# Exchange Rate Disconnect in General Equilibrium: A Teaching Note


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**Abstract**

This short note presents a simplified version of the exchange rate disconnect model from Itskhoki and Mukhin (2019a,b), which can be easily solved in class on the board. We show how the model explains the main exchange rate puzzles and discuss other solutions from the literature.


## 1 Setup

The world consists of two symmetric economies – home (Europe) and foreign (U.S., denoted with a $*$) – each with its own nominal unit of account, in which all local prices are expressed. The nominal exchange rate $\mathcal{E}_t$ is the price of dollars in terms of euros, i.e. higher $\mathcal{E}_t$ corresponds to a depreciation of home currency.

A representative home household receives an exogenous endowment of home goods $Y_t$, chooses consumption $C_t$ and invests in local and foreign nominal bonds $B_t$ and $B_t^*$ to maximize expected utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

subject to the budget constraint

$$P_t C_t + \frac{B_{t+1}}{R_t} + \frac{\mathcal{E}_t B_{t+1}^*}{e^{\psi_t} R_t^*} = P_{Ht} Y_t + B_t + \mathcal{E}_t B_t^* + T_t,$$

where $R_t$ and $R_t^*$ are the gross nominal interest rates, $P_t$ is the ideal price index, and for simplicity we assume that the law of one price holds $P_{Ht}^* = P_{Ht}/\mathcal{E}_t$, so that all endowment is sold at price $P_{Ht}$. The final consumption basket is a CES aggregator of local and foreign goods with elasticity $\theta$ and a home bias $1-\gamma > 1/2$:

$$C_t = \left[(1-\gamma)C_{Ht}^{\frac{\theta-1}{\theta}} + \gamma C_{Ft}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}.$$

Asset markets are incomplete. There are two nominal bonds paying in different currencies, yet only the foreign bonds are traded internationally. Moreover, there is a financial wedge $\psi_t$ between the effective returns on foreign bonds for home vs. foreign households. One microfoundation of the $\psi_t$ shock arises from a limits-to-arbitrage model of the financial sector, intermediating the international trade in bonds (see Itskhoki and Mukhin 2019a,b, Gabaix and Maggiori 2015), with the resulting profits of the financial sector $T_t = \frac{\mathcal{E}_t B^*_{t+1}}{R^*_t}(e^{-\psi_t}-1)$ returned lump-sum to the households.[1]

The problem of the foreign households is symmetric, except that they can only trade foreign bonds and are not subject to the $\psi_t$ shock, so that their budget constraint is given simply by $P^*_t C^*_t + B^*_{t+1}/R^*_t = P^*_{Ft}Y^*_t + B^*_t$ (in units of foreign currency). The market clearing for home and foreign goods requires that

$$Y_t = C_{Ht} + C^*_{Ht} \qquad \text{and} \qquad Y^*_t = C_{Ft} + C^*_{Ft}.$$

The monetary policy in each country stabilizes inflation, so that consumer price indices can be normalized to one, $P_t = P^*_t = 1$. All prices are fully flexible. Both financial shocks $\psi_t$ and shocks to the relative log endowment $\log(Y_t/Y^*_t)$ follow AR(1) processes with the autoregressive coefficient $\rho$ and innovations $\varepsilon^\psi_t$ and $\varepsilon^y_t$.

## 2 Equilibrium system

Define the real exchange rate (RER) and the terms-of-trade (ToT) as

$$\mathcal{Q}_t \equiv \frac{\mathcal{E}_t P^*_t}{P_t}, \qquad \mathcal{S}_t \equiv \frac{P_{Ft}}{\mathcal{E}_t P^*_{Ht}} = \frac{\mathcal{E}_t P^*_{Ft}}{P_{Ht}},$$

where the last equality holds due to the law of one price. We use capital letters for levels and small letters for log-deviations from the symmetric steady state.[2] It is convenient to partition

[1]If international financial transactions are intermediated by risk-averse (or financially-constrained) intermediaries, the equilibrium requires departures from the uncovered (or covered) interest rate parity, $\psi_t$, with these deviations fluctuating over time (e.g., in response to shocks to noise-trader and fundamental demand for bonds). There are many alternative interpretation of the $\psi_t$ shock, ranging from complete-market models of endogenous risk-premia (e.g., Verdelhan 2010, Colacito and Croce 2013, Farhi and Gabaix 2016) to models of heterogeneous beliefs and expectational errors (e.g., Evans and Lyons 2002, Gourinchas and Tornell 2004, Bacchetta and van Wincoop 2006).

