What do Financial Markets say about the Exchange Rate?

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Asset Market View of the Exchange Rate

- Under complete markets, home and foreign SDFs pin down the exchange rate:

$$\underbrace{\Delta s_{t+1}}_{\text{change in FX}} = \underbrace{m^*_{t+1}}_{\text{log foreign SDF}} - \underbrace{m_{t+1}}_{\text{log home SDF}}$$

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  - *Macro:* m_{t+1} representative Home household SDF, e.g. $m_{t+1} = -\gamma \Delta c_{t+1}$
  - *Finance:* representation of risk-return relation among assets, e.g. $m_{t+1} = \boldsymbol{\lambda}_t' \boldsymbol{r}_{t+1}$

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- For **every economy** that satisfies no arbitrage — **General AMV of FX**

Components of the Exchange Rate

- FX depreciation rate:

$$\Delta s_{t+1} = \underbrace{E_t \Delta s_{t+1}}_{\text{expected depriciation}} + \underbrace{\widetilde{\Delta s}_{t+1}}_{\text{surprise depriciation}}$$

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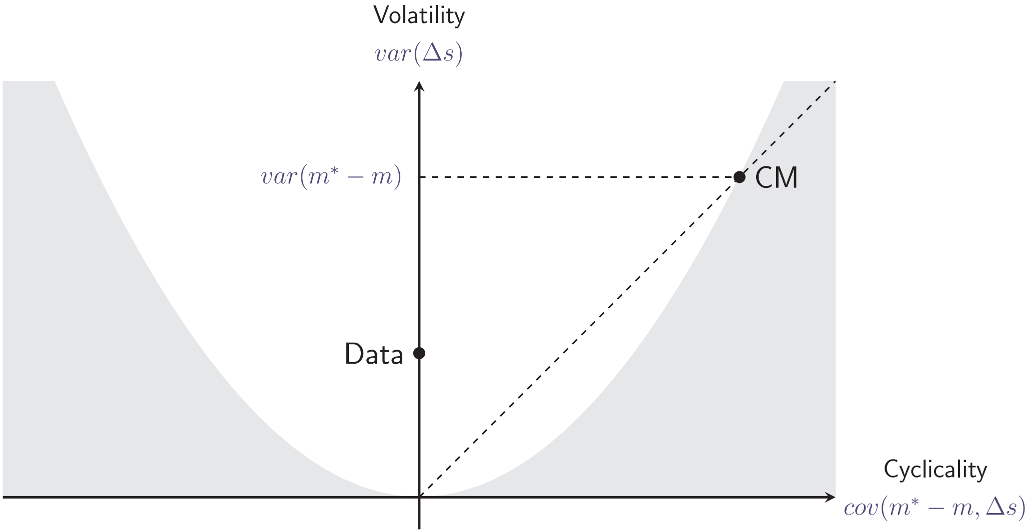
- FX decomposition:

$$\Delta s_{t+1} = \underbrace{E_t \Delta s_{t+1}}_{\approx 2\%} + \overbrace{\underbrace{v^G_{t+1}}_{\approx 20\%} + \underbrace{v^L_{t+1}}_{\approx 20\%} + \underbrace{u_{t+1}}_{\approx 60\%}}^{\widetilde{\Delta s}_{t+1}}$$

Graphical Representation



Market structure and Shock structure

1. **Market structure**:
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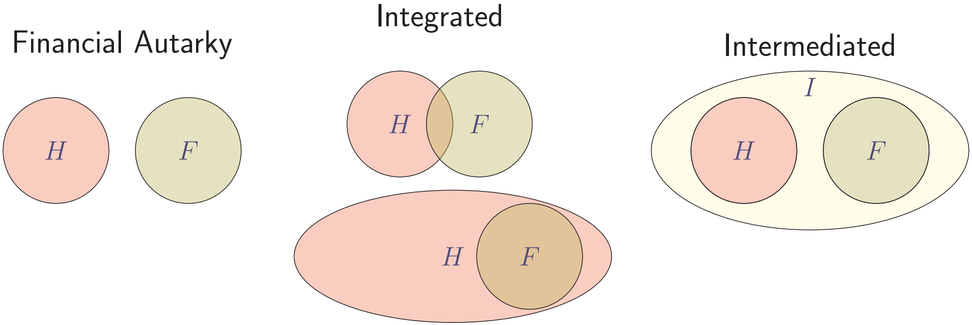
- E.g., complete markets: all agents trade all risks (states) with each other.
  Departures along two dimensions:
  1. not all risks (states) are traded
     — *cf.* risks vs states of the world
  2. not all agents can trade assets
     — (partial) integration vs intermediation

