{T}_e = -\frac{1}{\sigma}\log\left(\frac{\underline{h}'(\theta)}{\check{h}_e(\theta)}\right) \tag{A55}$$

From this equation $\check{h}_e(\theta)$ can be written as

$$\check{h}_e(\theta) = e^{\sigma \check{T}_e}\underline{h}'(\theta) \tag{A56}$$

The value of the strategy is equal to ($h_t = e^{-\sigma t}\check{h}_e(\theta) = e^{-\sigma(t-\check{T}_e)}\underline{h}'(\theta)$)

$$\begin{aligned} J^F_{FE}(h) &= \int_0^{\check{T}_e} e^{-(r+\delta)t}(R(h_t) - w(h_t)h_t - f_d)dt \\ &= \left[\frac{1-e^{-(r+\delta+\beta\sigma)\check{T}_e}}{r+\delta+\beta\sigma}\right]\frac{\Theta_d^{1-\beta}\check{h}_e(\theta)^\beta}{1+\beta} - \left[\frac{1-e^{-(r+\delta+\sigma)\check{T}_e}}{r+\delta+\sigma}\right]\frac{(b_u+xb)}{2}\check{h}_e(\theta) - \left[\frac{1-e^{-(r+\delta)\check{T}_e}}{r+\delta}\right]f_d \\ &= \left[\frac{1-e^{-(r+\delta+\beta\sigma)\check{T}_e}}{r+\delta+\beta\sigma}\right]\frac{\Theta_d^{1-\beta}\underline{h}'(\theta)^\beta}{1+\beta}e^{\beta\sigma\check{T}_e} - \left[\frac{1-e^{-(r+\delta+\sigma)\check{T}_e}}{r+\delta+\sigma}\right]\frac{(b_u+xb)}{2}e^{\sigma\check{T}_e}\underline{h}'(\theta) \\ &- \left[\frac{1-e^{-(r+\delta)\check{T}_e}}{r+\delta}\right]f_d \end{aligned} \tag{A57}$$

FOC with respect to $\check{T}_e$

$$\begin{aligned} &e^{-(r+\delta)\check{T}_e}\frac{\Theta_d^{1-\beta}\underline{h}'(\theta)^\beta}{1+\beta} + \left[\frac{1-e^{-(r+\delta+\beta\sigma)\check{T}_e}}{r+\delta+\beta\sigma}\right]\frac{\Theta_d^{1-\beta}\underline{h}'(\theta)^\beta}{1+\beta}\beta\sigma e^{\beta\sigma\check{T}_e} \\ &- e^{-(r+\delta)\check{T}_e}\frac{(b_u+xb)}{2}\underline{h}'(\theta) - \left[\frac{1-e^{-(r+\delta+\sigma)\check{T}_e}}{r+\delta+\sigma}\right]\frac{(b_u+xb)}{2}\sigma e^{\sigma\check{T}_e}\underline{h}'(\theta) - e^{-(r+\delta)\check{T}_e}f_d = 0 \end{aligned} \tag{A58}$$

When $\overline{T}_e(\theta) < \check{T}_e(\theta)$, it means that the firm starts with the employment level lower than $\check{h}_e(\theta)$ and FE option is not possible. When $\overline{T}_e(\theta) \leq 0$, the firm exists on impact.