[2]There is a continuum of steady states, which can be parametrized by a net foreign asset position $B^*$. The symmetric one corresponds to $B^* = 0$. It is straightforward to ensure uniqueness by adding an arbitrary small

the equilibrium conditions into four blocks — prices, goods market, asset market, and the country budget constraint.

1. **Price block** The ideal price indexes take the familiar form:

$$p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft},$$
$$p_t^* = (1-\gamma)p_{Ft}^* + \gamma p_{Ht}^*.$$

   Substitute these expressions into the definition of RER to obtain the relationship between RER and ToT:

$$q_t = (1-\gamma)(e_t + p_{Ft}^* - p_{Ht}) + \gamma(e_t + p_{Ht}^* - p_{Ft}) = (1-2\gamma)s_t, . \tag{1}$$

   which uses the assumption of the law of one price.

2. **Goods market** Substitute the optimal CES demand for home goods:

$$C_{Ht} = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\theta} C_t \qquad \text{and} \qquad C_{Ht}^* = \gamma\left(\frac{P_{Ht^*}}{P_t^*}\right)^{-\theta} C_t^*,$$

   and similarly for foreign goods into the market clearing conditions and linearize to obtain:

$$y_t = (1-\gamma)\big[c_t - \theta(p_{Ht} - p_t)\big] + \gamma\big[c_t^* - \theta(p_{Ht}^* - p_t^*)\big],$$
$$y_t^* = (1-\gamma)\big[c_t^* - \theta(p_{Ft}^* - p_t^*)\big] + \gamma\big[c_t - \theta(p_{Ft} - p_t)\big].$$

   Take the difference between the two and solve for the relative consumption:

$$c_t - c_t^* = \frac{1}{1-2\gamma}(y_t - y_t^*) - \frac{2\gamma\theta}{1-2\gamma}(s_t + q_t). \tag{2}$$

   Due to home bias, the relative consumption is proportional to relative output corrected by the expenditure switching term.

3. **Asset market** There are three asset pricing equations — one for the domestic bond and two for the foreign bond:

$$\mathbb{E}_t\Theta_{t+1}R_t = 1, \quad \mathbb{E}_t\Theta_{t+1}R_t^*\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}e^{\psi_t} = 1, \quad \mathbb{E}_t\Theta_{t+1}^*R_t^* = 1,$$

transaction cost as in Schmitt-Grohé and Uribe (2003), which emerges endogenously in the microfounded model of the financial sector in Itskhoki and Mukhin (2019a,b).

where $\Theta_{t+1} \equiv \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}$ and $\Theta^*_{t+1} \equiv \beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\frac{P^*_t}{P^*_{t+1}}$ are the nominal SDFs of home and foreign households. Linearize these equations to get

$$\begin{aligned} i_t &= \mathbb{E}_t\big[\sigma\Delta c_{t+1} + \pi_{t+1}\big], \\ i^*_t + \psi_t &= \mathbb{E}_t\big[\sigma\Delta c_{t+1} + \pi_{t+1} - \Delta e_{t+1}\big], \\ i^*_t &= \mathbb{E}_t\big[\sigma\Delta c^*_{t+1} + \pi^*_{t+1}\big], \end{aligned}$$