Market structure



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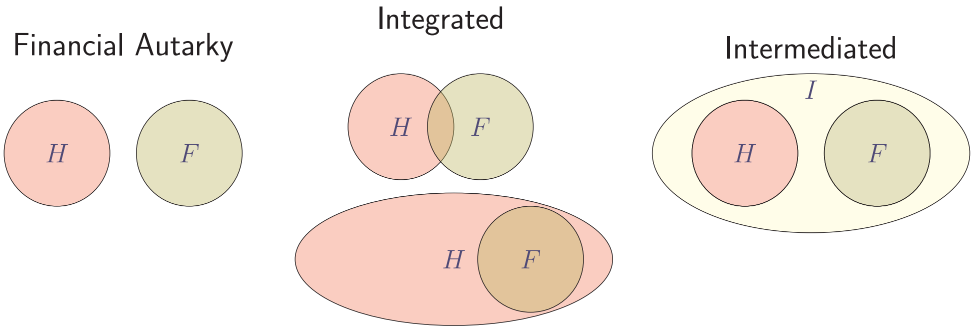

- H set of assets with returns $\{r_{t+1}\}$ in home currency priced by m_{t+1}, includes r_{ft}
- F set of assets with returns $\{r^*_{t+1}\}$ in foreign currency priced by m^*_{t+1}, includes r^*_{ft}
- I combined set of assets $\{r_{t+1}, r^*_{t+1} + \Delta s_{t+1}\}$ that must satisfy no arbitrage:

$$\forall r_{p,t+1} \in I \ : \quad var_t(r_{p,t+1}) = 0 \quad \Rightarrow \quad E_t(r_{p,t+1}) = r_{f,t}$$

Building FX from finance I: FX risk, $\widetilde{\Delta s_{t+1}}$

- **Step 1**: Given SDFs (m_{t+1}, m^*_{t+1}) and traded returns $\{r_{t+1}\}$, $\{r^*_{t+1}\}$, construct:

$$v^G_{t+1} \;=\; \text{projection of } \; \tilde{m}^*_{t+1} - \tilde{m}_{t+1} \; \text{ on } \; \{\epsilon^G_{t+1}\} \equiv \{\tilde{r}_{t+1}\} \cap \{\tilde{r}^*_{t+1}\}$$

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  which requires that the exchange rate surprise satisfies:

$$\widetilde{\Delta s}_{t+1} = v^G_{t+1} + \eta_{t+1} \qquad \text{such that} \qquad \eta_{t+1} \perp \{\epsilon^G_{t+1}\}$$

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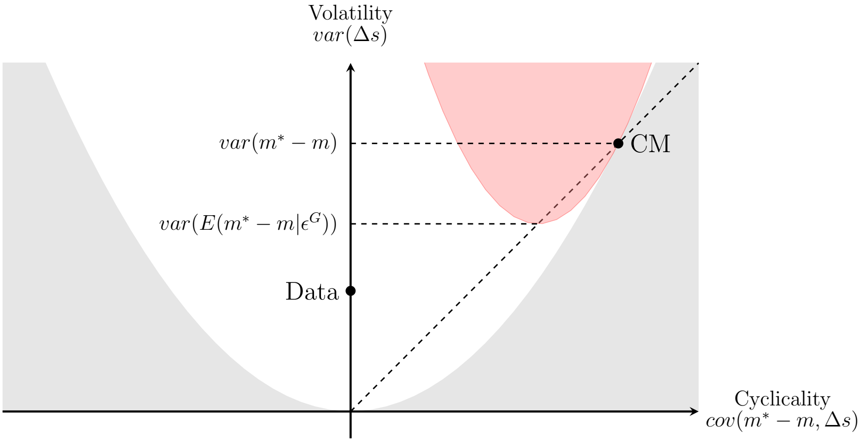
  3. Implications for FX risk: $\mathrm{var}_t(\Delta s_{t+1}) \geq \mathrm{var}(v^G_{t+1}) = var(\tilde{m}^*_{t+1} - m_{t+1} | \{\epsilon^G_{t+1}\})$

Illustration: FX volatility and cyclicality



- Proposition 1 implies a joint constraint on FX volatility and cyclicality:

$$\text{var}_t(\Delta s_{t+1}) \geq \text{var}(v^G_{t+1}) + \frac{\left(\text{cov}_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}) - \text{var}(v^G_{t+1})\right)^2}{\text{var}_t(m^*_{t+1} - m_{t+1}) - \text{var}(v^G_{t+1})}$$

