What do Financial Markets say about the Exchange Rate?

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Finance and the Exchange Rate

This paper: a general framework for understanding how finance interacts with the exchange rate

The Two Roles of Finance

1. **Sharing macroeconomic risks across countries (smoothing of shocks):** households line up their marginal rates of substitution, pinning down the exchange rate (Cole Obstfeld 1991, Backus Smith 1993, ...)

2. **Source of shocks:** shocks hit the financial sector, and in turn affect the exchange rate (Gabaix Maggiori 2015, Itskhoki Mukhin 2021, Jiang Krishnamurthy Lustig Sun 2022, ...)

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Conjectures based on existing models:

- *The two roles cannot exist simultaneously* → **NO**
- *Each role faces intrinsic challenges* → **NO**

→ Show you a market structure in which both roles coexist without the challenges

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- The role of financial transmission of shocks arises as a complement to the risk-sharing perspective

Special Case: Risk-Sharing View of the Exchange Rate

- Simplest risk-sharing model: **complete and integrated markets**, home and foreign SDFs pin down the exchange rate:

$$\underbrace{\Delta s_{t+1}}_{\substack{\text{change in FX} \\ \text{£ appreciation}}} = \underbrace{m^*_{t+1}}_{\substack{\text{log foreign SDF} \\ \text{£ state price}}} - \underbrace{m_{t+1}}_{\substack{\text{log home SDF} \\ \text{\$ state price}}}$$

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- Exclude other role: no room for financial shocks beyond what we learn from households

General Risk-Sharing View

1. **Innovations:** exchange rate innovations coincide with differential SDF innovations when projected on risks that investors in both countries can trade $\boldsymbol{\epsilon}^{g}_{t+1}$

$$\text{proj}(\Delta s_{t+1}|\boldsymbol{\epsilon}^{g}_{t+1}) = \text{proj}(m^{*}_{t+1} - m_{t+1}|\boldsymbol{\epsilon}^{g}_{t+1})$$

2. **Expectation:** expected depreciation $E_t \Delta s_{t+1}$ pinned down by household SDFs if and only if exchange rate is spanned by asset returns

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- Standard models are polar cases:
  - Complete and integrated markets: asset market view $\rightarrow$ FX pinned down by risk-sharing
  - Extreme segmentation: no risks traded by households, intermediary only trades risk-free bonds $\rightarrow$ FX determined by financial shocks

What We Learn From The General Framework

- **Macroeconomic risk-sharing is not always associated with puzzles**
  - Puzzles remain when
    - *Changing what is traded only*: incomplete but integrated markets
    - *Changing who is trading only*: intermediated but rich set of assets
  - Incomplete and intermediated markets: puzzles can disappear even with substantial risk-sharing
- **Financial sector shocks do not need extreme segmentation**
  - intermediaries can trade any assets
  - households can trade their local assets if returns weakly correlated across countries and to the exchange rate
- **"Happy middle" empirically plausible**
  - stocks and bonds weakly correlated across countries and with the exchange rate