The no-arbitrage conditions for home households investing in the two types of bonds (the first two equations) can be combined to get a modified UIP condition:

$$\mathbb{E}_t\Delta e_{t+1} = i_t - i^*_t - \psi_t.$$

The risk-sharing conditions for home and foreign households trading the international bond, emerges from the last two equations:

$$\mathbb{E}_t\big[\sigma(\Delta c_{t+1} - \Delta c^*_{t+1}) - \Delta q_{t+1}\big] = \psi_t. \tag{3}$$

Thus, there are two deviations from the Backus-Smith perfect risk sharing condition, $\sigma(\Delta c_{t+1} - \Delta c^*_{t+1}) = \Delta q_{t+1}$: (i) because of incomplete markets, the condition holds only in expectation rather than for any realization of shocks; (ii) furthermore, $\psi_t$ introduces a wedge in this "average" risk sharing.[3]

4. **Country budget constraint** Combine the household budget constraint with the transfers $T_t$, market clearing conditions, and $B_t = 0$ (zero net supply at home) to obtain

$$\frac{\mathcal{E}_t B^*_{t+1}}{R^*_t} = \mathcal{E}_t B^*_t + \Big[\mathcal{E}_t P^*_{Ht} Y_t - P_{Ft} C_{Ft}\Big],$$

where the term in brackets corresponds to the home net exports. Normalize all terms by the steady-state GDP, $P_H Y = PC$, and take the first-order approximation:

$$\beta b^*_{t+1} = b^*_t + \gamma\big[\underbrace{e_t + p^*_{Ht} - p_{Ft} + c^*_{Ht} - c_{Ft}}_{=\theta q_t + (\theta-1)s_t - (c_t - c^*_t)}\big],$$

which can be rewritten in the intertemporal form as

$$b^*_t + \gamma\sum_{j=0}^{\infty}\beta^j\Big[\theta q_{t+j} + (\theta-1)s_{t+j} - (c_{t+j} - c^*_{t+j})\Big] = 0. \tag{4}$$

[3] Notice that if the wedge were specific to foreign bonds, with both households getting the same effective returns on the asset, it would not affect the equilibrium exchange rate, and would be fully absorbed by the foreign interest rate $i^*_t$. Similarly, if this were a wedge to home household stochastic discount factor, it would not appear in the UIP condition, and would be fully absorbed by the home interest rate $i_t$.

# 3 Equilibrium

The system of equations (1)-(4) fully describes the equilibrium allocation. The general approach to solving the model is to use static block (1)-(2) to substitute out the ToT $s_t$ and the relative consumption $c_t - c_t^*$ from the dynamic equations (3)-(4) and then solve the latter for the RER $q_t$ and the country's net foreign asset position $b_t^*$. The latter can be done using the method of undetermined coefficients or the Blanchard-Kahn method.

To keep things as simple as possible, we focus on the Cole and Obstfeld (1991) parametrization with $\sigma = \theta = 1$.[4] Start by guessing the general solution to the risk-sharing condition (3):[5]

$$q_t - (c_t - c_t^*) = m_t + \frac{\psi_t}{1-\rho}, \quad \text{where } \Delta m_t = u_t \sim \text{i.i.d.} \tag{5}$$

Thus, in contrast to the case of complete markets, there is an endogenous wedge in risk sharing between countries, with a permanent component $m_t$ and a transitory component $\psi_t$. The unknown innovation to the martingale $u_t$, in turn, is pinned down by the country's budget constraint, while the level of $m_t$ depends on the previous history of shocks, i.e. on the accumulated asset position $b_t^*$. Substitute condition (5) into equation (4) and take conditional expectation $\mathbb{E}_t[\cdot]$:

$$b_t^* + \gamma\Big[\frac{m_t}{1-\beta} + \frac{\psi_t}{(1-\rho)(1-\beta\rho)}\Big] = 0.$$

Now take conditional expectation $\mathbb{E}_{t-1}[\cdot]$ of the last expression

$$b_t^* + \gamma\Big[\frac{m_{t-1}}{1-\beta} + \frac{\rho\psi_{t-1}}{(1-\rho)(1-\beta\rho)}\Big] = 0.$$