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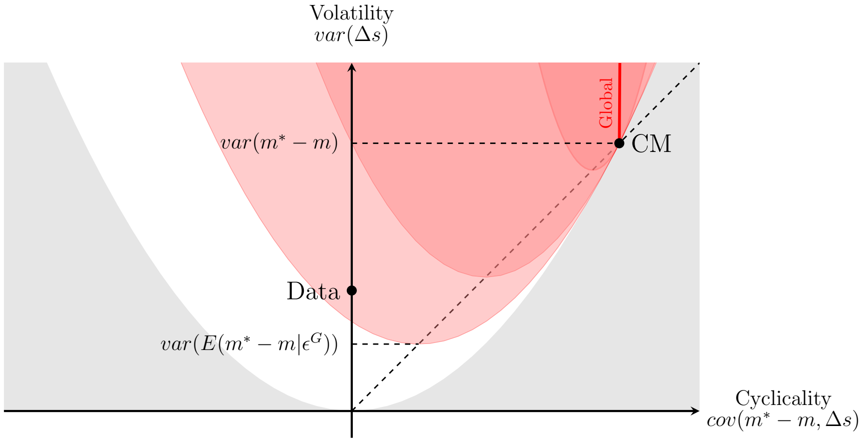

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- Step 1 (Proposition 1) requires that:

$$proj\left[\widetilde{\Delta s}_{t+1}|\{\epsilon^G_{t+1}\}\right] \;=\; proj\left[\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}|\{\epsilon^G_{t+1}\}\right]$$

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- Example: Arrow security for state $h^{t+1} = (h_{t+1}, h^t)$, traded directly or via intermediary:

$$\underbrace{m^*_{t+1}(h^{t+1})}_{\text{£ price of } h_{t+1}} \; - \; \underbrace{m_{t+1}(h^{t+1})}_{\text{\$ price of } h_{t+1}} \; = \; \underbrace{\Delta s_{t+1}(h^{t+1})}_{\text{\$ depreciation in } h_{t+1}}$$

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- Proposition 1 generalizes this to any globally traded risk $\epsilon^G_{t+1} \in \{\tilde{r}_{t+1}\} \cap \{\tilde{r}^*_{t+1}\}$

  — buy ϵ^G_{t+1} in \$ and sell ϵ^G_{t+1} in £ prices $\widetilde{\Delta s}_{t+1} | \epsilon^G_{t+1}$ by no arbitrage (zero risk portfolio)

Large Share of Global Shocks I: spanned risk

- Spanned risk: $\tilde{m}_{t+1}, \tilde{m}^*_{t+1} \in \{\epsilon^G_{t+1}\} = \{\tilde{r}_{t+1}\} \cap \{\tilde{r}^*_{t+1}\}$,
- Therefore:

$$proj\left[\widetilde{\Delta} s_{t+1} | \{\epsilon^G_{t+1}\}\right] = \tilde{m}^*_{t+1} - \tilde{m}_{t+1} = v^G_{t+1}$$

- e.g., arises in models with small number of macro risks that are traded
  - RBC models, including rare disaster and LR risk models
- Implication: exacerbates complete market problems:

$$\mathrm{var}_t(m^*_{t+1} - m_{t+1}) = \mathrm{cov}_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}) \leq \mathrm{var}_t(\Delta s_{t+1})$$

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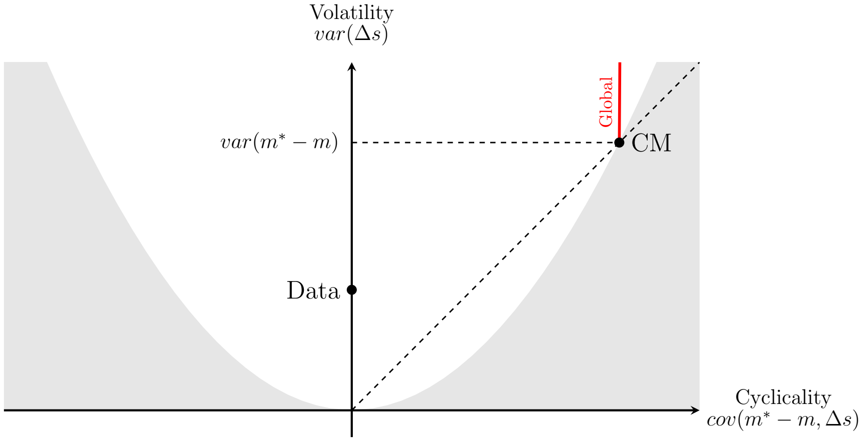

Large Share of Global Shocks II: spanned FX risk

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$$\widetilde{\Delta s}_{t+1} = v^G_{t+1} = proj\left[\tilde{m}^*_{t+1} - \tilde{m}_{t+1} | \{\epsilon^G_{t+1}\}\right] \quad \text{and} \quad \eta_{t+1} = v^L_{t+1} + u_{t+1} = 0$$