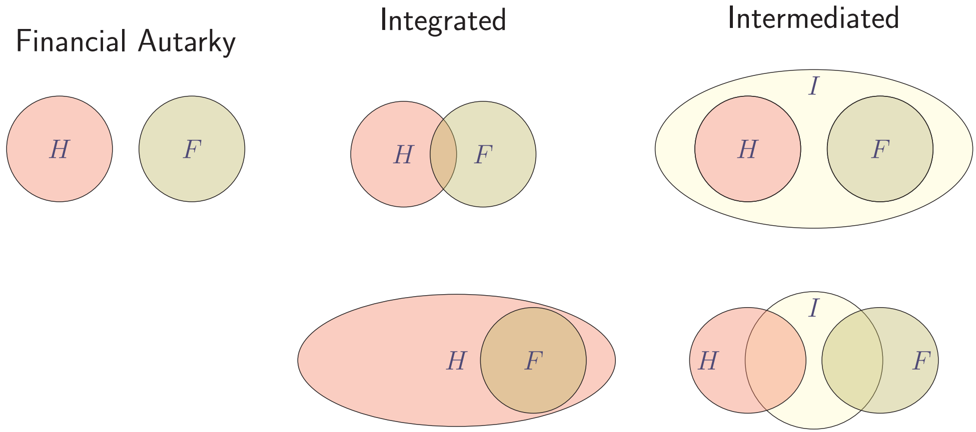
Outline

General International Asset Markets

- H: assets that home households (with m_{t+1}) can freely invest in
- F: assets that foreign households (with m^*_{t+1}) can freely invest in
  - Freely invest in = Euler equation. Exclude assets with binding borrowing constraints, short-sale constraints, adjustment costs, and convenience yields ...

- I: international assets over which international no-arbitrage must hold
  - (Partial) Integration: $I = H$ or $I = F$
  - Intermediation: I assets the intermediary (with m^I_{t+1}) can trade
  - In general, could be interaction of many intermediaries, other traders can be present, ...

Market Structures



Mathematical Representation

- $\boldsymbol{r}_{t+1}$: returns of assets in $H \cap I$ in home currency, risk free $r_{f,t}$
- $\boldsymbol{r}^*_{t+1}$: returns of assets in $F \cap I$ in foreign currency, risk free $r^*_{f,t}$
- International returns $\boldsymbol{r}^I_{t+1} = \{\boldsymbol{r}_{t+1}, \boldsymbol{r}^*_{t+1} + \Delta s_{t+1}\}$

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- *Assumption 1*: **Euler equations** in local markets
  - m_{t+1} prices $\boldsymbol{r}_{t+1}$ (require knowledge)
  - m^*_{t+1} prices $\boldsymbol{r}^*_{t+1}$ (require knowledge)
- *Assumption 2*: **No international arbitrage** in $\boldsymbol{r}^I_{t+1}$

  $\Leftrightarrow$ $\exists\ m^I_{t+1}$ that prices $\boldsymbol{r}^I_{t+1}$ (do not require knowledge, some $m^I_{t+1} := m^{I*}_{t+1} - \Delta s_{t+1}$)

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$\rightarrow$ **Characterize all joint restrictions involving Δs_{t+1} from assumptions 1 and 2**

A Useful Decomposition

Three types of shocks

- **Globally-traded shocks**: all those spanned by both $\{\boldsymbol{r}_{t+1}\}$ and ($\cap$) $\{\boldsymbol{r}^*_{t+1}\}$
  - ϵ^g_{t+1} must affect returns in the two countries
  - investors must have access to a trading strategy in each country that isolates ϵ^g_{t+1} from other sources of risk
- **Locally-traded shocks**: can be spanned by either $\{\boldsymbol{r}_{t+1}\}$ or ($\cup$) $\{\boldsymbol{r}^*_{t+1}\}$ and $\perp \boldsymbol{\epsilon}^g_{t+1}$
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Decompose the FX depreciation rate:

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Lemma 1: (a) fully integrated: $\widetilde{\Delta s}_{t+1} = g_{t+1}$; (b) partially integrated: $u_{t+1} = 0$.

The General Risk-Sharing View

1. **Proposition 1: FX innovations** coincide with differential SDF innovations when projected on global shocks

$$\text{proj}(m^*_{t+1} - m_{t+1} | \boldsymbol{\epsilon}^g_{t+1}) = \text{proj}(\Delta s_{t+1} | \boldsymbol{\epsilon}^g_{t+1}) = g_{t+1}$$

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- g_{t+1} can be fully constructed from (m_{t+1}, m^*_{t+1}) and $\{\boldsymbol{r}_{t+1}, \boldsymbol{r}^*_{t+1}\}$ without any knowledge of properties of Δs_{t+1}
- no constraints on exposure of exchange rate to other risks, ℓ_{t+1} and u_{t+1}