With binding minimum wage, the present value of the strategy is equal to

$$\begin{aligned} J^F_{FE}(h) &= \int_0^{T_{m,e}} e^{-(r+\delta)t}(R(h_t) - w_m h_t - f_d)dt + \int_{T_{m,e}}^{\check{T}_e} e^{-(r+\delta)t}(R(h_t) - w(h_t)h_t - f_d)dt \\ &= \left[\frac{1-e^{-(r+\delta+\beta\sigma)T_{m,e}}}{r+\delta+\beta\sigma}\right]\Theta_d'^{1-\beta}\underline{h}'(\theta)^\beta e^{\beta\sigma\check{T}_e} - \left[\frac{1-e^{-(r+\delta+\sigma)T_{m,e}}}{r+\delta+\sigma}\right]w_m\underline{h}'(\theta)e^{\sigma\check{T}_e} \\ &+ \left[\frac{e^{-(r+\delta+\beta\sigma)T_{m,e}} - e^{-(r+\delta+\beta\sigma)\check{T}_e}}{r+\delta+\beta\sigma}\right]\frac{\Theta_d'^{1-\beta}\underline{h}'(\theta)^\beta}{1+\beta}e^{\beta\sigma\check{T}_e} \\ &- \left[\frac{e^{-(r+\delta+\sigma)T_{m,e}} - e^{-(r+\delta+\sigma)\check{T}_e}}{r+\delta+\sigma}\right]\frac{(b_u+xb)}{2}\underline{h}'(\theta)e^{\sigma\check{T}_e} - \left[\frac{1-e^{-(r+\delta)\check{T}_e}}{r+\delta}\right]f_d \end{aligned} \tag{A59}$$

Now, FOC with respect to $\check{T}_e$ is given by

$$\left[\frac{1-e^{-(r+\delta+\beta\sigma)T_{m,e}}}{r+\delta+\beta\sigma}\right]\Theta_d'^{1-\beta}\underline{h}'(\theta)^\beta\beta\sigma e^{\beta\sigma\check{T}_e}-\left[\frac{1-e^{-(r+s)T_{m,e}}}{r+s}\right]w_m\underline{h}'(\theta)\sigma e^{\sigma\check{T}_e}$$
$$+\frac{e^{-(r+\delta+\beta\sigma)T_{m,e}}}{r+\delta+\beta\sigma}\frac{\Theta_d'^{1-\beta}\underline{h}'(\theta)^\beta}{1+\beta}\beta\sigma e^{\beta\sigma\check{T}_e}+\frac{r+\delta}{r+\delta+\beta\sigma}e^{-(r+\delta)\check{T}_e}\frac{\Theta_d'^{1-\beta}\underline{h}'(\theta)^\beta}{1+\beta}$$
$$-\frac{e^{-(r+s)T_{m,e}}}{r+s}\frac{(b_u+xb)}{2}\underline{h}'(\theta)\sigma e^{\sigma\check{T}_e}-\frac{r+\delta}{r+s}e^{-(r+\delta)\check{T}_e}\frac{(b_u+xb)}{2}\underline{h}'(\theta)-e^{-(r+\delta)\check{T}_e}f_d=0$$

or, equivalently

$$\left[\frac{\Theta_d'^{1-\beta}\underline{h}'(\theta)^\beta}{r+\delta+\beta\sigma}\right]\left(\left(1-\frac{\beta}{1+\beta}e^{-(r+\delta+\beta\sigma)T_{m,e}}\right)\beta\sigma e^{\beta\sigma\check{T}_e}+(r+\delta)\frac{e^{-(r+\delta)\check{T}_e}}{1+\beta}\right)-e^{-(r+\delta)\check{T}_e}f_d$$
$$-\left[\frac{1-e^{-(r+s)T_{m,e}}}{r+s}\right]w_m\underline{h}'(\theta)\sigma e^{\sigma\check{T}_e}-\frac{1}{r+s}\frac{(b_u+xb)}{2}\underline{h}'(\theta)e^{\sigma\check{T}_e}\left(e^{-(r+s)T_{m,e}}\sigma-(r+\delta)e^{-(r+s)\check{T}_e}\right)=0 \tag{A60}$$

Assume that firms that eventually stay and serve domestic market prefer option FD to D. It means that $\check{T}<\bar{T}$, or equivalently, $h_t(\theta)>\check{h}'(\theta)$ for all θ at time period $t=0$. The employment level $\check{h}'(\theta)\geq\check{h}_e'(\theta)$ for all productivity levels (can be derived from present value equations), and, therefore, for each firm it is true that $h_t(\theta)>\check{h}_e'(\theta)$. This means that all firms that eventually exit the market prefer option FE to option E. This concludes the proof.