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- Implications:
  - partially ameliorates complete market problems:

$$\text{var}_t(\Delta s_{t+1}) = \text{cov}_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1}) \leq \text{var}_t(m^*_{t+1} - m_{t+1})$$

  - as we see next, necessarily leads to the risk premium puzzle

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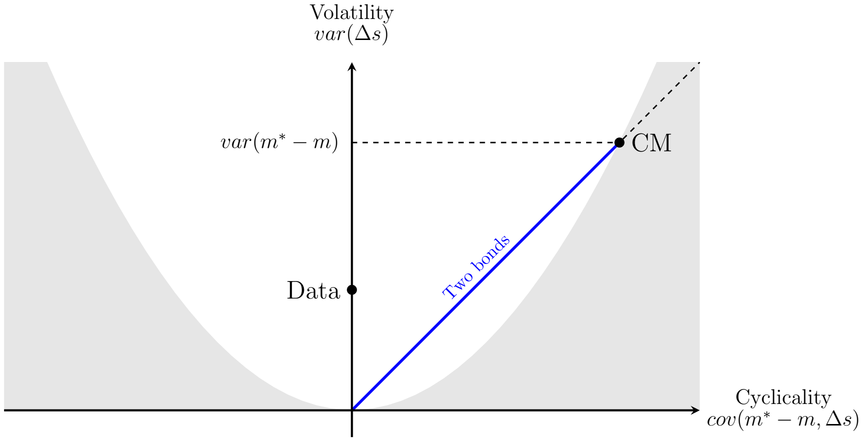

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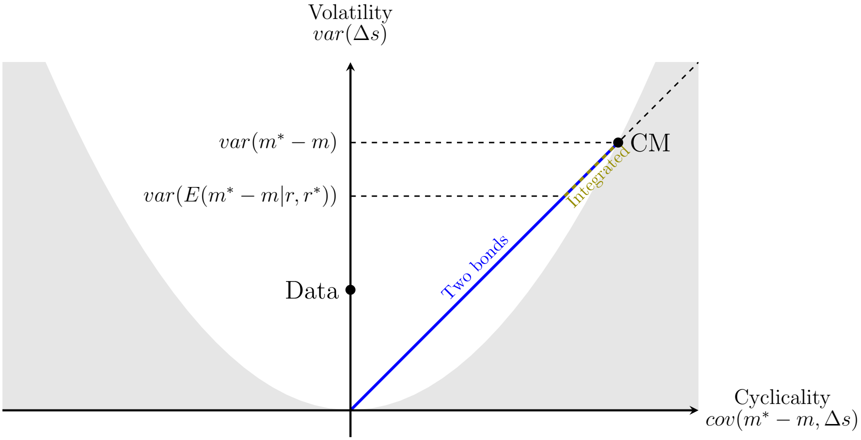

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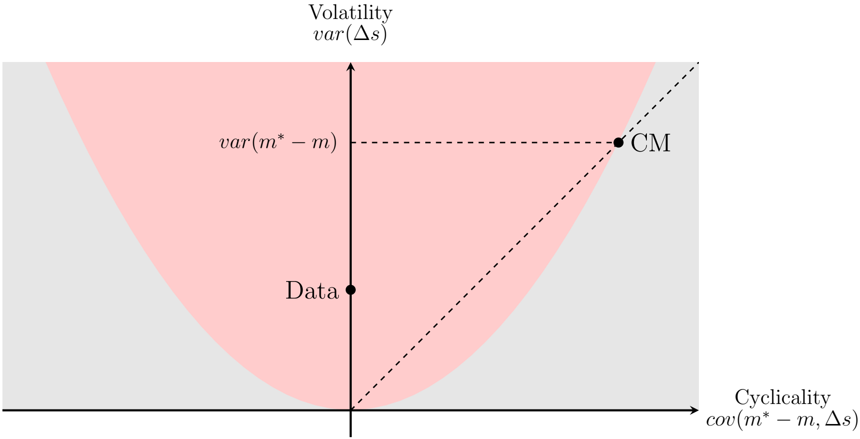
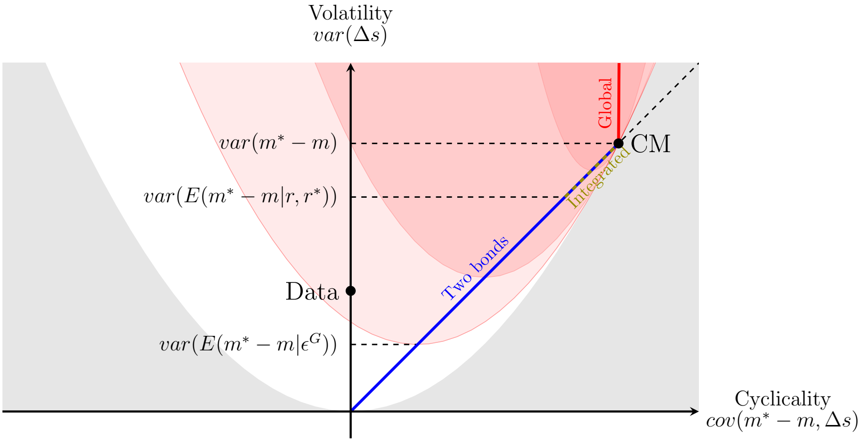