▸ sketch of a proof

The General Risk-Sharing View

2. **Proposition 2: Expectation** $E_t \Delta s_{t+1} = \delta_t + \psi_t$, where $\delta_t = E_t r_{p,t+1} - E_t r^*_{p,t+1}$

$$\delta_t = \underbrace{r_{ft} - r^*_{ft}}_{\text{UIP}} - \underbrace{cov_t(m_{t+1}, \Delta s_{t+1})}_{\text{exchange rate risk premium}} - \underbrace{\frac{1}{2} var_t(\Delta s_{t+1})}_{\text{convexity}} + \theta_t,$$

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(a) if exchange rate innovations are spanned by assets, $u_{t+1} = 0$, then $E_t \Delta s_{t+1}$ is pinned down by m and m^*, $\psi_t = 0$, similarly to the compete-markets case

(b) If exchange rate innovations are unspanned, $u_{t+1} \neq 0$, $E_t \Delta s_{t+1}$ is unconstrained by household SDFs, that is $\psi_t \neq 0$

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- As long as these constraints are satisfied, any exchange rate process can be obtained by choosing remaining properties of financial sector: intermediary health or regulation, noise trader shocks, convenience yields, ...

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Currency Puzzle Moments

In many macro-finance models:

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For general market structures:

- **Proposition:** The volatility and cyclicality of the exchange rate must satisfy

$$\overbrace{var_t(\Delta s_{t+1})}^{\text{volatility}} \geq var_t(g_{t+1}) + \frac{\Big(\overbrace{cov_t(m^*_{t+1} - m_{t+1}, \Delta s_{t+1})}^{\text{cyclicality}} - var_t(g_{t+1})\Big)^2}{var_t(m^*_{t+1} - m_{t+1}) - var_t(g_{t+1})}$$

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Complete and Integrated Markets

$$\text{proj}(\Delta s_{t+1}|\boldsymbol{\epsilon}^{g}_{t+1}) = \text{proj}(m^{*}_{t+1} - m_{t+1}|\boldsymbol{\epsilon}^{g}_{t+1}) = g_{t+1}$$

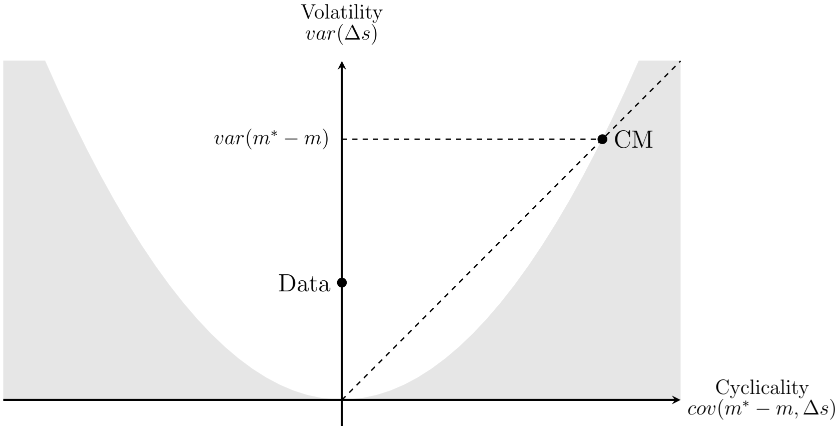
- **Complete and integrated markets**: everybody can trade everything with each other
  $\rightarrow$ all shocks are global shocks

$$\Delta s_{t+1} = m^{*}_{t+1} - m_{t+1}$$

  - *Cyclicality puzzle*: low or negative covariance of FX with relative economic conditions
  - *Volatility puzzle*: $var(\Delta s_{t+1}) \ll var(m^{*}_{t+1} - m_{t+1})$

Volatility and Cyclicality Puzzles

Assume that m and m^* are such that the complete markets setting leads to puzzles



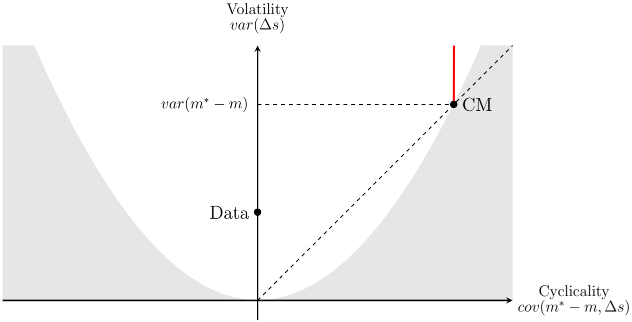
▸ Cauchy-Schwartz inequality

When Does Risk-Sharing Lead to the Puzzles?