Taking the difference between the two equations, one gets the innovation as a function of $\varepsilon_t^\psi$:

$$u_t = -\frac{1-\beta}{(1-\rho)(1-\beta\rho)}\varepsilon_t^\psi. \tag{6}$$

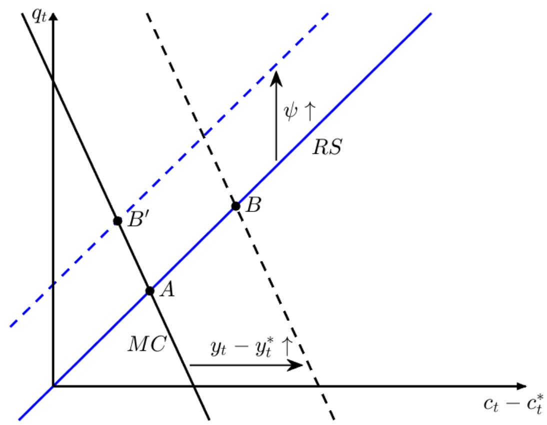
Therefore, (5) together with (6) characterize the equilibrium comovement of the real exchange rate with consumption, which jointly satisfy the risk sharing condition and the budget constraints of the countries, thus generalizing the famous Backus-Smith condition, which applies under complete markets.

As shown in Figure 1, the equilibrium values $q_t$ and $c_t - c_t^*$ can then be found as the intersection of the risk-sharing condition (5) and the market clearing condition (2). We combine them to solve for $q_t$ and $c_t - c_t^*$ explicitly as a function of primitives, namely the endowment

[4] While this case allows to minimize the number of parameters, all qualitative results below go through independently of the value of $\sigma$ and $\theta$.

[5] Indeed, one can verify that $\mathbb{E}_t\Big[\Delta m_{t+1} + \frac{\psi_{t+1}-\psi_t}{1-\rho}\Big] = -\psi_t$, as required by (3).

Figure 1: Equilibrium in good and asset markets




Note: the market clearing (MC) curve shows the value of $c_t - c_t^*$ and $q_t$ that ensure equilibrium in the goods market, while the risk-sharing (RS) curve corresponds to the equilibrium in asset markets.

shocks and the financial shocks (or risk sharing wedges in (5))

$$
\begin{aligned}
q_t &= (1-2\gamma)(y_t - y_t^*) + (1-2\gamma)^2\Big[m_t + \frac{\psi_t}{1-\rho}\Big], \\
c_t - c_t^* &= (1-2\gamma)(y_t - y_t^*) - 4\gamma(1-\gamma)\Big[m_t + \frac{\psi_t}{1-\rho}\Big],
\end{aligned}
\tag{7}
$$

where recall the characterization of the martingale innovation $\Delta m_t$ in (6).

Note that the endowment shock $y_t - y_t^*$ shifts only the equilibrium curve in the goods market, while the financial shocks $\psi_t$ shifts only the equilibrium curve in the asset market. Intuitively, there are two reasons for an exchange rate depreciation ($q_t \uparrow$): (i) a relative abundance of domestic goods ($y_t - y_t^* \uparrow$), or (ii) a relatively high demand for foreign assets ($\psi_t \uparrow$).[6]

# 4 Puzzles

**Disconnect:** empirically, exchange rates are an order of magnitude more volatile than other macro variables and with the correlation close to zero (Meese and Rogoff 1983). Out of all macro aggregates, we focus on consumption: the results also apply to GDP once output is endogenized, while the correlation with inflation is discussed separately in the PPP section.

As equations (7) make clear, to reproduce the disconnect from the data, the model has to satisfy two conditions:

1. A shock that drives exchange rates, but has limited direct effect on macro variables. The endowment shock clearly does not satisfy this criteria as even in a closed economy limit

[6]Going beyond the endowment economy, a relative abundance of domestic goods ($y_t - y_t^* \uparrow$), in turn, can be either due to high productivity (e.g., in a flexible-price production economy) or due to low markups (e.g., in response to a monetary expansion in a model with nominal rigidities).