Summary: alternative market and shock structures



- Small share of global shocks v_{t+1}^G requires **both**:
  1. Sparce markets: lack of integration or non-traded FX risk (intermediation or one bond)
  2. Sparce shocks: limited cross-border risk spanning (unspanned risks)

Building FX from ground up II: FX return $E_t \Delta s_{t+1}$

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1. if $R^2 = 1$, then $\widetilde{\Delta s}_{t+1} = v_{t+1}$ is spanned and $u_{t+1} = 0$; note that $v_{t+1} = v^G_{t+1} + v^L_{t+1}$
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     - $(1 - \omega) u_{t+1} = -\sum_{j=0}^{\infty} (E_{t+1} - E_t)[\psi_{t+j+1}]$, where ω is LR persistence of u_{t+1}

A Look at the Data

- Econometrician's interpretation of H and F
  - Use data on asset returns in their origin currency $\{r_{t+1}\}$ and $\{r^*_{t+1}\}$
  - How much can we learn about FX from the risk-return relation of financial assets?

- Data:
  - G10 countries, from 1988 to 2022, monthly
  - Exchange rates (Bloomberg)
  - Equity indices (MSCI): Large+Mid Cap, Value, Growth, 10 industries
  - Sovereign bonds (central banks): maturities 2 to 10 years

Is The Exchange Rate Spanned?

Estimate and report R^2 for various subset of returns:

$$\Delta s_{t+1} = \alpha + \underbrace{\beta' \boldsymbol{r}_{t+1} + \beta^{*\prime} \boldsymbol{r}^*_{t+1}}_{\text{global + local component}} + \underbrace{u_{t+1}}_{\text{unspanned component}}$$

Dependent Variable	AU	CA	DE	JP	NO	NZ	SE	CH	UK
Bonds									
10Y	0.25	0.33	7.49	5.36	4.73	1.05	4.79	4.01	0.92
All Maturities	7.23	7.89	15.72	10.15	13.66	5.67	13.95	11.52	13.65
Stocks									
Mkt	21.67	26.56	6.96	4.44	11.24	16.56	16.20	12.34	12.71
Mkt + Value/Growth	21.60	27.98	6.75	5.06	12.47	17.16	15.91	12.71	13.68
Mkt + Value/Growth + Ind.	35.07	41.61	18.55	22.78	29.41	24.53	24.00	19.61	26.88
Bond + Equity	36.74	45.05	26.79	29.13	36.64	27.95	30.62	25.28	33.80
N	419	395	419	419	406	419	414	419	419

Is The Exchange Rate Spanned? No

Estimate and report R^2 for various subset of returns:

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Financial FX disconnect $\Rightarrow E_t \Delta s_{t+1}$ not constrained

Identifying Global Shocks

- **Undirected approach**: Canonical correlation analysis
  - Look for portfolios of domestic and foreign assets that are maximally correlated
  - Strict global shocks: correlation of 1
  - Not much relation between assets: maximum correlation between 64% and 90%
  - Generous approach: include all pairs with correlation above 60%

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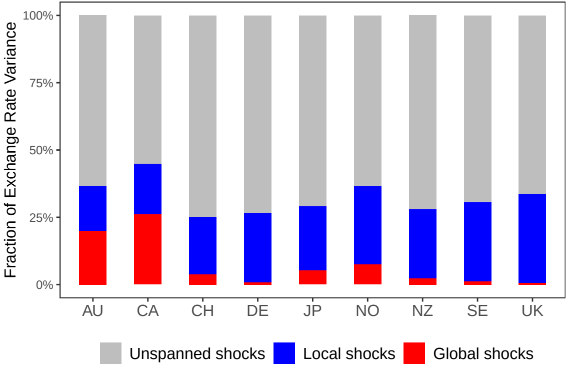
  - Generous approach: include all pairs with correlation above 60%
- **Directed approach**: Use variables known to relate to global cycles
  - VIX, Global Financial Cycle (Miranda-Aggripino and Rey), Excess Bond Premium (Gilchrist and Zakrajsek)
  - Implicit strong assumption: could find assets to replicate them in each country

Do Global Shocks Explain the Exchange Rate?