Key feature: High $var_t(g_{t+1})$

- Case 1: $g_{t+1} = m^* - m_{t+1}$

1. **Who trades?** Imperfect integration or intermediation but rich enough asset space
2. **Spanned SDFs**: assets in *each* country span *both* SDFs



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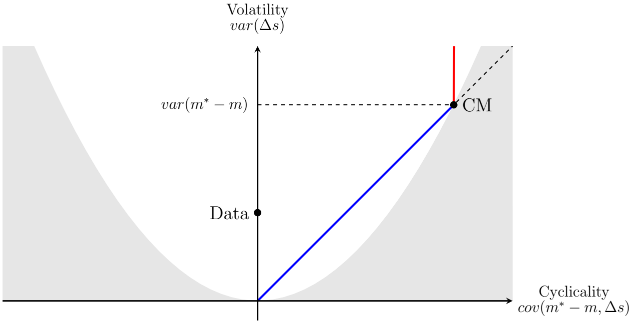
- Case 2: spanning the FX risk $g_{t+1} = \Delta s_{t+1}$

1. **What is traded?** Incomplete but integrated markets
2. **Spanned FX risk**: assets in *each* country span FX
   - When FX is traded directly or spanned by traded macro shocks



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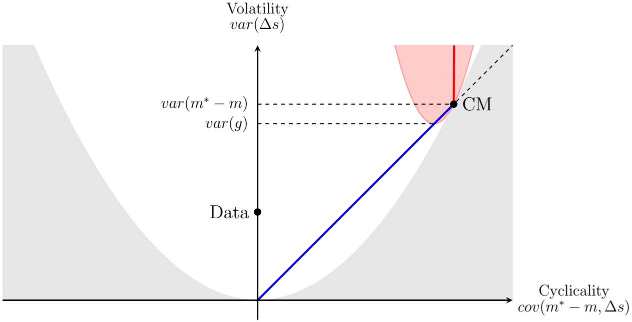
- Case 2: spanning the FX risk $g_{t+1} = \Delta s_{t+1}$
- Case 3: intermediation with high enough risk sharing

- Remove both integration and completeness: *intermediated and incomplete*
- If enough risks in common, $var_t(g_{t+1})$ is high and puzzles still occur by continuity



Do Financial Shocks Need Extreme Segmentation?

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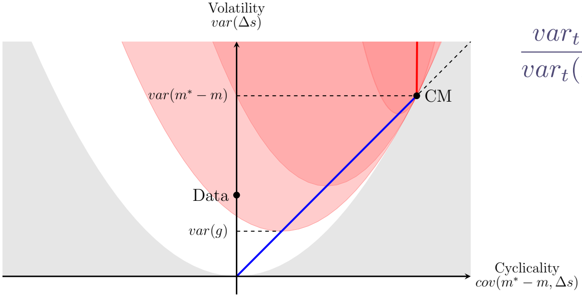
- *Workhorse models of financial shocks*: households only trade risk free assets, intermediary only bears currency risk $\rightarrow$ no global shocks
- **Relaxing segmentation:** add trading opportunities without creating global shocks
  - Intermediary can trade arbitrary sophisticated contracts: access all local markets, trade derivatives, ...
  - Households can each trade their local assets if their returns are not related

Combining Risk-Sharing and Financial Shocks

Intermediation with asset returns are related across countries

- As long as g_{t+1} is low enough, volatility cyclicality tradeoff is loose enough:



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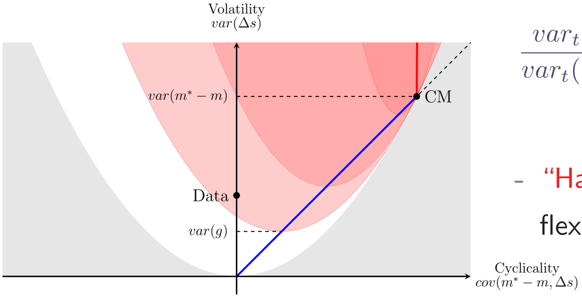

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- "Happy middle": some risk-sharing, but enough flexibility in financial shocks to avoid the puzzles

Outline

1. The General Risk-Sharing View
2. Exchange Rate Across Market Structures
3. Empirical Constraints

Empirical Questions

Converse problem when looking at data

1. Observe FX Δs and asset returns $\boldsymbol{r}$ and $\boldsymbol{r}^*$
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- No market-structure free test
- For some market structures, no need to look at the data to know restrictions:
  - Complete and integrated, then $m^* - m = \Delta s$
  - Extreme segmentation, then all relations with m^* and m are feasible

Empirical Questions

- Assume markets are intermediated and households in each country can trade their local assets
  - G10 countries, 1988-2022, monthly
  - *Equity indices* (MSCI): Large+Mid Cap, Value, Growth, 10 industries
  - *Sovereign bonds* (central banks): maturities 2 to 10 years

1. Do these returns span the exchange rate $\Leftrightarrow$ constraint on expected depreciation rate
2. Are there common shocks between the two sets of local returns and do they explain the exchange rate $\Leftrightarrow$ constraint on exchange rate shocks ($=$ global shocks)

Is The Exchange Rate Spanned?

Estimate and report R^2 for various subset of returns:

$$\Delta s_{t+1} = \alpha + \underbrace{\beta' \boldsymbol{r}_{t+1} + \beta^{*\prime} \boldsymbol{r}^*_{t+1}}_{\text{global + local component}} + \underbrace{u_{t+1}}_{\text{unspanned component}}$$

Dependent Variable	AU	CA	DE	JP	NO	NZ	SE	CH	UK
Bonds									
10Y	0.25	0.33	7.49	5.36	4.73	1.05	4.79	4.01	0.92
All Maturities	7.23	7.89	15.72	10.15	13.66	5.67	13.95	11.52	13.65
Stocks									
Mkt	21.67	26.56	6.96	4.44	11.24	16.56	16.20	12.34	12.71
Mkt + Value/Growth	21.60	27.98	6.75	5.06	12.47	17.16	15.91	12.71	13.68
Mkt + Value/Growth + Ind.	35.07	41.61	18.55	22.78	29.41	24.53	24.00	19.61	26.88
Bond + Equity	**36.74**	**45.05**	**26.79**	**29.13**	**36.64**	**27.95**	**30.62**	**25.28**	**33.80**
N	419	395	419	419	406	419	414	419	419

Is The Exchange Rate Spanned? No

Estimate and report R^2 for various subset of returns:

$$\Delta s_{t+1} = \alpha + \underbrace{\beta' \boldsymbol{r}_{t+1} + \beta^{*\prime} \boldsymbol{r}^*_{t+1}}_{\text{global + local component}} + \underbrace{u_{t+1}}_{\text{unspanned component}}$$

Dependent Variable	AU	CA	DE	JP	NO	NZ	SE	CH	UK
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Financial FX disconnect $\Rightarrow$ flexibility in $E_t \Delta s_{t+1}$

Do Global Shocks Explain the Exchange Rate?