$\gamma \approx 0$, it has large direct effect on consumption. In contrast, financial shocks only affect exchange rates, but not other macro aggregates. Thus, as long as the volatility induced by $\psi_t$ dominates the one from $y_t - y_t^*$, the model can simultaneously reproduce a high volatility of $q_t$ relative to $c_t - c_t^*$ and a low correlation between the two.

2. A muted transmission of exchange rate volatility into other macro variables. Although several ingredients – including pricing to market, low elasticities of substitution, intermediates and sticky prices – can help with this, the first-order parameter is the openness of the economy $\gamma$. The home bias is indeed much more pronounced for large developed economies ($\gamma \approx 0.15$ for U.S., Eurozone, Japan), which exhibit the strongest exchange rate disconnect.

**Backus-Smith:** in contrast to the predictions of a model with complete markets and CRRA preferences, in the data, $\text{cor}(q_t, c_t - c_t^*) \lesssim 0$ (Backus and Smith 1993). Thus, while to the first-order the correlation between consumption and exchange rates is zero, it tends to be mildly negative in most samples. Figure 1 shows how the model rationalizes this finding: the equilibrium values of $q_t$ and $c_t - c_t^*$ are determined by the intersection of the risk-sharing condition (5) with the market clearing condition (2). Under complete markets, the former curve is fixed, while productivity shocks shift the latter one (from A to B), which generates a positive correlation between $q_t$ and $c_t - c_t^*$. In contrast, in this model, $\psi_t$ shifts the risk-sharing curve (from A to B′), which generates a negative Backus-Smith correlation. The lower is $\gamma$, the more vertical is the market-clearing curve and the higher is the pass-through of $\psi_t$ in $q_t$ and the lower in $c_t - c_t^*$.

**UIP:** the Fama regression estimates tend to have negative slope coefficient and a low $R^2$. The latter fact follows directly from a low predictability of exchange rates established below. Consider the case with only financial shocks and no endowment shocks. Given that the monetary policy stabilizes the CPI, the modified UIP condition from above can the be used to express the interest rates as:

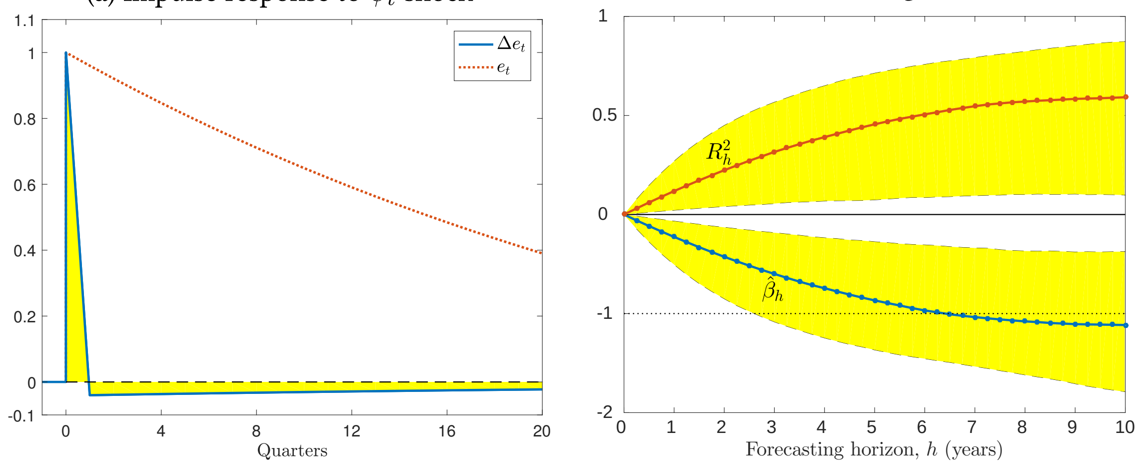
$$i_t - i_t^* = \mathbb{E}_t\big(\Delta c_{t+1} - \Delta c_{t+1}^*\big) = 4\gamma(1-\gamma)\psi_t.$$