Estimate fraction of variance due to global shocks

$$\Delta s_{t+1} = \alpha + \underbrace{\beta^{G\prime} \epsilon^G_{t+1}}_{\text{global component}} + \xi_{t+1}.$$

Decompose variance of exchange rate: global shocks, local shocks, unspanned shocks



Do Global Shocks Explain the Exchange Rate? Not Much

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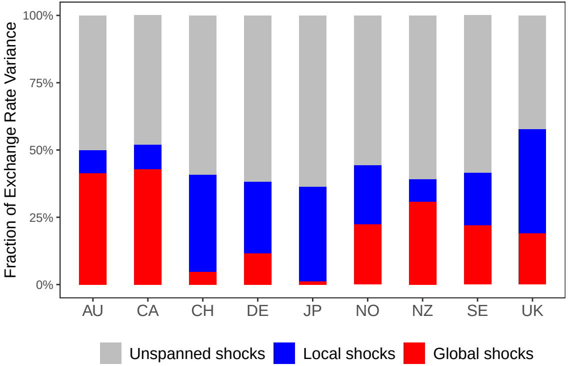

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  - FX risk becomes tightly constrained and close to complete markets if:
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    - Enough internationally traded assets to span SDFs

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  - FX expectation (risk premium) close to complete markets
  - FX risk becomes tightly constrained and close to complete markets if:
    - Both representative households can trade the exchange rate, or
    - Enough internationally traded assets to span SDFs
- **Intermediated markets**
  - Constraints depend on empirical properties of returns: exactly what we measured in data
  - FX expectation: spanning of exchange rate $\rightarrow$ No
  - FX shocks: presence of common shocks $\rightarrow$ Not much
    - Difficult to find global shocks, explain even less of FX variation

      $\Rightarrow$ FX variation not constrained by domestic and foreign pricing kernels

Recipe for a Realistic Model of Exchange Rates

- **Intermediated markets**
- **Small global shocks** ($\sim$10–20%)
  - constrained by the presence of the many other assets
  - must be positively related to relative household SDF $m^* - m$
- **Some local shocks** ($\sim$10–30%)
  - negatively related to relative SDF $m^* - m$ to offset global shocks and solve cyclicality puzzle
- **Mostly unspanned shocks** ($\sim$50–80%)
  - respect the financial FX disconnect and macro FX disconnect
  - allow FX risk premium dynamics (within Sharpe ratio bounds)
  - e.g., demand shocks in specialized FX markets

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  - not just in our world, but what could be in any alternative world (with no arbitrage)

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Conclusion

- A general characterization of implications of financial markets for the exchange rate
  - not just in our world, but what could be in any alternative world (with no arbitrage)
- What do financial markets say about the exchange rate (FX)?
  - Not much: properties of $\{r_{t+1}\}$ and $\{r^*_{t+1}\}$ do not pin down the FX Δs_{t+1}
- Upside:
  - Intermediated market structure particularly well-suited to fit the data
  - Need more information than asset prices to understand the exchange rate:
    - who is trading? who is holding FX risk?
    - demand shocks in specialized FX markets

APPENDIX

Related Literature

- *Euler equations as diagnostics:* Hansen and Jagannathan; Alvarez and Jermann
- *FX puzzles:* Backus and Smith; Brandt, Cochrane and Santa-Clara; Fama
- *Equlibrium explanations of FX puzzles:* Verdelhan; Colacito and Croce; Farhi and Gabaix
- *Departures from complete markets:* Lustig and Verdelhan
- *FX bond disconnect:* Chernov and Creal
- *Intermediated markets and FX:* Gabaix and Maggiori; Itskhoki and Mukhin; Jiang, Krishnamurthy and Lustig; Gourinchas, Ray and Vayanos

Assets

- Two countries: Home and Foreign ($*$)
  - H is the set of portfolios of home assets $\boldsymbol{r}_{t+1}$; risk-free $r_{ft} \in H$
  - F is the set of portfolios of foreign assets $\boldsymbol{r}^*_{t+1}$ in foreign currency; $r^*_{ft} \in F$
  - Returns expressed in each country's currency
  - Log-normal returns and use log-linear portfolio algebra (Campbell Viceira 2002)

- Examples
  - *Autarky:* H has domestic stocks and bonds, F has foreign stocks and bonds
  - *Integrated markets:* H and F have the same assets (converted in local currency)
  - *Complete markets:* all Arrow-Debreu claims