Estimate fraction of variance due to global shocks

$$\Delta s_{t+1} = \alpha + \underbrace{\beta^{g\prime} \epsilon^{g}_{t+1}}_{\text{global component}} + \xi_{t+1}.$$

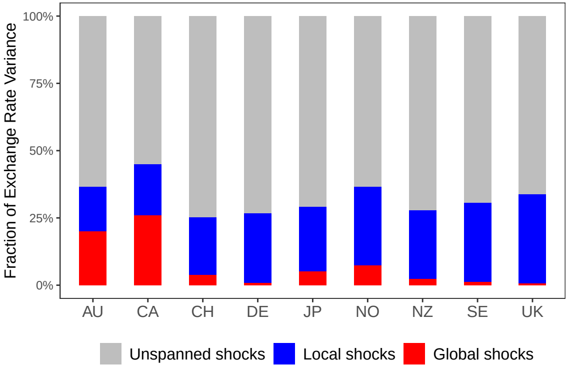
Decompose variance of exchange rate: global shocks, local shocks, unspanned shocks

Do Global Shocks Explain the Exchange Rate? Not Much

Estimate fraction of variance due to global shocks

$$\Delta s_{t+1} = \alpha + \underbrace{\beta^{g\prime} \epsilon^{g}_{t+1}}_{\text{global component}} + \xi_{t+1}.$$

Decompose variance of exchange rate: global shocks, local shocks, unspanned shocks



Takeaways

- **An empirically plausible structure**: intermediation + local trading of stocks and bonds

$$\Delta s_{t+1} = \underbrace{E_t \Delta s_{t+1}}_{\text{small}} \quad + \quad \underbrace{g_{t+1}}_{\approx 10\%} \quad + \quad \underbrace{\ell_{t+1}}_{\approx 30\%} \quad + \quad \underbrace{u_{t+1}}_{\approx 60\%}$$

  - Most of exchange rate variation is coming from risks that are not shared
  - A substantial role for risk-sharing
  - *Caveat*: not a proof or a test that this is the correct market structure

Conclusion

A general analysis of finance and the exchange rate:
macroeconomic risk sharing vs. financial transmission of shocks

- Two simple conditions fully map out risk-sharing restrictions on FX across market structures
  - Key concepts: globally traded shocks, and exchange rate spanning

1. The finance-FX puzzles can only be avoided by abandoning both complete markets and integration $\neq$ abandoning risk-sharing altogether
2. Extreme segmentation is not necessary for a large role of financial shocks
3. A "happy middle" market structure with both roles: households in local markets and sophisticated multi-market intermediaries

APPENDIX

Complete and integrated markets

- The classic relation $\Delta s_{t+1} = m^*_{t+1} - m_{t+1}$ can be written in terms of
  - innovations $\widetilde{\Delta s}_{t+1} = \widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}$
  - and means (under conditional log-normality)

$$
\begin{aligned}
E_t(\Delta s_{t+1}) &= E_t(m^*_{t+1}) - E_t(m_{t+1}) \\
&= \log E_t(M^*_{t+1}) - \log E_t(M_{t+1}) - \frac{1}{2} var_t(m^*_{t+1}) + \frac{1}{2} var_t(m_{t+1}) \\
&= r_{ft} - r^*_{ft} - \frac{1}{2} var_t(m_{t+1} + \Delta s_{t+1}) + \frac{1}{2} var_t(m_{t+1}) \\
&= r_{ft} - r^*_{ft} - cov_t(m_{t+1}, \Delta s_{t+1}) - \frac{1}{2} var_t(\Delta s_{t+1})
\end{aligned}
$$