EI: Exit on Impact

The firm exits on impact if

$$\overline{T}_e\leq 0 \tag{A61}$$

which is equivalent to

$$h(\theta)\leq\underline{h}'(\theta) \tag{A62}$$

So the firm at the margin has $h(\theta)=\underline{h}'(\theta)$ and, therefore, according to the definition of $\underline{h}'(\theta)$, has $\varphi(\underline{h}'(\theta),\theta)=0$. Then, to describe all the firms that exit on impact we can involve the following condition

$$\max_h\varphi(h,\theta)\leq 0 \tag{A63}$$

FOC with respect to h

$$\frac{\beta}{1+\beta}\Theta_d^{1-\beta}h^{\beta-1}-\frac{b_u+xb}{2}=0$$

from which

$$h=\left(\frac{\beta}{1+\beta}\frac{2}{b_u+xb}\right)^{\frac{1}{1-\beta}}\Theta_d$$

Therefore,

$$\begin{aligned}\max_h \varphi(h,\theta) &= \frac{1}{1+\beta}\Theta_d^{1-\beta}h^{\beta} - \frac{(b_u+xb)}{2}h - f_d \\ &= \frac{1}{1+\beta}\Theta_d\left(\frac{\beta}{1+\beta}\frac{2}{b_u+xb}\right)^{\frac{\beta}{1-\beta}} - \frac{(b_u+xb)}{2}\left(\frac{\beta}{1+\beta}\frac{2}{b_u+xb}\right)^{\frac{1}{1-\beta}}\Theta_d - f_d \\ &= \frac{1}{1+\beta}\Theta_d\left(\frac{\beta}{1+\beta}\frac{2}{b_u+xb}\right)^{\frac{\beta}{1-\beta}} - \frac{\beta}{1+\beta}\left(\frac{\beta}{1+\beta}\frac{2}{b_u+xb}\right)^{\frac{\beta}{1-\beta}}\Theta_d - f_d \\ &= \frac{1-\beta}{1+\beta}\Theta_d\left(\frac{\beta}{1+\beta}\frac{2}{b_u+xb}\right)^{\frac{\beta}{1-\beta}} - f_d\end{aligned}$$

where $\Theta_d = Q^{-\frac{\beta-\zeta}{1-\beta}}\theta^{\frac{\beta}{1-\beta}}$. Then, the condition to exit on impact is

$$\frac{1-\beta}{1+\beta}\left(\frac{\beta}{1+\beta}\frac{2}{b_u+xb}\right)^{\frac{\beta}{1-\beta}} Q^{-\frac{\beta-\zeta}{1-\beta}}\theta^{\frac{\beta}{1-\beta}} - f_d \leq 0 \tag{A64}$$

Incumbents Productivity Cutoffs

1. $\bar{\theta}'_x$: firms choose X when $\theta \geq \bar{\theta}'_x$. Let's define

$$J^F_{DD}(h(\theta),\theta) = 1_{\{\bar{T}\leq\check{T}\}}J^F_D(h(\theta),\theta) + 1_{\{\bar{T}>\check{T}\}}J^F_{FD}(h(\theta),\theta)$$

   Then cutoff $\bar{\theta}'_x$ can be defined as

$$J^F_X(h(\bar{\theta}'_x),\bar{\theta}'_x) = J^F_{DD}(h(\bar{\theta}'_x),\bar{\theta}'_x)$$

2. $\bar{\theta}'_d$: firms choose D/FD when $\theta \in [\bar{\theta}'_d, \bar{\theta}'_x)$. Analogously to the previous part we can define

$$J^F_{EE}(h(\theta),\theta) = 1_{\{\bar{T}_e\leq\check{T}_e\}}J^F_E(h(\theta),\theta) + 1_{\{\bar{T}_e>\check{T}_e\}}J^F_{FE}(h(\theta),\theta)$$

   Then, the cutoff is

$$J^F_{DD}(h(\bar{\theta}'_d),\bar{\theta}'_d) = J^F_{EE}(h(\bar{\theta}'_d),\bar{\theta}'_d)$$