Stochastic Discount Factors

- **Assumption 1:** Domestic log SDF m_{t+1} prices all assets in H, foreign log SDF m^*_{t+1} prices all assets in F:

$$\forall r_{t+1} \in H: \quad E_t \exp(m_{t+1} + r_{t+1}) = 1$$

$$\forall r^*_{t+1} \in F: \quad E_t \exp(m^*_{t+1} + r^*_{t+1}) = 1$$

- Interpretations
  1. *Macro:* m_{t+1} representative Home household SDF, e.g. $m_{t+1} = -\gamma \log(C_{t+1}/C_t)$
  2. *Finance:* representation of risk-return relation among assets, e.g. $m_{t+1} = \boldsymbol{\lambda}_t' \boldsymbol{r}_{t+1}$

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$$\boldsymbol{r}^I_{t+1} = (\boldsymbol{r}_{t+1}, \boldsymbol{r}^*_{t+1} + \Delta s_{t+1})$$

- **Assumption 2** No arbitrage opportunities in the set of international portfolios I:

$$\forall r_{p,t+1} \in I,\ var_t(r_{p,t+1}) = 0 \quad \Rightarrow \quad E_t(r_{p,t+1}) = r_{f,t}.$$

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  - Integrated markets (e.g., $I = H$): m_{t+1} ensures no arbitrage
  - Intermediated markets ($I \supset H$): some m^I_{t+1} trades assets in I and ensures no arbitrage

Global and Local Shocks

General exchange rate decomposition:

$$\Delta s_{t+1} = E_t \Delta s_{t+1} + v^G_{t+1} + v^L_{t+1} + u_{t+1}$$

1. Expected depreciation $\delta_t = E_t \Delta s_{t+1}$
2. Depreciation shocks $\widetilde{\Delta s}_{t+1} = \Delta s_{t+1} - E_t \Delta s_{t+1}$
   - **Traded shocks**: spanned by $\{\boldsymbol{r}_{t+1}\}$ and $\{\boldsymbol{r}^*_{t+1}\}$
     - **Globally traded**: $v^G_{t+1} \in \{\epsilon^G_{t+1}\}$ can be spanned *separately* in H and F
     - **Locally traded**: v^L_{t+1} can be spanned by one asset set but not the other
   - **Unspanned shocks**: $u_{t+1} \perp \{\boldsymbol{r}_{t+1}, \boldsymbol{r}^*_{t+1}\}$

Exchange Rate Shocks, $\widetilde{\Delta s}_{t+1} = \Delta s_{t+1} - E_t \Delta s_{t+1}$

- Depreciation rate coincides with difference of SDF projected on globally traded shocks

- **Proposition 1** For $\{\boldsymbol{\epsilon}^G_{t+1}\} = \{\boldsymbol{r}_{t+1}\} \cap \{\boldsymbol{r}^*_{t+1}\}$:

$$E(\widetilde{\Delta s}_{t+1}|\boldsymbol{\epsilon}^G_{t+1}) = E(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1}|\boldsymbol{\epsilon}^G_{t+1}) = v^G_{t+1},$$

  that is $\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1} = v^G_{t+1} + \epsilon_{t+1}$ and $\widetilde{\Delta s}_{t+1} = v^G_{t+1} + \eta_{t+1}$.

  - No restriction for exposure to local shocks or to unspanned shocks, $\eta_{t+1} = v^L_{t+1} + u_{t+1}$
  - Restriction on conditional volatility and conditional Backus-Smith correlation
  - Generalization: $|cov(\widetilde{m}^*_{t+1} - \widetilde{m}_{t+1} - \widetilde{\Delta s}_{t+1}, r_{t+1})| \leq B\sqrt{1 - corr_t(r_{t+1}, r^*_{t+1})}$

Assets and Portfolios

Two technical assumptions:

- Vector of log returns:

$$\boldsymbol{r}_{t+1} = (r_{1,t+1}, \ldots, r_{N,t+1}) \sim MVN(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

- Campbell-Viceira (2002) approximation for log portfolio excess returns relative to a risk-free rate r_{ft}:

$$\begin{aligned} r_{p,t+1} - r_{ft} &= \log\left(\boldsymbol{w}_t' e^{\boldsymbol{r}_{t+1} - r_{ft}}\right) \\ &\approx \boldsymbol{w}_t'(\boldsymbol{r}_{t+1} - r_{ft}) + \frac{1}{2}\boldsymbol{w}_t' \operatorname{diag}(\boldsymbol{\Sigma}_t) - \frac{1}{2}\boldsymbol{w}_t'\boldsymbol{\Sigma}_t\boldsymbol{w}_t \end{aligned}$$