▸ Back

Global shocks

- Notation: $\widetilde{x}_{t+1} \equiv x_{t+1} - E_t x_{t+1}$
- The set of global shocks is $\boldsymbol{\epsilon}^g_{t+1} = \{\epsilon^g_{t+1} | \exists \boldsymbol{\lambda} \in \mathbb{R}^N, \boldsymbol{\lambda}^* \in \mathbb{R}^{N^*} : \epsilon^g_{t+1} = \boldsymbol{\lambda}' \widetilde{\boldsymbol{r}}_{t+1} = \boldsymbol{\lambda}^{*\prime} \widetilde{\boldsymbol{r}}^*_{t+1}\}$
- Example 1: A mix of foreign and domestic assets
  - $\boldsymbol{r}_{t+1} = (r_{ft}, r_{1,t+1}, r_{2,t+1}, r^*_{ft} + \Delta s_{t+1}, r^*_{1,t+1} + \Delta s_{t+1})$
  - $\boldsymbol{r}^*_{t+1} = (r^*_{ft}, r^*_{1,t+1}, r^*_{2,t+1}, r_{ft} - \Delta s_{t+1}, r_{1,t+1} - \Delta s_{t+1})$
  - H investor can construct a portfolio with a return $r^*_{1,t+1} - r^*_{ft}$ by buying the foreign risky asset 1 and by selling the foreign risk-free asset, both converted into domestic currency
  - F investor can construct a portfolio with a return $r_{1,t+1} - r_{ft}$
  - Both $\widetilde{r}_{1,t+1}$ and $\widetilde{r}^*_{1,t+1}$ are in the set of $\boldsymbol{\epsilon}^g_{t+1}$
- Example 2: N risky assets in each country
  - $\widetilde{r}_{i,t+1} = \alpha_i \epsilon_{t+1} + \beta_i \epsilon_{i,t+1}$
  - $\widetilde{r}^*_{i,t+1} = \alpha^*_i \epsilon_{t+1}$
  - ϵ_{t+1} is global if at least one $\beta_i = 0$ or $N \to \infty$

▸ Back

Assets and Portfolios

Two technical assumptions:

- Vector of log returns:

$$\boldsymbol{r}_{t+1} = (r_{1,t+1}, \ldots, r_{N,t+1}) \sim MVN(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

- Campbell-Viceira (2002) approximation for log portfolio excess returns relative to a risk-free rate r_{ft}:

$$\begin{aligned} r_{p,t+1} - r_{ft} &= \log\left(\boldsymbol{w}_t' e^{\boldsymbol{r}_{t+1} - r_{ft}}\right) \\ &\approx \boldsymbol{w}_t'(\boldsymbol{r}_{t+1} - r_{ft}) + \frac{1}{2}\boldsymbol{w}_t' \operatorname{diag}(\boldsymbol{\Sigma}_t) - \frac{1}{2}\boldsymbol{w}_t'\boldsymbol{\Sigma}_t\boldsymbol{w}_t \end{aligned}$$

▸ Back

Proof of Proposition 1

1. **Quanto property**
   - Trading simple returns across borders induces exchange rate risk

$$\log(e^{r^*_{t+1}+\Delta s_{t+1}}) = r^*_{t+1} + \Delta s_{t+1}$$

   - Trading excess returns across borders only induces a quanto adjustment:

$$\log\left(e^{r_{ft}} + (e^{r^*_{t+1}} - e^{r^*_{ft}})e^{\Delta s_{t+1}}\right) \approx r_{ft} - r^*_{ft} + r^*_{t+1} + cov_t(r^*_{t+1}, \Delta s_{t+1})$$

Proof of Proposition 1

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2. **No international arbitrage**: consider $r_{t+1} \in H$ and $r^*_{t+1} \in F$ that each replicate a global shock, go long-short *in excess returns*:

$$\underbrace{r_{\text{diff},t+1}}_{\text{no risk so mean 0}} = \underbrace{(r_{t+1} - r_{ft})}_{cov(m,\epsilon^g)} - \underbrace{(r^*_{t+1} - r^*_{ft})}_{cov(m^*,\epsilon^g)} - \underbrace{cov_t(r^*_{t+1}, \Delta s_{t+1})}_{cov(\epsilon^g,\Delta s)}$$

▸ Back

The Quanto Property

- Conversion of excess return does not introduce FX risk ▸ Back
- Correlation of excess returns on U.S. industry portfolios in dollars and foreign currency

$$corr\left(e^{r_{t+1}} - e^{r_{f,t}}, \left(e^{r_{t+1}} - e^{r_{f,t}}\right) e^{\Delta s_{t+1}}\right)$$