The Fama slope coefficient is then equal to

$$\beta^{\text{Fama}} = \frac{\text{cov}(\Delta e_{t+1}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} = \frac{\text{cov}(\mathbb{E}_t \Delta e_{t+1}, i_t - i_t^*)}{\text{var}(i_t - i_t^*)} = -\frac{(1-2\gamma)^2}{4\gamma(1-\gamma)}.$$

Thus, the coefficient is unambiguously negative and converges to $-\infty$ as $\gamma \to 0$. Given that $\beta^{\text{Fama}} = 1$ for productivity shocks, the actual number will be somewhere in between and under realistic calibration turns to be mildly negative as in the data.

Figure 2: Properties of the equilibrium exchange rate process




Note: (a) impulse response of both $\Delta e_t$ and $e_t$ to a $\psi_t$-shock innovation; (b) $\hat{\beta}_h$ and $R_h^2$ from the predictive regression $\mathbb{E}\{e_{t+h} - e_t | q_t\} = \alpha_h + \beta_h q_t$, at different horizons $h \geq 1$.

**Predictability:** exchange rates follow a random-walk like process in the data. For simplicity, consider the autarky limit $\gamma \approx 0$ and no productivity shocks:

$$q_t = m_t + \frac{\psi_t}{1-\rho}.$$

Define the lag operator as $\mathbb{L}x_t = x_{t-1}$ and apply it to the equation above:

$$\begin{aligned}(1-\rho\mathbb{L})(1-\mathbb{L})q_t &= (1-\rho\mathbb{L})(1-\mathbb{L})\Big[m_t + \frac{\psi_t}{1-\rho}\Big] \\ &= (1-\mathbb{L})\frac{\varepsilon_t^{\psi}}{1-\rho} + (1-\rho\mathbb{L})u_t \\ &= \frac{\beta}{1-\beta\rho}\Big(1 - \frac{1}{\beta}\mathbb{L}\Big)\varepsilon_t^{\psi}.\end{aligned}$$

Thus, the RER follows ARIMA(1,1,1) process with the autoregressive coefficient $\rho$ and a moving average coefficient $1/\beta$.

Intuitively, the equilibrium in asset markets (the UIP) pins down the *slope* of $q_t$ (investors care about future *changes* in exchange rates rather than the absolute value), while the goods market (country's budget constraint) determine the level of $q_t$. In equilibrium, a positive demand shock of home households for foreign bonds ($\psi_t > 0$) has to be counteracted by higher returns on domestic bonds induced by expected exchange rate appreciation. This lowers country's future net exports, which according to the country's budget constraint has to be compensated by higher exports today. Therefore, the exchange rate needs to depreciate on impact when the shock arrives (see Figure 2a).

As agents become more patient and the shock becomes more persistent $\beta\rho \to 1$, both AR and MA components converge to one and cancel out from the equation above, meaning that the RER becomes a random walk with an unbounded volatility. At the same time, away from that limit, there is some predictability in exchange rates (Figure 2b), consistent with the recent evidence from Eichenbaum, Johannsen, and Rebelo (2018). Finally, note that the random-walk like dynamics of $q_t$ is not specific to $\psi_t$ shocks and also holds for $y_t - y_t^*$ shocks.

**PPP:** all measures of the RER exchange rate (including CPI-, PPI-, and wage-based) co-move closely with the nominal exchange rate and hence, have very high volatility and persistence (Rogoff 1996). Given a volatile random-walk like $q_t$ derived above, three additional ingredients of the model allow it to solve the PPP puzzle:

1. *Inflation targeting* by the monetary policy ensures that in the medium-run $p_t, p_t^* \approx 0$ and therefore, $e_t = q_t$. In contrast, most of the previous literature on the PPP puzzle assumes monetary shocks as the key drivers of nominal exchange rates and introduces nominal rigidities to dampen the arising wedge between real and nominal exchange rates (see e.g. Carvalho and Nechio 2011).