Back

Global and Local Shocks

- Decomposition in globally traded shock $\boldsymbol{\epsilon}_{t+1}^{G}$ and local shocks $(\boldsymbol{\epsilon}_{t+1}, \boldsymbol{\epsilon}_{t+1}^{*})$:

$$\tilde{\boldsymbol{r}}_{t+1} = \boldsymbol{P}\boldsymbol{\epsilon}_{t+1} + \boldsymbol{P}^{G}\boldsymbol{\epsilon}_{t+1}^{G},$$

$$\tilde{\boldsymbol{r}}_{t+1}^{*} = \boldsymbol{P}^{*}\boldsymbol{\epsilon}_{t+1}^{*} + \boldsymbol{P}^{G*}\boldsymbol{\epsilon}_{t+1}^{G}$$

1. Globally-traded shocks can be replicated in each country:

$$\boldsymbol{\epsilon}_{t+1}^{G} = \boldsymbol{A}^{G}\tilde{\boldsymbol{r}}_{t+1} = \boldsymbol{A}^{G*}\tilde{\boldsymbol{r}}_{t+1}^{*}$$

2. Local shocks:

$$\boldsymbol{\epsilon}_{t+1} = \boldsymbol{A}\tilde{\boldsymbol{r}}_{t+1}, \quad \boldsymbol{\epsilon}_{t+1}^{*} = \boldsymbol{A}^{*}\tilde{\boldsymbol{r}}_{t+1}^{*} \quad : \quad (\boldsymbol{\epsilon}_{t+1}, \boldsymbol{\epsilon}_{t+1}^{*}) \perp \boldsymbol{\epsilon}_{t+1}^{G}$$

▸ Back

Proof of Proposition 1

1. **Quanto property**
   - Trading simple returns across borders induces exchange rate risk

$$\log(e^{r^*_{t+1}+\Delta s_{t+1}}) = r^*_{t+1} + \Delta s_{t+1}$$

   - Trading excess returns across borders only induces a quanto adjustment:

$$\log\left(e^{r_f} + (e^{r^*_{t+1}} - e^{r^*_{f,t}})e^{\Delta s_{t+1}}\right) \approx r_f - r^*_f + r^*_{t+1} + cov_t(r^*_{t+1}, \Delta s_{t+1})$$

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2. **No international arbitrage**: consider $r_{t+1} \in H$ and $r^*_{t+1} \in F$ that each replicate a global shock, go long-short *in excess returns*:

$$\underbrace{r_{\text{diff},t+1}}_{\text{no risk so mean 0}} = \underbrace{(r_{t+1} - r_{ft})}_{cov(m,\epsilon^G)} - \underbrace{(r^*_{t+1} - r^*_{ft})}_{cov(m^*,\epsilon^G)} - \underbrace{cov_t(r^*_{t+1}, \Delta s_{t+1})}_{cov(\epsilon^G,\Delta s)}$$

Back

The Quanto Property

- Conversion of excess return does not introduce FX risk ▸ Back
- Correlation of excess returns on U.S. industry portfolios in dollars and foreign currency

$$corr\left(e^{r_{t+1}} - e^{r_{f,t}}, \left(e^{r_{t+1}} - e^{r_{f,t}}\right) e^{\Delta s_{t+1}}\right)$$

	AU	CA	DE	JP	NO	NZ	SE	CH	UK
US Market	99.88	99.94	99.95	99.96	99.87	99.90	99.92	99.94	99.94
US Value	99.90	99.95	99.96	99.96	99.87	99.91	99.92	99.95	99.95
US Growth	99.87	99.93	99.94	99.96	99.88	99.90	99.92	99.94	99.94
US Oil, Gas, Coal	99.90	99.96	99.97	99.98	99.92	99.92	99.94	99.96	99.96
US Basic Material	99.81	99.90	99.92	99.95	99.85	99.88	99.90	99.93	99.93
US Consumer Discretionary	99.91	99.95	99.95	99.96	99.9	99.91	99.92	99.95	99.95
US Consumer Products, Services	99.93	99.97	99.97	99.97	99.92	99.93	99.94	99.96	99.96
US Industrials	99.86	99.93	99.94	99.96	99.84	99.90	99.90	99.94	99.94
US Health Care	99.90	99.96	99.95	99.96	99.88	99.93	99.93	99.95	99.96
US Financials	99.91	99.95	99.95	99.94	99.87	99.93	99.91	99.92	99.94
US TeleCom	99.87	99.93	99.95	99.95	99.9	99.91	99.93	99.96	99.95
US Technology	99.88	99.93	99.94	99.96	99.89	99.91	99.92	99.94	99.94
US Utilities	99.84	99.92	99.94	99.96	99.85	99.88	99.91	99.96	99.94