3. $\check{\theta}'_d$: firms choose E when $\theta \in [\check{\theta}'_d, \bar{\theta}'_d)$. In this case,

$$\overline{T}_e(\check{\theta}'_d) = \check{T}_e(\check{\theta}'_d)$$

4. $\underline{\theta}'_d$: firms choose FE when $\theta \in [\underline{\theta}'_d, \check{\theta}'_d)$. When $\theta < \underline{\theta}'_d$ firms choose EI. From equation (A64)

$$\frac{1-\beta}{1+\beta}\left(\frac{\beta}{1+\beta}\frac{2}{b_u+xb}\right)^{\frac{\beta}{1-\beta}} Q'^{-\frac{\beta-\zeta}{1-\beta}}\underline{\theta}'^{\frac{\beta}{1-\beta}}_d - f_d = 0$$

B.3 Value of Workers

Employed by firms D/FD firms

As discussed above, the value of unemployed worker is pinned down by (from equation (6))

$$J^U = \frac{b_u+xb}{r} \tag{A65}$$

Let's denote $T = \min\{\bar{T}, \check{T}\}$. Then, the value of the worker employed by the firm that stays in domestic market is equal to

$$J^E_{D/FD} = \int_0^{T_m} e^{-(r+s)t}\left[w_m + sJ^U\right]dt + \int_{T_m}^{T} e^{-(r+s)t}\left[w_t + sJ^U\right]dt + e^{-(r+s)T} J^E_{D/FD}(\bar{h}'_d(\theta)) \tag{A66}$$

where the dynamic wage is

$$w_t = \frac{\beta}{1+\beta}\Theta_d'^{1-\beta}\bar{h}_d'^{\beta-1}(\theta)e^{-(\beta-1)\sigma(t-T)} + \frac{b_u + xb}{2} \tag{A67}$$

and the new steady-state value of employment is given by

$$J^E_{D/FD}(\bar{h}'_d(\theta)) = \frac{1}{r+s}(w + sJ^U) \tag{A68}$$

Then, from (A66),

$$\begin{aligned} J^E_{D/FD} &= \frac{1-e^{-(r+s)T_m}}{r+s}w_m + \frac{e^{-(r+\delta+\beta\sigma)T_m} - e^{-(r+\delta+\beta\sigma)T}}{r+\delta+\beta\sigma}\frac{\beta}{1+\beta}\Phi^{\frac{\beta-1}{\beta}}e^{(\beta-1)\sigma T} \\ &+ \frac{e^{-(r+s)T_m} - e^{-(r+s)T}}{r+s}\frac{b_u+xb}{2} + \frac{1}{r+s}sJ^U + \frac{e^{-(r+s)T}}{r+s}w \end{aligned} \tag{A69}$$

Employed by E/FE firms

Analogous to the previous case, define $T_e(\theta) = \min\{\bar{T}_e(\theta), \check{T}_e(\theta)\}$. Then, the value of employed worker is given by

$$J^E_{E/FE}(\theta) = \int_0^{T_m} e^{-(r+s)t}\left[w_m + sJ^U\right]dt + \int_{T_m}^{T_e(\theta)} e^{-(r+s)t}\left[w_t + sJ^U\right]dt + e^{-(r+s)T_e(\theta)}J^U \tag{A70}$$

where the wage is defined analogous to (A67). Then, from (A70),

$$\begin{aligned} J^E_{E/FE}(\theta) &= \frac{1-e^{-(r+s)T_m}}{r+s}w_m + \frac{e^{-(r+\delta+\beta\sigma)T_m} - e^{-(r+\delta+\beta\sigma)T_e(\theta)}}{r+\delta+\beta\sigma}\frac{\beta}{1+\beta}\Theta_d'^{1-\beta}(\theta)\underline{h}'^{\beta-1}(\theta)e^{(\beta-1)\sigma T_e(\theta)} \\ &+ \frac{e^{-(r+s)T_m} - e^{-(r+s)T_e(\theta)}}{r+s}\frac{b_u+xb}{2} + \frac{1-e^{-(r+s)T_e(\theta)}}{r+s}sJ^U + e^{-(r+s)T_e(\theta)}J^U \end{aligned} \tag{A71}$$

which now depends on firm's productivity level.