2. *High home bias* $\gamma \approx 0$ implies that PPI$\approx$ CPI and the two measures of the RER co-move closely as well:

$$q_t = (1-\gamma)(e_t + p_{Ft}^* - p_{Ht}) + \gamma(e_t + p_{Ht}^* - p_{Ft}) = (1-\gamma)q_t^{PPI} - \gamma s_t$$

$$\Rightarrow \; q_t^{PPI} = \frac{1}{1-2\gamma} q_t$$

3. *Small productivity shocks* (relative to exchange rate volatility) guarantee that the wedge between wage-based and PPI-based RERs is small as well. See the paper for a model with endogenous production.

Finally, in contrast to equation (1), which implies $\text{std}(s_t) > \text{std}(q_t)$, the data suggests $\frac{\text{std}(s_t)}{\text{std}(q_t)} \approx \frac{1}{3}$ (Atkeson and Burstein 2008). While we do not address this puzzle in this note, the full model with the deviations from the law of one price can reproduce this moment as well.

# 5 Related papers

**Engel and West (2005)** propose an asset-pricing perspective on the exchange rates. To see their point, complement the model with money-in-the-utility resulting in money demand

$$m_t - p_t = \sigma c_t - \chi i_t.$$

Express the interest rate from this condition, substitute into the UIP condition, iterate it forward and define $\delta \equiv \frac{\chi}{1+\chi}$ to obtain

$$e_t = \delta \sum_{j=0}^{\infty} \delta^j \mathbb{E}_t \psi_{t+j} + (1-\delta) \sum_{j=0}^{\infty} \delta^j \mathbb{E}_t f_{t+j}, \quad \text{where } f_t \equiv \big[(m_t - m_t^*) - \sigma(c_t - c_t^*) + q_t\big].$$

Thus, just like asset prices, the equilibrium exchange rate depends on expectations about future risk premia and macro fundamentals. The Engel-West theorem then states that if either (i) $\psi_t = 0$ and $f_t \sim I(1)$ or (ii) $\psi_t \sim I(1)$ and $f_t \sim I(k),\ k = 0, 1$, then as $\delta \to 1$, $e_t$ converges to a random walk.[7]

Similarly to the theorem, our unpredictability result also emphasizes the persistence of shocks and a low discounting, but it is the limit $\beta\rho \to 1$ that guarantees that exchange rates follow a random walk, while the value of $\chi$ plays no role. In contrast to the partial equilibrium approach of EW that takes $\psi_t$ and $f_t$ as given, the general equilibrium model implies a cointegration between the two: dropping the monetary shocks, $\mathbb{E}_t \Delta f_{t+1} = -\psi_t$. Thus, if $\psi_t \sim I(k)$ then $f_t \sim I(k+1)$ and condition (ii) cannot be satisfied. For endowment shocks, on the other hand, $\mathbb{E}_t \Delta f_{t+1} = 0$, so $e_t = f_t$ is a random walk independently from the value of $\delta$. Finally, the two models have exactly the same predictions for the monetary shocks $m_t - m_t^*$.

**Corsetti, Dedola, and Leduc (2008)** present *two* explanations to the Backus-Smith puzzle that do not rely on $\psi_t$ shocks. The first one relies on elasticity of substitution between home and foreign goods. For simplicity, assume financial autarky, full home bias $\gamma \to 0$, and allow for general values of $\theta$. Substitute the market clearing condition (2) into the net exports to get

$$q_t = \frac{1}{2\theta - 1}(y_t - y_t^*).$$

[7] The heuristic proof of this result goes as follows. Given the linearity of the process, consider separately $\psi_t$ and $f_t$ components. For the latter, we have

$$e_t = (1-\delta)\mathbb{E}_t \sum_{j=0}^{\infty} \delta^j \left( f_t + \sum_{i=1}^{j} \Delta f_{t+i} \right) = f_t + \sum_{j=1}^{\infty} \delta^j \mathbb{E}_t \Delta f_{t+j}$$