	AU	CA	DE	JP	NO	NZ	SE	CH	UK
US Market	99.88	99.94	99.95	99.96	99.87	99.90	99.92	99.94	99.94
US Value	99.90	99.95	99.96	99.96	99.87	99.91	99.92	99.95	99.95
US Growth	99.87	99.93	99.94	99.96	99.88	99.90	99.92	99.94	99.94
US Oil, Gas, Coal	99.90	99.96	99.97	99.98	99.92	99.92	99.94	99.96	99.96
US Basic Material	99.81	99.90	99.92	99.95	99.85	99.88	99.90	99.93	99.93
US Consumer Discretionary	99.91	99.95	99.95	99.96	99.9	99.91	99.92	99.95	99.95
US Consumer Products, Services	99.93	99.97	99.97	99.97	99.92	99.93	99.94	99.96	99.96
US Industrials	99.86	99.93	99.94	99.96	99.84	99.90	99.90	99.94	99.94
US Health Care	99.90	99.96	99.95	99.96	99.88	99.93	99.93	99.95	99.96
US Financials	99.91	99.95	99.95	99.94	99.87	99.93	99.91	99.92	99.94
US TeleCom	99.87	99.93	99.95	99.95	99.9	99.91	99.93	99.96	99.95
US Technology	99.88	99.93	99.94	99.96	99.89	99.91	99.92	99.94	99.94
US Utilities	99.84	99.92	99.94	99.96	99.85	99.88	99.91	99.96	99.94

Non-linear Case

- **Innovations** If R^*_{t+1} is globally traded

$$cov_t\left(\frac{M^*_{t+1}}{E_t M^*_{t+1}} - \frac{M_{t+1}}{E_t M_{t+1}} - e^{\Delta s_{t+1}}, R^*_{t+1}\right) = \underbrace{cov_t^{\mathbb{Q}}(R^*_{t+1}, e^{\Delta s_{t+1}})}_{\text{risk-neutral quanto}} - \underbrace{cov_t^{\mathbb{P}}(R^*_{t+1}, e^{\Delta s_{t+1}})}_{\text{true quanto}}$$

- Right-hand side is close to 0
  - in limit towards diffusion processes
  - if quanto risk is small relative to FX risk

▸ Back

Cauchy-Schwartz inequality

- General restriction

$$var_t(\Delta s_{t+1}) \geq \frac{cov_t^2(m^*_{t+1} - m_{t+1}, \Delta s_{t+1})}{var_t(m^*_{t+1} - m_{t+1})}$$

▸ Back to puzzles

- The role of global shocks

$$var_t(\Delta s_{t+1}) \geq var(g_{t+1}) + \frac{\left(cov_t(m^*_{t+1} - m_{t+1}, \Delta s_{t+1}) - var(g_{t+1})\right)^2}{var_t(m^*_{t+1} - m_{t+1}) - var(g_{t+1})}$$

▸ Back to risk-sharing and shocks

Spanned SDFs

- Variance:

$$var_t(\Delta s_{t+1}) = var_t(m^*_{t+1} - m_{t+1}) + var_t(\ell_{t+1} + u_{t+1}) \geq var_t(m^*_{t+1} - m_{t+1})$$

- Cyclicality:

$$\begin{aligned} cov_t(m^*_{t+1} - m_{t+1}, \Delta s_{t+1}) &= cov_t(g_{t+1}, g_{t+1} + \ell_{t+1} + u_{t+1}) \\ &= var_t(g_{t+1}) = var_t(m^*_{t+1} - m_{t+1}) \end{aligned}$$

▸ Back

Spanned FX

- Variance:

$$var_t(\Delta s_{t+1}) = var_t(\text{proj}(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}|\boldsymbol{\epsilon}^g_{t+1})) \leq var_t(m^*_{t+1} - m_{t+1})$$

- Cyclicality:

$$cov_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}) = cov_t(\Delta s_{t+1}, \text{proj}(m^*_{t+1} - m_{t+1}|\boldsymbol{\epsilon}^g_{t+1})) = var_t(\Delta s_{t+1})$$

▸ Back