and hence, $\mathbb{E}_{t-1}\Delta e_t = (1-\delta)\mathbb{E}_{t-1}\Delta f_t + (1-\delta)\sum_{j=1}^{\infty} \delta^j \mathbb{E}_{t-1}\Delta f_{t+j} \to 0$ as $\delta \to 1$ when $f_t \sim I(k)$, $k \leq 1$ (so that $\sum_{j=1}^{\infty} \delta^j \mathbb{E}_{t-1}\Delta f_{t+j}$ is finite). The argument for risk-premium shocks is slightly different. As $\delta \to 1$, the unexpected innovation to $e_t$ becomes unbounded when $\psi_t \sim I(1)$:

$$\Delta e_t - \mathbb{E}_{t-1}\Delta e_t = \frac{\delta}{1-\delta}(\mathbb{E}_t - \mathbb{E}_{t-1})\psi_t + \sum_{j=1}^{\infty} \frac{\delta^{j+1}}{1-\delta}(\mathbb{E}_t - \mathbb{E}_{t-1})\Delta\psi_{t+j}.$$

Thus, when goods are strong complements, $\theta < \frac{1}{2}$, a positive productivity shock increases demand for foreign goods so much that the RER *appreciates* and generates a negative correlation with the relative consumption.

The second explanation from CDL emphasizes the wealth effects. Consider again the closed-economy limit, no $\psi$ shocks, but allow for general values of $\theta$ and $\sigma$. Following the same steps as for the benchmark model, one can solve for $u_t$. The key assumption is that $y_t - y_t^*$ follows an integrated process. E.g. when $\Delta y_t - \Delta y_t^*$ is an AR(1) process, we get

$$\frac{\mathrm{d}q_t}{\mathrm{d}\varepsilon_t^y} = \frac{1}{1-\beta\rho}\left[\frac{1}{2\theta-1} - \beta\rho\sigma\right].$$

It follows, if $(2\theta-1)\sigma\beta\rho > 1$, the RER appreciates in response to a positive shock and generates a negative Backus-Smith correlation. Intuitively, a positive growth shock makes home consumers want to borrow against future income leading to $nx_t < 0$, which is supported by the appreciation of $q_t$. A higher substitution $\theta$ between home and foreign goods strengthens this effect as does a lower inter-temporal substitution $1/\sigma$. Note also that having a random walk for $y_t - y_t^*$ is not enough and one needs a more persistent process for $y_t - y_t^*$.

**Benigno and Thoenissen (2008)** use the Balassa-Samuelson effect to explain the Backus-Smith puzzle. To see the point, add a non-tradable sector. Equations (2) and (4) still hold, except that all variables are now replaced with their analogs for tradables. Assuming the Cobb-Douglas aggregator between tradables and non-tradables and the share of the latter equal $1-\delta$, the optimal demand implies

$$(c_{Tt} - c_{Tt}^*) - (c_{Nt} - c_{Nt}^*) = (p_{Nt} - e_t - p_{Nt}^*) - (p_{Tt} - e_t - p_{Tt}^*) = q_{Tt} - q_{Nt}.$$

Suppose there are no shocks in non-tradable sector, so that $c_{Nt} - c_{Nt}^* = y_{Nt} - y_{Nt}^* = 0$. Then taking the autarky limit $\delta, \gamma \approx 0$, the RER is equal to $q_t = q_{Nt} = q_{Tt} - (y_{Tt} - y_{Tt}^*)$. Substitute this into the budget constraint and assume financial autarky:

$$nx_t \propto (2\theta-1)q_{Tt} - (c_{Tt} - c_{Tt}^*) = (2\theta-1)q_t + 2(\theta-1)(y_{Tt} - y_{Tt}^*) = 0.$$

Thus, as long as $\theta > 1$, a positive shock to $y_{Tt} - y_{Tt}^*$ increases consumption and *appreciates* the RER generating a negative Backus-Smith correlation. This mechanism is, however, hardly consistent with the evidence from Engel (1999) on the small role of relative shocks between tradables and non-tradables in driving the exchange rates.

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